FOREST MEASUREMENT

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BY

HAROLD C. BELYEA
Assistant Professor of Forest Engineering
New York State College of Forestry, Syracuse University

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PREFACE

For some years the writer has felt that there is both need and demand for a textbook which would set forth the fundamentals of forest mensuration in a form sufficiently complete to be adaptable for assimilation by students of undergraduate rank approaching a consideration of this important subject for the first time. It must be confessed that the task of writing such a book has not been found easy. The subject is so complicated, its developments so devious, its applications so practical, and its progress of recent years so rapid and far-reaching, that the task of presentation in simple form for the consumption of elementary students, the decision as to topics, and the degree of detail with which each should be treated, have all presented problems of considerable perplexity.

That this book has solved each and all with absolute infallibility is the least of its claims. In view of the fact that the texts available in the subject are either highly technical and exhaustive in treatment, or else of absolute practicality in their synoptic method of presentation, coupled with the fact that all are quite out of date with the more recent developments, particularly in the fields of volume and yield computation, it has seemed to the writer that this contribution, which endeavors to hold to the middle ground, might be of definite assistance to teachers, students and practicing foresters. To clear up the subject for easier digestion, to bring its methods up to date, to make it as brief and concise as possible without sacrifice of important detail, have been its major objectives. To this end it has seemed best to stick closely to the main theme and leave to others the further development of its ramifications and the advocacy of modifications. Inasmuch as the U. S. Forest Service absorbs so many of the technically trained men from the forest schools, the procedure generally acceptable in the Service has in a great many cases established the criterion.

Some criticism, perhaps more or less justifiable, may be offered at the inclusion of a chapter on elementary statistics in a book of this type. It should be understood that there has never been the slightest attempt nor intention to cover the whole subject of statistics within the limits of a single chapter. Furthermore, the discussion has purposely been centered around the development, application and further treatment of but a single phase — the standard deviation, a most important one in gaging the reliability of any series of biometric data. Considering the more recent advances and the present status of the general work in the field of forest measurement, it is difficult to see how some considerations of this subject could be overlooked or omitted. It is the belief of the writer that the time is rapidly approaching when it will be necessary for all forestry students of collegiate grade to take a complete course in statistical methods. This, at the present, is a requisite in at least one forest school in the country and could well be imitated by others.

Every writer owes much to those investigators and authors who have pioneered before him. This holds particularly true in this as in all fields of forestry. The Bibliography which is appended by no means covers the subject. It does list, however, those sources which have most influenced the compilation of this book. The works to which the author owes the most, and which have been most freely consulted and drawn upon, are indicated by an asterisk.

Acknowledgments are due to the late Dean Franklin Moon and to Prof. Nelson C. Brown of the New York State College of Forestry for encouragement and advice, to Messrs. T. S. Woolsey, Jr., of New Haven, Conn., E. F. McCarthy, Director of the Central States Forest Experiment Station, U. S. Forest Service, of Columbus, Ohio, and to Prof. R. R. Fenska of the New York State College of Forestry, for reviewing and criticizing earlier drafts of the manuscript. Particular appreciation is held for the exhaustive review and keen criticism of the final draft by Prof. Thorvald S. Hanson of the Department of Forestry, University of Minnesota, and Director of the Forest Experiment Station at Cloquet, Minn.; and by Major W. G. Wright, Forester for Price Brothers and Company, Limited, of Quebec, Canada.

The author also desires to express his appreciation to Mr. Clyde Webb, Logging Engineer, U. S. Forest Service, Missoula, Mont., and to Mr. U. S. Swartz, Logging Engineer, U. S. Forest Service, Ogden, Utah, for their kindly review and criticism of Chapter V, and for the valuable suggestions which they offered for improvement in text and form.

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FOREST MEASUREMENT

CHAPTER I

INTRODUCTION TO FOREST MEASUREMENT

1. Definition and Scope. — The branch of forestry which considers the measurement of forest products is usually termed *forest mensuration.* It is defined as the science which deals with the determination of the volume of stands, trees, and logs; and with the study of growth and yield of trees and stands.* It thus concerns itself not only with the probabilities of present exploitation but also with the possibilities of future utilization.

Forest measurement lies at the basis of all practical work in forestry. It enters every transaction that involves the exchange, purchase, sale, and appraisal of forests and forest products. By it the volume of timber standing in a forest is determined before cutting, and it is again the basis of measurement after the trees are felled. A most important phase is the study of growth which seeks to estimate in terms of some given unit the yields of wood material derivable from stands of different species at various periods of time.

2. Timber and Lumber. — Standing trees ready for harvesting, that is, ready to be cut and converted into marketable products, are known as **Timber** or, more properly, **Merchantable Timber.** Often timber is spoken of as **Stumpage,** that is, merchantable timber *standing* on the stump. The term " merchantable timber " is a descriptive term implying such size, distribution, character, amount, and location, as to render the processes of converting it into manufactured products economically feasible. " Unmerchantable timber " is to be understood as stumpage too young, too small, too scattered, of inferior species or too remote from centers of manufacture and distribution to secure from its removal a profitable return. The measurement of standing timber for the purpose of determining its volume is known as **Timber Estimating.**

When trees are felled and cut into sections of size convenient

* This definition is one which has been adopted by the Society of American Foresters. Journal of Forestry, Vol. XV, No. 1, Jan. 1917, p. 78.

for handling in the various steps of the logging and milling operations, the portions of the main stem which are accepted as usable are known as **Logs.*** The minimum dimensions of the sections of trees which may be so accepted vary mainly with production costs as influenced by economic conditions prevailing in the industry in that region at the time, and the maximum dimensions will be fixed by the physical capacities of the manufacturing machinery.

The measurement of logs for the purpose of determining their volume is known as **Scaling.**

The process of enumerating the individual items or the dimensions of individual items which are measured for the purpose of determining their volume is known as **Tallying.** Tallying is

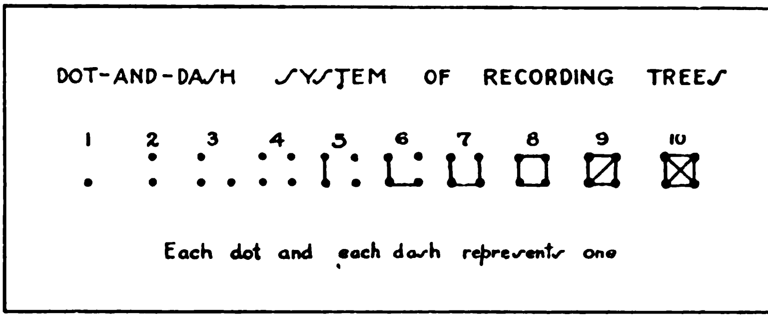


FIG. 1. — "Dot and Dash" System of Tallying.

always carried out according to a prearranged plan, with standard forms or charts, and with such simplicity as will achieve skill and speed in the actual operations of measurement and accuracy and efficiency in the subsequent computation of volume.

Any system of tallying which is clear, concise, compact, and comprehensive will serve. The method which is most commonly used by foresters in North America is known as the " dot and dash " system. Reference is made to Fig. 1 for its explanation.

3. Units of Dimension. — The correlative of tallying is measuring — the measurement generally of such dimensions as will enable computation of volume. In Europe and in British India,

* At the present time the shortest length accepted as a log is probably 6 feet. Individual pieces shorter than this are spoken of as " billets " or " bolts," and are usually cut to meet the specifications laid down by the manufacturers of some special products such as shingles, spokes, pulp, cooperage stock, etc.

the prevailing custom in measuring timber and forest products is to determine circumference, or girth, and length. In North America, *diameter* and *length* are preferred. In Europe length is commonly measured in meters and circumference in centimeters. In the United States and Canada we prefer to measure length in feet and tenths of feet, and diameter in inches and tenths of inches. The fractional tenths of feet and inches* are preferred for simplicity in calculation.

4. Dimensions of Diameter in Standing Trees. — Every tree is enclosed within a cortex of corky material called *bark*. Bark as a rule is of very little use or value† to wood-using industries, and is usually discarded in the processes of manufacture. Bark thickness has no influence upon the measurement of length, but profoundly affects measurements of diameter. In the case of redwood and Douglas fir, the bark may comprise as much as 20 per cent‡ or more of the volume and from 15 to 20 per cent of the diameter according to the age of the tree and the point of measurement. The volume of trees is almost always computed and recorded as of *inside* the bark.

Diameter Breast High. — This is the most convenient and most commonly used measurement for classifying trees on the basis of their size. It is taken and recorded as outside the bark at a point on the tree stem just 4½ feet above the ground. Its usual abbreviation is "D.B.H.o.b." (Diameter outside the bark Breast High) and the point of measurement is known as the D.B.H. point. It is always an average measurement — the mean of the major and the minor axes. Its use is preferred for the following reasons:

1. It provides a point of uniform convenience for measurement by the forester which can be accepted as a standard, and which avoids the unnecessary fatigue involved in repeated measurement at some lower or higher point.

* The usual practice in the lumber mills is to cut dimensions, particularly of thickness, in fractional values of inches based on quarters, eighths, and sixteenths.

† The bark of some trees, notably eastern hemlock, which can be used as a source of tanning extract for leather is an admitted exception. Cases are on record where the bark has been considered of more value for this purpose than the wood for construction purpose. Thousands of trees stripped of their bark have been left to rot in the woods.

‡ The Relation of Bark to Diameter and Volume in Redwood, by J. E. Pemberton, Jr., Jour. of For., Vol. XXII, No. I, Jan. 1924, p. 24.

2. Ordinarily it is high enough up the tree stem to avoid eccentricities in the taper or form of the stem caused by root swell or fluting of the root buttresses.
3. A uniform point of measurement is set and a standardization of this diameter measurement of standing trees is achieved.
4. Its use, rather than a diameter measurement on the stump, is indicated by the fact that stumps are never cut at uniform height, and such standardization would at once be lost.
5. Were a standardization of stump height achieved, the value of such diameter measurements would be completely upset by a change in utilization standards demanding either higher or lower stumps.

D.B.H. measurements are usually classified according to full inch values, that is, a 1 inch interval, the fractional parts of the inch from 0.6 below to 0.5 above being thrown into the one class known as diameter class. For example, all trees with a D.B.H. measurement between and including 9.56 inches to 10.55 inches* would be classed as 10 inch trees. With very large trees, D.B.H. classes may be grouped on a 2 inch, 3 inch, or even 4 inch interval.

The Value of $d\frac{1}{2}$. — This is not *half* a diameter dimension but a *full* diameter measurement taken at a point *half way in a length dimension*, namely that of total height in the tree. As most commonly used it is taken on the tree stem at a point just half way between the D.B.H. and the tip of the tree. It is estimated or measured and recorded in inches and tenths of inches.

5. Dimensions of Length in Standing Trees. — The terms length and height are used more or less interchangeably, the same distance, for example, being measured as height when the tree, or portion thereof, is standing upright as is called length when the tree is felled and on the ground. Its abbreviation or symbol is *H* or *L*. Several dimensions of length in the standing tree are commonly observed which have significant value, namely, total height, used length, merchantable length, and clear length.

Total Height. — This is the full distance from the ground to the top of the stem. It, like all other distances in length, is measured in feet and tenths of feet. Tree heights are classified into height

* Often, in scaling, and in some cases in measuring, the diameter classification is based on the inch and all fractional measurements thereof. For example trees 9.0 to 9.9 are called 9.0 inch trees, and 10.0 to 10.9 are called 10 inch trees, etc.

classes, sometimes with a 5 foot interval — that is, 50, 55, 60, 65, 70, etc. and sometimes, as with taller trees on a 10 foot interval, 120, 130, 140, etc. In determining the class to which a tree belongs it is customary to throw everything from 0.6 of a foot above the half way mark below up to 0.5 of a foot above the half way mark above into the single height class. For example, all trees from 135.6 to 145.5 would be considered as belonging to the 140 foot height class.

Used Length. — This is that length of the main stem which is actually cut into and used as logs for the production of wood material. It is measured in feet and tenths of a foot.

Merchantable Length. — This differs from the used length in that it is more apt to be a statement of ideal rather than actual conditions. It is a judgment of what *should be* utilized rather than what is really being done. Merchantable lengths measured in feet and tenths of feet are generally grouped into height classes on a 2 foot, a 5 foot, or a 10 foot interval.

Clear Length. — This is a quality measurement taken to determine how much of the main stem is *clear* or free from limbs and therefore capable of yielding the largest amount of knot-free, clear, high grade lumber. It is the distance measured on the standing tree from the ground to the lowest limb, live or dead. It is measured and recorded in feet and tenths of a foot.

6. Dimensions of Diameter in Logs. — Diameter dimensions in logs are always read or reduced to their inside bark values, giving rise to the abbreviation d.i.b. — diameter inside bark. The underlying reason is that these diameter dimensions are used for the computation of volume, and the volume desired is that of the wood inside the bark. It should always be a mean diameter. Three significant diameter dimensions are always recognized in logs: top diameter, butt diameter, and middle diameter.

Top Diameter (symbol d). — The diameter inside of the bark in inches and tenths of an inch at the smaller end, or the end toward the top when the log was standing in its normal position in the tree.

Butt Diameter (symbol D). — The diameter inside the bark in inches and tenths of an inch on the larger end of the log, or the end toward the *butt* when the log was standing in its normal* position in the tree.

* It is well to note that in any log sequence in a tree a small d on one log is of the same value as large D on the log immediately above it.

Middle Diameter (symbol $d\frac{1}{2}$). — A diameter dimension measured outside the bark but reduced to its corresponding value inside the bark, taken at a point half way between the butt and top ends of the log. It is read and recorded in inches and tenths of an inch. This is the least common of the diameter dimensions commonly measured on logs, but it is of importance in several methods of calculating the contents.

7. Dimensions of Length in Logs. — Although the length of each log is of main importance, it should be noted that the sum of the several log lengths plus those of the stump and top should always equal the total height of the tree.

Log Length. — This is the length from the larger to the smaller end of the log parallel to the main axis of the stem measured in feet and tenths of a foot. Its symbol or abbreviation is *L*.

In lumbering, logs are usually cut to a uniform length that has been agreed upon between the owner and the operator. This length has varied in different parts of the country and at different times from 6 to 40 feet. By custom, however, 16 feet has been understood as the *standard log length* and, unless otherwise stated, will be understood to be the measurement when the term implying a log length is used. For example, a " 1 log tree " will mean a tree which contains one 16 foot log; a "2 log tree " one which contains two 16 foot logs; a " $1\frac{1}{2}$ log tree," one which contains one 16 foot log, and a half of the next 16 feet or an 8 foot log above; and so on.

The lowest log in the tree, that is, the one immediately above the stump, is known as the " butt log " or the " 1st log "; the one above that is known as the " 2nd log"; and the one above that as the " 3rd log"; and so on.

Length of Top. — The portion of the main stem of the tree above the smallest diameter dimension considered merchantable and between that and the tip of the tree is known as the top section or, more briefly, " the top." It is measured in feet and tenths of a foot.

Stump Height. — A measurement of the distance from the average ground level to the horizontal plane of the point of section on the stump or the stump cut. It is measured in feet and tenths of a foot.

The stump and the top are the portions of the tree left in the woods as a waste. A measurement of their dimensions contributes little or no knowledge concerning the amount of wood material in the tree utilized by the logger, although their volume is an important factor in considering total volume.

8. Basal Area. — It is the area in *square feet* corresponding to a diameter dimension in inches taken at any point on any portion of a tree's stem between root and tip. As commonly stated, it is a misnomer due to the implication that it is the area at the *base* of the tree only. The term *sectional area* might be better, but " basal area " is so woven into American forestry literature and thought that it seems hopeless to try to change it. Sectional area is seldom measured directly although there is no reason why the diameter measuring instruments could not be so calibrated as to state the corresponding basal area directly. The usual practice is to measure the diameter, and to look up the equivalent area value in a set of tables* which show for various diameter values in inches and tenths of an inch the areas of the corresponding circles in square feet.

* See Table XXIV.

CHAPTER II

THE INSTRUMENTS USED IN FOREST MEASUREMENT

9. The Need of Instruments. — Instruments are needed for the measurement of dimensions, especially of diameter and length. Many that are used for such a purpose are common to other professions, especially plane surveying. There are several, however, which owe their development and adaptation to the problems of forestry, such as the measurement of the volume and increment in trees, logs, and forests.

The more important forestry measurement instruments are:

A. Instruments for measuring dimensions of diameter:
1. Calipers,
2. Diameter tape,
3. Biltmore stick,
4. Dendrometers.

B. Instruments for measuring dimensions of length:
1. Hypsometers,
2. Measuring tape,
3. Measuring pole.

C. Instruments for measuring volume:
1. Scale sticks.

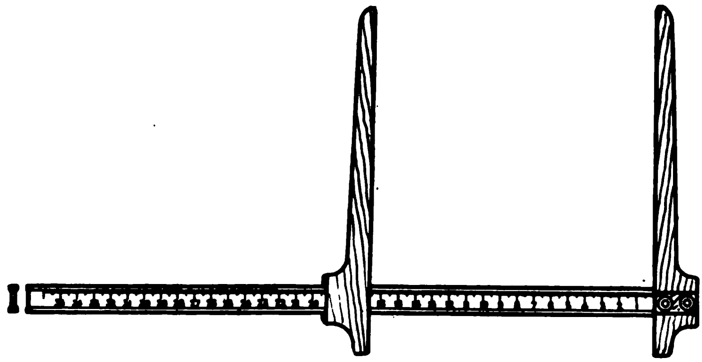
D. Instrumental aids for measuring increment:
1. Increment borer,
2. Bark measurer.

INSTRUMENTS FOR MEASURING DIMENSIONS OF DIAMETER

10. Calipers. — Calipers are used to measure the breast height diameter of standing trees and the diameter dimensions of logs at other points than at their ends. They consist of a wooden beam having scales on the two flat sides graduated to inches and tenths. This beam has at one end a fixed arm, held in place by a pair of brass bolt nuts. There is also a sliding arm fitted with a brass sleeve permitting it to travel loosely along the beam. This sliding arm should be in such adjustment that when any weight or force is brought against its inner face, as when it is brought against a tree trunk, it swings to a position of true right angles to the beam, and parallel to the inner face of the fixed arm. Calipers should never be taken to the field unless they are first tested for adjustment.*

The following cautions should be observed in using calipers:

1. The measurement of a tree for diameter is to be taken at a constant height above the ground. This is generally at the



Courtesy of Keuffel & Esser

FIG. 2. — Tree Calipers.

breast height point, but may be at any other convenient point arbitrarily chosen. Care must be taken to keep the measurements at this height as there is a tendency to let the calipers slip down on the tree, thus recording larger measurements than those of actuality.

2. To use the calipers, approach the tree or log, and, opening the jaws, thrust in the calipers until the beam rests against the stem; bring both of the arms snug against the stem and read the diameter at the *inner* face of the sliding arm.

3. The measurement should be the true average diameter of the tree at that point. If the circumference of the tree has deviated from a true circle, two measurements must be taken, and the mean of the two is to be accepted as the diameter.

Advantages.

1. Diameters can be read directly to a tenth of an inch, thus making the instrument applicable for precise scientific work.
2. The points of bearing of the arms on the tree are always in sight and any irregularities can be avoided.

* The adjusting screw may be found on the outer edge of the sliding arm just above the brass sleeve.

3. The firm closure of the arms against the tree bole crushes out any irregularities due to stiff, scaly, or friable bark.
4. Calipers are light and portable, the 36 inch calipers weighing only about 2 pounds, and can easily be transported in the woods.
5. They are adaptable for use in the hands of unskilled labor.
6. They can be taken apart and packed for transportation.

Disadvantages.

1. On account of the factor of size calipers are not adaptable to Pacific Coast timber.
2. Calipers are not so convenient as the Biltmore stick or the tape in rough, bushy country, due to the arms catching in the brush.
3. Two measurements are necessary on every tree in order to get the mean diameter.
4. There is a tendency toward carelessness on the part of the user in allowing the calipers to slip down, thus incurring inaccuracy.
5. There is a tendency for the movable arm to swell and bind in wet or damp weather.
6. There is also a tendency for the beam to get coated with pitch and bind the sleeve.
7. In winter the sleeve will get clogged with frozen snow, thus preventing the use of the instrument.

11. The Diameter Tape. — The diameter tape finds its best use in the large sized trees of the Pacific Coast. It is a regular tape either of steel or cloth,* usually graduated on one side in inches and tenths. On the reverse side, the tape is so graduated that, for a given circumference, the corresponding diameter can be read directly. The theory on which this is based is merely the ratio existing between the diameter and circumference of a circle.

On the whole, except in very large timber, the diameter tape is not applicable for general use for the following reasons:

1. Its use slows up the work of measuring diameters and, where possible, will give way to faster methods.

2. It is not the most accurate method of obtaining diameter measurements due to:

* A convenient and cheap instrument can be made from a common cloth tape with the diameters printed on the back of the tape in India ink at the points of corresponding circumference measurement. For northeastern trees such a tape should be at the most about 6 feet long. For the very large timber of the Pacific Coast it needs to be much longer. It is in this type of timber that the diameter tape best demonstrates the speed with which, in the hands of a trained observer, it can be used.

a. Its deviation from the true circumference of the tree caused by the tape passing over the irregularities on the surface of the tree.

b. The circumference of a tree is rarely a perfect circle and may contain less sectional area than does a circle of the corresponding diameter; hence there will be a less basal area per tree and per acre under this method.

c. The tendency of the tape to slip down on the bole of the tree may indicate a larger diameter than that actually existing at the point of measurement.

3. The observer does not see the full circumference of the tree and has no knowledge of the proximity of knots, burls, or swellings of branches which affect the true diameter values.

4. A difference in tension of the tape due to elastic or brittle bark affects the true diameter values.

5. The values read on the tape, not being directly correlated to such of the points of measurement as may be seen, are capable of being misread.

6. It has been determined in actual experience with trees 8 inches in diameter and over, that it takes just about as long to make one diameter reading with a tape as two caliper readings on the same tree and at the same level but at right angles to one another.

12. The Biltmore Stick. — The Biltmore Stick, devised by Dr. C. A. Schenk of the Biltmore Forest School, measures the diameters of trees on the basis of the relations of similar triangles. This stick is so constructed and graduated that if it is held in horizontal position at a known distance from the eye against the bole of a tree, so that the line of sight from the left-hand edge of the tree trunk just intersects its zero end, the line of sight from the other edge of the tree bole will intercept the stick at a graduation value equivalent to its true diameter in inches at that point. The intercepted distance cut off on the stick will be less than the full diameter by an appreciable difference, and this difference becomes greater as the diameters increase. Consequently, the graduations on the stick for successively larger diameters fall closer and closer. The graduations of the scale as measured off from the zero end of the stick can be calculated from the following formula:

$$S = \frac{d(a - t)}{\sqrt{a(d + a)}}$$

where S = the scale value of the graduations in inches,
a = the arm reach of the observer in inches,
d = the diameter of the tree in inches.
t = thickness of stick in inches.

The values of the graduations are thus dependent upon the distance between the eye and the tree bole. This varies with different men, and experience has shown that efficiency in its use can be attained only when the graduations are based on the ordinary reach of the observer. A table of graduation distances corresponding to different diameter values at different arms' lengths is presented in Table XXX.

The advantages of the Biltmore stick are:

1. It is light, straight, and without cumbersome parts, and can be used in the thickest brush.
2. Diameter values can be read at a glance.
3. It is especially adapted to use with large sized western trees where calipers would prove extremely burdensome, and time does not permit use of the tape.
4. It can be used more rapidly than either the calipers or diameter tape.
5. Its use is preferred by the cruiser who is estimating diameters by the eye and desires a light, convenient instrument for occasional check.
6. In the hands of a skilled and careful man it is a surprisingly accurate instrument.

The disadvantages of the instrument are:

1. The Biltmore stick is not as exact as the diameter tape or the calipers. It probably is the least accurate of the diameter-determining instruments.
2. The position of the stick against the tree must be in a horizontal plane at exact right angles to the line of sight from the eye to the geometric center of the tree.
3. The tree measured with a Biltmore stick must be erect and standing in a truly vertical plane.
4. The stick is not capable of being graduated or read in tenths of inches, and hence is not adaptable to precise work.
5. For correct results, the tree at the point where the stick is held should be circular in cross-section. This error is variable and is much greater than with any diameter tape or caliper measurement.
6. When D.B.H. values are read at a point on the tree bole 4½ feet above the ground, *the eye of the observer must be on a level with*

the stick. For proper use, the cruiser must lower his eye to the level of the stick and not raise the stick to the eye level point.

7. The eye must be at that proper distance from the stick for which the stick is graduated. There is a continual tendency to shorten this distance through the flexing of the elbow. This difficulty must be watched.
8. When a stick is used calling for a longer eye distance than the ordinary reach of the cruiser, errors due to inaccuracy and inefficiency are incurred.
9. There is no such thing as a standard Biltmore stick. On the other hand, calipers and diameter tapes have no personal limitations and are standard to everybody using them.

13. Dendrometers. — These are instruments for measuring diameter dimensions in the upper portions of the stems of standing trees, at heights beyond the reach of the observer equipped with calipers, or Biltmore stick, or diameter tape. Generally they are a combined height measuring and diameter measuring instrument, adapted to measuring diameters indirectly through principles based on the relations of the sides of similar triangles. Although a number of hand instruments of this type are available, those which make any great pretensions to accuracy are equipped with telescopic lenses of moderate magnifying power and with tripod support.

Probably the best of these is the one invented by a Swedish forester, Liljenstrom. It consists of a telescope mounted in a manner similar to a transit mounting and equipped with graduation circles in both the horizontal and vertical planes. Within the telescope tube, another graduated scale is fixed so as to superimpose, but not obscure, the field of vision. The horizontal distance from the tree to the instrument must be accurately measured. Readings obtained on the telescope scale are translated to terms of diameters by means of a set of tables prepared for a schedule of known distances. Readings on the vertical scale referred to an appropriate table enable the determination of the height of the tree, or the height of any desired diameter point.

A somewhat different type of instrument has been devised by Donald Bruce.* In principle its construction is akin to that of the ordinary calipers. A fixed arm at one end of a beam, which also

* A New Dendrometer, by Donald Bruce. University of California Publications in Agricultural Sciences, Vol. III, No. 4, Nov. 27, 1917, pp. 55–61.

carries a movable arm, is fitted with two small mirrors on the points of the arms. These mirrors are adjusted at a contra-facing 45 degree angle so as to reflect the two lines of sight from the ends of the desired diameter. Since these two lines of sight are parallel, a reading of the distance between the two arms on the scale of the graduated beam directly derives the desired measurement.

Generally speaking, the use of such instruments will be confined to the examination of stands of very valuable timber, such as redwood, Douglas fir, etc. and to the highly technical problems undertaken by experiment stations and other investigation agencies as will demand an accurate measurement of upper diameters in standing trees. These instruments have little or no place in the rougher technique of practical timber estimating. If upper dimensions are desired, the forester will train himself to estimate them by eye within a reasonable degree of accuracy.

INSTRUMENTS FOR MEASURING DIMENSIONS OF LENGTH

14. Hypsometers. — The dimension of height in standing trees is obtained in mensuration work by the use of hypsometers. They are height measuring instruments based on the relations of the sides and angles of similar triangles. Two principles are involved in their development and application:

A. Geometric Principles, as based on the relations of the similar sides of similar triangles. The more important and commonly used of the hypsometers developed on this basis are:

1. The Klausner Hypsometer,
2. The Faustman Hypsometer,
3. The Chrysten Hypsometer,
4. The Merritt Hypsometer.

B. Trigonometric Principles, as based on the relations existing between the sides and trigonometric functions of right angled triangles. Two important hypsometers are based on this principle:

1. The Improved Abney Level,
2. The U. S. Forest Service Hypsometer.

A great many other hypsometers have been constructed on these same bases, some of which have only an academic value. Those listed above are the most commonly used.

Five Phases of Hypsometer Measurement. In using hypsometers, two measurements are necessary in order to get the height of a standing tree. This is due to the usual position of the eye

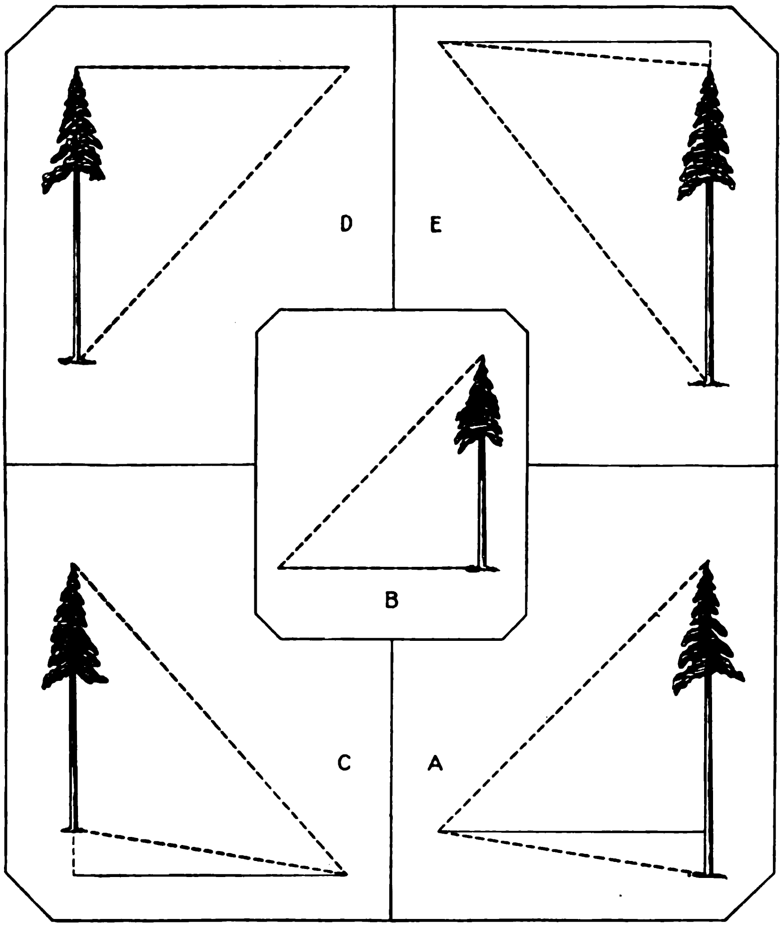


FIG. 3. — The Five Phases of Hypsometer Measurement. The Dotted lines Represent Lines of Sight to the Base and Tip of the Tree. The Light Solid Line is the Horizontal Plane at the Position of the Eye of the Observer.

relative to the horizontal plane of the base or top of the tree. When the level of the eye intersects the tree at any point between its base and tip, the two readings must be added. When the level of the eye is either above the top or below the base, the two readings must be subtracted. The sum or the difference gives the total height. These principles can best be illustrated by reference to Fig. 3.

Hypsometers Based on Geometric Principles

15. The Klausner Hypsometer. — This instrument consists of a flat metal rule 6 inches long, at one end of which there is hinged a sighting rule of slightly greater length. Each of these rules has an objective sighting device at the far end. At the joint there is fitted a movable peep sight, or eye piece revolving in a vertical plane, so as to serve both objectives. The sighting rule can be raised or lowered by means of a thumb screw attached just below the joint. The base rule is graduated in divisions to correspond with units of horizontal measurement from the tree to the observer. Attached to the base rule is a sliding device carrying a thin strip of metal kept in a vertical position by means of a weight and hinge joint. This strip is graduated and forms the height scale of the instrument. The instrument is also equipped with a threaded clamp and head so that it can be fitted on a tripod.

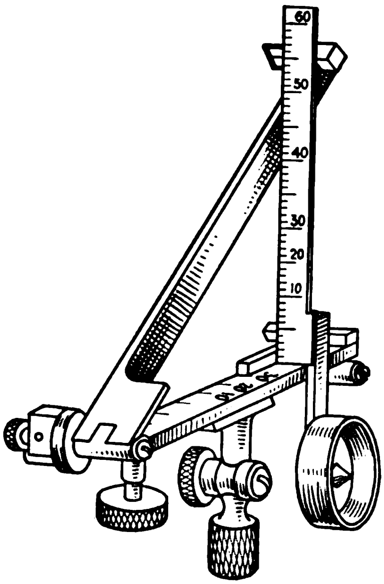


FIG. 4. — The Klausner Hypsometer.

The instrument is set up in such a position that both the top and base of the tree can be seen. The slope distance from the base of the tree to the instrument is then measured and set off by the slide on the distance scale. Both of the objectives are then sighted on the tree, and the sighting objective is raised or lowered to cut the top of the tree. The height of the tree is then read directly.

An objection to its use as generally constructed is that it is adaptable only to short trees. This difficulty can be overcome by varying the value of the horizontal units, in which case the vertical units will be gaged at like value, yards or meters.

Advantages.

1. It is fitted to a tripod and hence avoids the errors due to shaking by vibration.
2. Only one observation is required at each set-up.
3. The height can be read directly from the altitude scale without further computation.
4. It is quite accurate.

Disadvantages.

1. It is not compact and is not easily carried in the field.
2. It is not preferred in rough country because of the additional burden of carrying the tripod.
3. It is a rather delicate instrument and may easily get out of adjustment.

16. The Faustman Hypsometer. — This instrument consists of a skeleton metal or solid wooden frame, equipped with a vertical

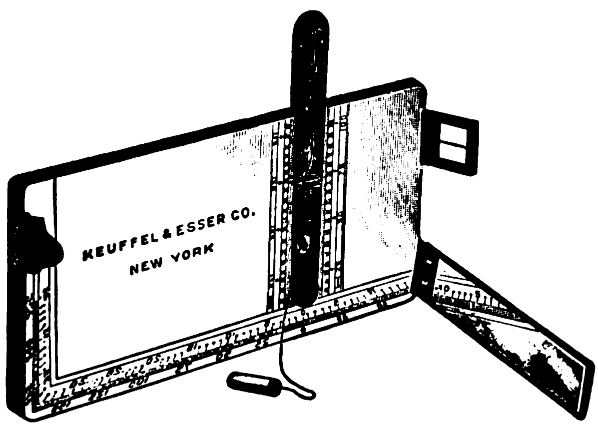


FIG. 5. — The Faustman Hypsometer.

slide on one side of its longitudinal center. The slide is provided at the ends with thumb notches marked respectively I and II, with transversely arranged index markings. A plumb line carrying a small brass plummet is attached at the central portion of the end marked II. At one end of the instrument is located an eyepiece and at the opposite end a cross hair, both of metal, and hinged so as to be folded flat when the instrument is not in use.

A long, narrow mirror is hinged to the end of the frame below the objective, so as to reflect right- and left-hand inverted scales engraved on the lower edge of the frame. These two scales meet in a zero at a point intersected by the plumb line when the instrument is held in level position. The right-hand scale progresses to 75 and the left-hand scale to 225. These figures represent the heights of the trees in the units assumed in the measurement of the horizontal distance. If this distance is in feet, then the height of the tree is read in feet; if in yards or meters, the height is then read in similar terms.

The vertical slide is movable and is set to indicate the measured distance of the observer from the tree. When the horizontal distance is less than 100 feet, index II should be adjusted in accordance with the right-hand scale, and when the distance is more than 100 feet, index I should be adjusted on the left-hand scale. This adjustment is such that the distance between the point of suspension of the plumb line and the lower edge of the instrument, and the distance from the tree form the corresponding sides of two similar triangles.

The instrument is held in the left hand of the observer. It is sighted at the top of the tree, and the height is read directly from the reflected image of the height scale in the mirror. This gives the distance that the top of the tree is above the horizontal plane of the line of sight. A second sight at the base of the tree is usually necessary. The sum or difference of the two is the height of the tree.

Advantages.

1. It is easily transported, and hence is adaptable to field work.
2. The results obtained from it are quite close enough except for the most accurate of scientific work.
3. The instrument can stand considerable usage, is easily repaired, simple in construction, and difficult to put out of permanent adjustment.

Disadvantages.

1. Unless the base line is carefully measured, there is a tendency for the error in height to be magnified.
2. When the position of the tree varies from the vertical plane there is a tendency for error to creep in.
3. Inaccurate readings may be taken owing to an unsteady plumb line due to:
   a. Shaking in and by the hand.
   b. Shaking of the instrument and plumb line due to wind.

4. There is a tendency among students to read the instrument too quickly before the plumb line comes to rest.
5. The thread of the plumb line may be easily broken and the plummet lost.
6. Two observations are usually needed on a tree in order to establish its height.

A more recent development of this instrument in Europe provides a metal frame equipped with a long slot in which the plummet swings. This provides shelter and protection to the plumb line from the force of the wind and a much higher degree of accuracy is attainable.

17. The Chrysten Hypsometer. — This consists of a strip of metal or wood as in the accompanying Fig. 6, with protruding edges at *a* and *b*. The instrument is held suspended in the field of vision and moved backward or forward until the lines of sight from the top and base of the tree just intersect respectively the upper and lower protruding lips of the instrument. The observer then notes how much on the scale of the instrument is cut off by the line of sight from the top of a 10 foot pole* standing directly in front of the tree under observation. If the graduations† on the face of the hypsometer are in units representing height in feet, the height of the tree can be read directly on the following equation, (see Fig. 6):

In the triangles EAB and ECB

$AB : ab :: CB : cb$

CB is known, the height of the pole 10 feet,

ab is the extreme length of the instrument between the protruding edges, also known and constant,

cb can be measured directly or read from the scale on the face of the instrument.

Hence AB (height of tree) equals $\frac{ab \times CB}{cb}$

* It makes no particular difference what length of pole is used, provided that the graduations on the hypsometer have been calculated and scaled off *for that particular length*. In the more or less open forests of the northeast a 6 foot pole will be found to be convenient. This is more easily carried through the woods than a pole of greater length, and is the length of the ordinary surveyor's range pole, which can be used for this purpose.

† In the Chrysten hypsometer the point of origin for graduation is the upper notch and it has a numerical value in feet equal to *the length of the pole*. Thus a 10 foot pole, for example, and a 10 foot tree will exactly coincide on

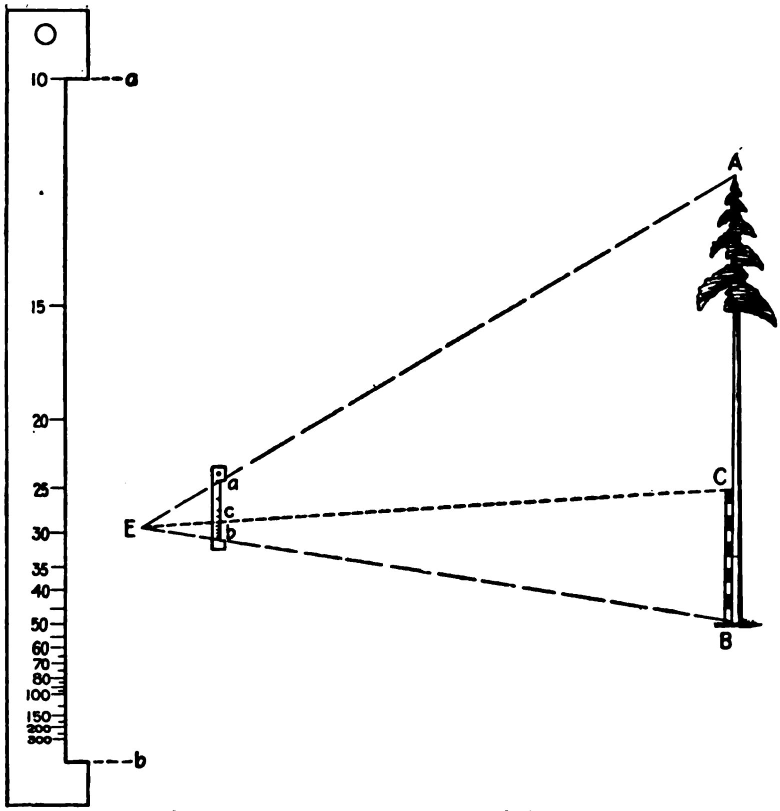


Fig. 6. — The Chrysten Hypsometer Graduated for a 10-Foot Pole, and Method of Use in the Field.

A convenient hand-made instrument may be made for eastern work with the graduation of a Biltmore stick for diameters on one side and the graduations of a Chrysten hypsometer for tree heights on the other. A 20 to 30 inch stick will take care of the ordinary run of eastern timber with 15 inches between the notches for the hypsometer. For western timber 35 inches or more of length would be desirable. See Table XXXI.

Advantages.

1. It is light, easily made, and easily transported in rough, bushy country.

the instrument. The graduation increases in numerical value *downward*. They also fall closer together. It is equally in error to graduate a Chrysten hypsometer from zero as it is to graduate it upward.

2. It is based on scientific principles, and with reasonable care, can be used for determining heights with a fair degree of accuracy.
3. The heights can be read directly.
4. There is no need of measuring the distance, and the instrument can be used from any point from which the top and base of the tree and the top of the pole can be seen.
5. Only one observation on each tree is necessary.
6. It is quicker than several other of the hypsometers and lends itself to conditions where speed is desired.
7. It is a particularly advantageous instrument for use where only occasional readings are taken to check estimated heights.

Disadvantages.

1. Extra care must be taken to hold the image of the tree exactly within the notches while reading the height values to the top of the pole.
2. Unless the pole is cut at the exact length for which the graduations are calculated and scaled, error will be incurred.
3. The tree observed must be standing erect and in a vertical plane.
4. The hypsometer must be held in a true vertical plane. To this end, *suspend* the instrument as a plummet from above.
5. It is a mistake to hold the instrument by a handle from below as the user invariably either inclines the instrument away from or toward himself, thus incurring error.
6. When the instrument is suspended from above, it is subject to some shaking from the wind.
7. Error in construction may be incurred by graduating the instrument with correctly computed and scaled graduations, but *in the wrong direction.*
8. As a rule, for heights of 50 feet and over it is impossible to graduate and read the scale closer than every five feet. Thus the instrument does not lend itself to precise work with large sized trees.
9. The pole is awkward to carry, and, as a further disadvantage, it must be placed against the bole of every tree before height measurement can be made.
10. Some skill is necessary to use the instrument with consistent accuracy.
11. It is not adapted to steady work due to the fatigue of constantly holding the arm in the required position.

18. The Merritt Hypsometer. — As usually constructed, it consists of a hardwood rule, one of whose faces is graduated as a hypsometer to show standard log lengths. The graduations of the stick are worked out on the basis that the instrument is to be held at a definite and predetermined distance from the tree and at a definite distance in front of the eye of the observer. The stick is raised or lowered until the line of sight from the base of the tree cuts the base of the stick. The distance or graduation cut by the line of sight from the top* of the tree is read. This is the desired height and will be in terms of standard log lengths which can be reduced to terms of feet by multiplying by 16. It can be used for either total or merchantable heights, but is designed particularly for the latter.

Advantages.

1. As usually constructed, it is found on the reverse side of the Standard Biltmore stick, and presents the same advantage of lightness and convenience.
2. There is the added advantage of combining two instruments in one.
3. Its special advantage is where speed is an objective and where an occasional observation is desired to check ocular estimation of heights.

Disadvantages.

1. It is the least accurate of all the hypsometers.
2. Unless the stick is held in a true vertical plane, error is incurred.
3. The tree observed should be erect, standing in the same plane as that in which the stick is held.
4. There is a tendency on the part of the observer to raise the stick when he looks up to the top of the tree and to lower the stick when he looks downward toward its base. Error may thus be incurred.
5. It is designed for determining merchantable height or length only, and that in terms of standard log lengths.
6. If total heights are desired, the fractional parts of log lengths cut off on the scale by the line of sight from the top of the tree must be accurately *estimated* and then converted into feet.

Hypsometers Based on Trigonometric Principles

19. The Improved Abney Level. — The improved form of this instrument, which is of larger and stouter construction than the standard form, was developed for use with a slope chain in topographic surveying. It is often used as a hypsometer. Two

* In the form of this hypsometer adopted by the Federal Land Bank of Springfield, Mass., for use in its timber examinations, the graduations are from the top of the stick and an opposite procedure is to be followed.

methods* of use are offered, the one solving the height of the tree by tangents and the other by sines.

METHOD OF TANGENTS. The observer seeks any convenient position from which both the top and the base of the tree may be seen. He then sights to the top of the tree, and by means of the clamp on the vertical arc of the instrument, sets, reads and records the vertical angle. The horizontal distance to the tree is measured. This distance multiplied by the natural tangent of the vertical angle gives the distance that the top of the tree is above the horizontal plane of the line of sight. A second reading to the base of the tree is necessary. The sum of these two gives the total height of the tree.

METHOD OF SINES. In this method the slope distance from the observer to the tree is measured. It is not necessary, however, to convert it to its true horizontal value. The vertical angles to the top and base of the tree are measured in *terms of degrees*. The height of the tree is then determined by application of the law of sines.

When the eye of the observer is above the level of the base of the tree, as in Fig. 7, the solution is in accordance with the following:

$$\text{Height of tree} = \frac{d(\sin X + Y)}{\cos X}$$

where d = the slope distance to the base of the tree.
X = the vertical angle to the top of the tree.
Y = the vertical angle to its base.

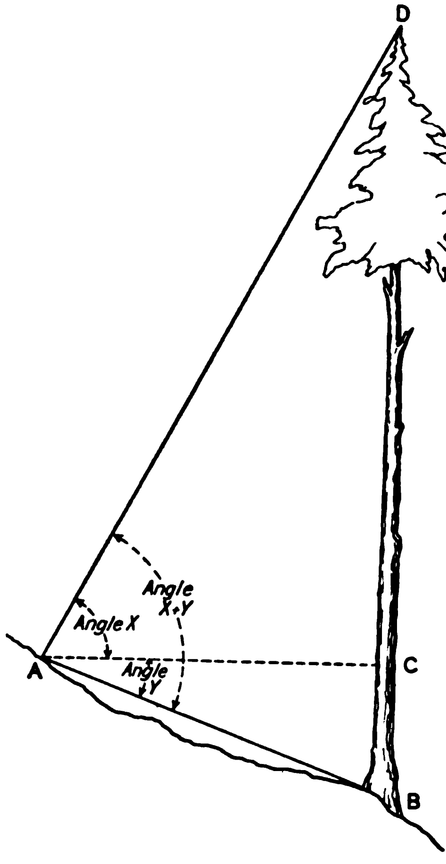
When the eye of the observer is below the base or above the tip, as in Case C in Fig. 3, the solution is:

$$\text{Height of tree} = \frac{d(\sin X - Y)}{\cos X}$$

If the observer carries into the field a slide rule on which the A and D scales are graduated with the logarithms of numbers, and the B and C scales with the logarithms of sines, the solution of the

* A third method might be suggested: Set the vertical arc of the Abney at 100 per cent (or 45°). Sight to the top of the tree. A measurement of the horizontal distance from the observer to the tree is equal to the height that the tree tip is above the horizontal plane of sight. A second reading to the base of the tree is necessary. The sum of the two gives the total height of the tree.

problem is a simple process involving but one setting of the slide rule. Mr. I. T. Haig of the Priest River Forest Experiment



Angle A = CAD to Top of Tree, and CAB to Base of Tree.

a = Desired Height

d = Slope Distance from Tree to Observer.

$$\frac{a}{\text{Sin } A} = \frac{d}{\text{Sin } D}$$

$$a = \frac{d.(\text{Sin } A)}{\text{Sin } D}$$

But, $\text{Sin } A = \text{Sin } (X + Y)$ ·

$\text{Sin } D = \text{Sin } (90° - X)$

$= \text{Cos } X$

Hence $a = \dfrac{d.(\text{Sin } X+Y)}{\text{Cos } X}$

Fig. 7. — Mathematical Proof of the "Method of Sines" in Determining Tree Height.

Station, U. S. Forest Service, who first worked out this method,* advocates the use of a circular slide rule on the same principle.

An objection to the method lies in the fact that ordinarily the improved Abney level is graduated in per cents which must be reduced to corresponding values in degrees in accordance with the

* Short Cuts in Measuring Tree Heights, by I. T. Haig, Jour. of For., Vol. XXIII, No. 11, Nov. 1925, p. 941.

slide rule graduations. It would be theoretically possible to graduate the slide rule in percentages but mechanical difficulties make it inadvisable.

Advantages.

1. The Abney is a small, compact instrument easily carried in the field.
2. It is based on scientific principles, and gives results within reasonable degrees of accuracy.
3. It does not require a high degree of skill or training to use it.
4. It is adaptable to use by the less highly trained members of a field party.
5. It is not subject to error from the effect of wind.
6. It does not get out of order easily.
7. When out of order it is capable of simple and quick adjustment.

Disadvantages.

1. The distance from the tree to the observer must always be measured.
2. Two observations must be made on each tree.
3. Readings can never be made directly but must be separately calculated or measured.

20. The U. S. Forest Service Hypsometer. — This instrument was developed from the military clinometer or grademeter. It consists of a shallow disk-like box $\frac{3}{4}$ of an inch deep by about $3\frac{3}{4}$ inches in diameter. A peep hole in one edge carries the line of sight across the diameter of the box to a small glazed window bearing a horizontal cross hair on the opposite edge. A weighted circle in the interior of the box swings in a vertical plane on the central axis of the box, the movement of which is controlled by a spring and button release. This circle bears graduations which are reflected to the eye of the observer by means of a small mirror, which does not obscure the line of sight, and permits its observation simultaneously with the sighted object. The graduations are in per cents. Heights must be computed on the basis of the distance measured.

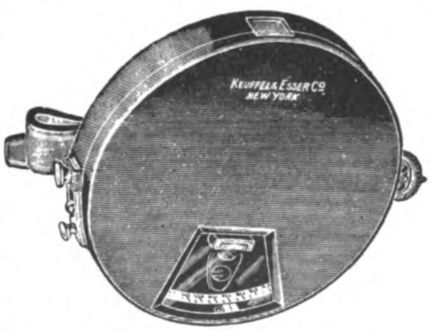


FIG. 8. — The U. S. Forest Service Hypsometer.

The instrument is a compact, durable, simple, easily operated hypsometer which deserves a wider use than it has hitherto received. It has the usual disadvantages of hypsometers in requiring two observations on every tree and the requisite measurement of horizontal distance. This is not easy in steep sloping country. To overcome this difficulty, a method has been proposed by McArdle and Chapman* to read tree heights from a set of charts based on the variables of slope angular measurement to the base and top of the tree and distance.

21. Relative Accuracy of Hypsometers. — As to which hypsometer will be used in a given case depends upon a number of factors, the most important of which are the previous training and personal prejudices of the forester, the degree of accuracy desired, the relative speed with which the work must be accomplished, the

TABLE I

RELATIVE ACCURACY OF VARIOUS HYPSOMETERS

Average Error from Transit Measurement Expressed as Per Cent of Total Height

Heights in Feet	Klausner Hypsometer	Improved Topographic Abney Level	Faustman Hypsometer	U. S. Forest Service Hypsometer
20	− 0.15	− 0.45	− 1.97	+ 1.10
30	− 0.09	− 0.45	− 1.33	+ 0.74
40	− 0.07	− 0.33	− 1.80	+ 0.60
50	− 0.11	− 0.25	− 0.40	+ 0.55
60	− 0.21	− 0.27	− 0.20	+ 0.55
70	− 0.40	− 0.41	− 0.16	+ 0.70
80	− 0.62	− 0.71	− 0.28	+ 1.03
90	− 0.78	− 0.89	− 0.51	+ 1.35
100	− 0.82	− 0.90	− 0.75	+ 1.45
110	− 0.72	− 0.80	− 0.79	+ 1.41
120	− 0.62	− 0.45	− 0.78	+ 1.25
Average for all Height Classes	− 0.42	− 0.54	− 0.72	+ 0.97

* The distance is measured and the slope angle determined as a per cent. Similarly the angle to the tip of the tree is determined as a per cent. By referring to a chart compiled on these variables, the tree height may be read directly. To eliminate the variable of distance, the chart may be worked out for 100 feet of *slope distance*. Heights are reduced or increased proportionally as the tree-observer slope distance varies from this. Measuring Tree Heights on Slopes, R. E. McArdle and Roy A. Chapman, Jour. of For., Vol. XXV, No. 7, Nov. 1927, p. 843.

density of the timber, the relative height of the trees, and the amount of underbrush on the ground. A recent study* shows that with care the average error need not vary greatly above 1 per cent. This is brought out in Table I.

Definite conclusions can be drawn as follows:

1. The average error for all height classes with all instruments is under 1 per cent.

2. All instruments show the least error with trees from 50 to 70 feet in height. Closer accuracy is also shown in the 120 foot class. This is believed to be due to the fact that the greater number of observations were taken from 1 chain (66 feet) and 2 chain (132 feet) set ups and that any problem of tree height

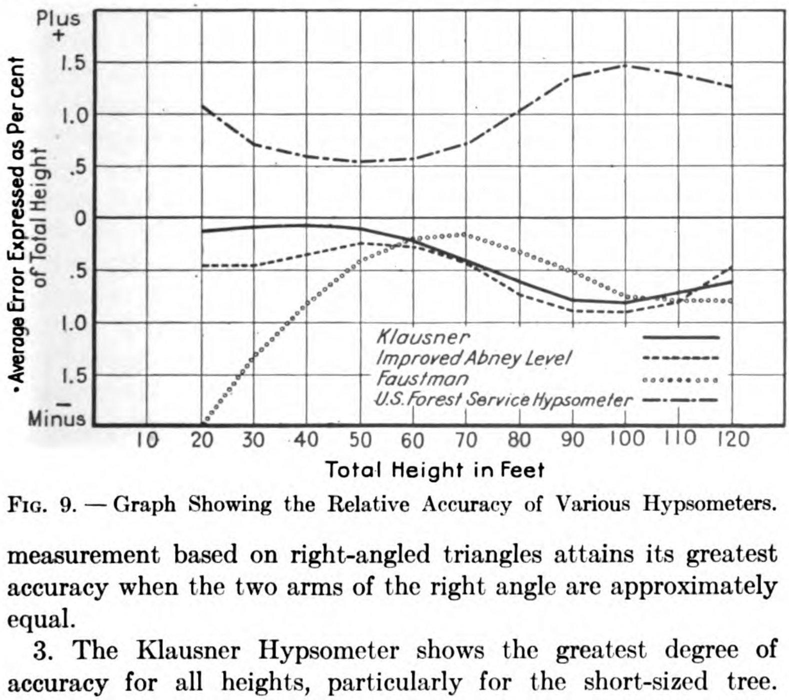


FIG. 9. — Graph Showing the Relative Accuracy of Various Hypsometers.

measurement based on right-angled triangles attains its greatest accuracy when the two arms of the right angle are approximately equal.

3. The Klausner Hypsometer shows the greatest degree of accuracy for all heights, particularly for the short-sized tree. It tends to underestimate slightly.

4. The improved Topographic Abney Level, though showing a greater variability than the Klausner, is an accurate and desirable

* A Study of the Relative Accuracy of Various Hypsometers with the Transit as a Standard, by C. C. Bennett and W. Seymour, Idaho Forester, Vol. XI, 1929, p. 25.

instrument and one that can be operated with great ease. Its tendency is to underestimate. To attain its highest accuracy it should be used with the support afforded by a Jacob staff fitted with ball-and-socket joint.

5. The greatest variation in errors by height classes is shown by the Faustman. Its tendency to underestimate is especially acute with trees less than 60 feet high, where preferably some other hypsometer should be used. On taller trees it shows a higher degree of accuracy than either the Abney or the Klausner.

6. The tendency of the U. S. Forest Service Hypsometer is to overestimate. The general trend of its curve is an inversion of that of the other three. It is probably the least accurate of the four.

7. In general where care is taken all four show a surprising degree of accuracy. The average error (unweighted) for all height classes is approximately from 0.53 per cent to not more than 0.7 per cent. On a tree 250 feet high this would mean a variation of less than 2 feet. If more accurate height data than these are desired, the tree can be felled and taped.

22. The Measuring Tape and Measuring Pole. — A steel or cloth tape, easily wound up and slipped into a pocket when not in use, is the simplest means of getting accurate measurement of length on felled trees and logs. A somewhat rougher, but nevertheless, quite accurate instrument used for determining length is the measuring pole. This is a square stick or light straight pole 8 feet long and graduated in feet and half feet. Many scalers prefer them to tapes in measuring the lengths of logs, for the purpose of reassuring themselves that the length specifications of the logging job are being maintained.

23. Cautions in Measuring Tree Heights. — The trees measured must be erect and in a vertical plane. This is especially true in measuring the heights of trees which lean either toward or away from the observer where errors involving a range of 30 per cent or more may be incurred according to the degree of inclination and the position of the observer. When it becomes necessary to measure the height of a leaning tree, a position should be chosen so that the vertical triangle of the tree is at right angles to the direction of inclination and a plumb line from the tip to the ground becomes the third side of the triangle. See Fig. 10.

Greater accuracy in measuring heights is maintained with the conical crowns of coniferous trees. A considerable amount of error is always incurred in measuring broad leafed trees due to their rounded or flat topped crown forms. It is almost a general rule that hypsometer measurements of standing broad leafed trees produce an overestimate of their height values. Practical woodsmen invariably measure windfalls or other felled trees as a check on their height determinations.

Three things are absolutely necessary to accurate height determination which cannot be over-emphasized:

1. Careful and accurate measurement of the required distance between the tree and the observer. This should be ordinarily in terms of true horizontal distance,

2. Care must be taken to hold the line of sight on the top of the tree while reading the height scale of the hypsometer. A steady hand and eye working in coördination are absolutely necessary,

3. Care must be taken to ascertain the position of the base of the tree. The observation on the base is just as important as that on the top. Check and re-check.

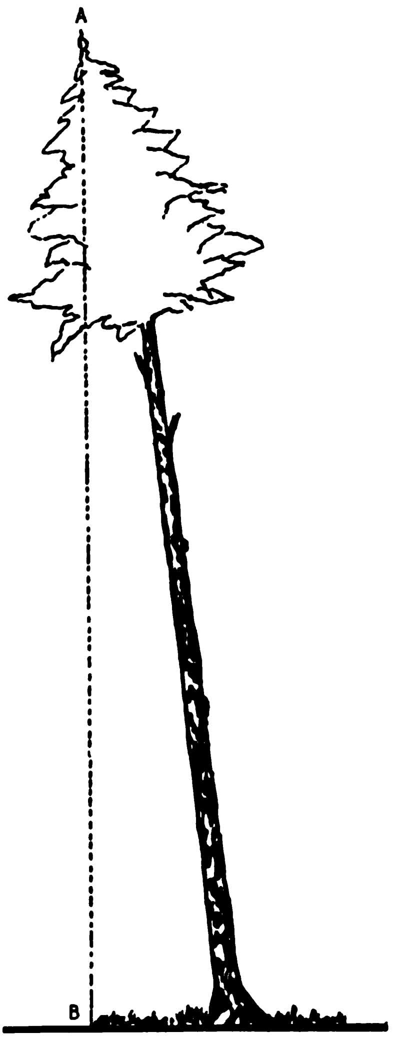


FIG. 10. — In Measuring the Height of a Leaning Tree, the Distance AB is the One which Must be Ascertained.

INSTRUMENTS FOR MEASURING VOLUME

24. The Scale Stick. — The scale stick is used to determine the volumes of logs. Scale sticks are usually made of hickory with a plain binding head of brass or iron. Along one edge, the stick is

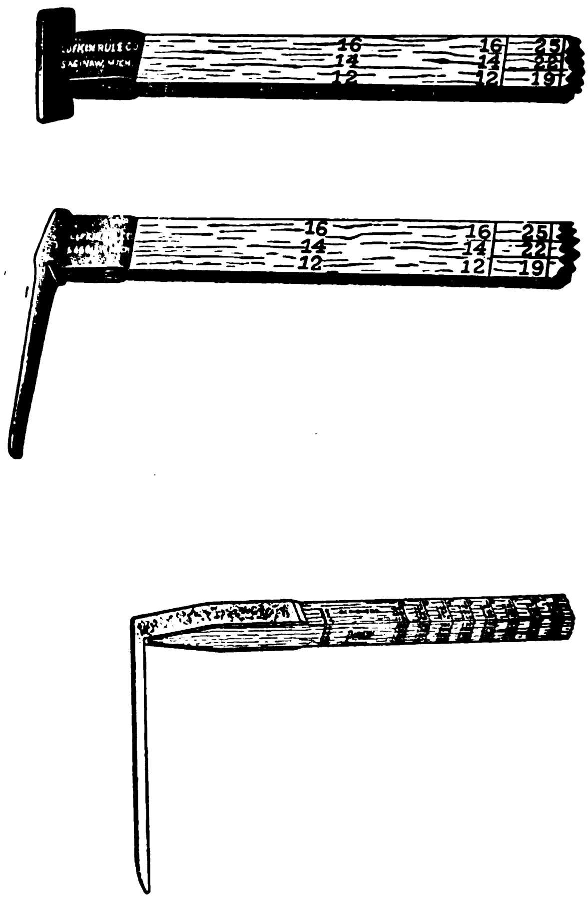


Fig. 11. — Scale Sticks.

graduated in inches. On the faces of the stick are shown the volumes in board feet corresponding to the different diameter values, and for various log lengths, generally from 8 to 16 feet.

The brass head of the stick is flanged; the advantage of such a head is that the rule may be quickly and accurately placed and caught on the end of the log. This catch type of head is only applicable directly when logs are peeled. When logs are scaled with the bark on, deductions must be made, or the rule laid more carefully on the log so that its zero coincides with the solid wood inside of the bark. Sometimes logs are nosed or sniped or pointed to facilitate removal from the woods and to prevent splitting in skidding or brooming in driving. With this type of scale stick a longer head must be used, one with a projecting metal arm at right angles to the stick, called the guide head. This acts exactly like the rigid arm on a caliper beam.

INSTRUMENTAL AIDS FOR MEASURING INCREMENT

25. The Increment Borer. — This is an instrument used for the purpose of studying the diameter increase in standing trees over a period of years. The one in most common use is known as the Swedish Increment Borer, having been developed by Swedish

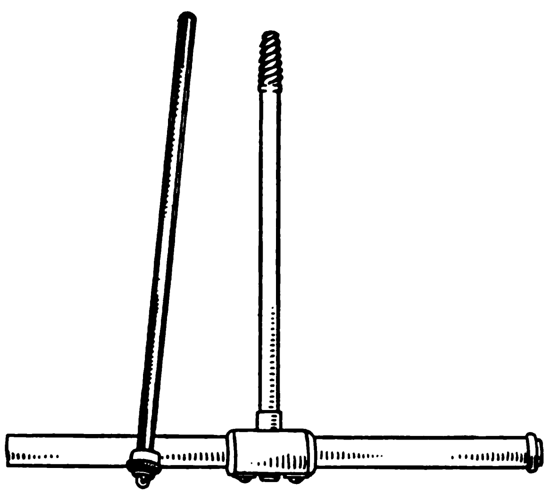


FIG. 12. — The Swedish Increment Borer.

foresters in their studies of growth and increment. In its improved form it consists of a hollow augur fitted with a detachable hollow handle of considerable length which serves the double purpose of giving leverage to the augur when in use and of serving as a receptacle for holding and carrying the augur and its fittings, when not in use. The augur is furnished with a long, narrow gouge or groove-shaped spatula wedge with a button head at one end and a set of shallow teeth on the curved up edges of the gouge at the other. The instrument is bored into the tree at the desired point in an endeavor to intersect the growing center of the tree in a plane perpendicular to its vertical axis. The spatula gouge is then thrust its full length between the core of the wood and the metal in the hollow portion of the augur. A short half turn of the augur is made backward to break off the core, which, by means of the spatula, is then withdrawn from the borer for further study.

Increment cores should be studied at once, and all significant data properly recorded at the time and point of observation. Several devices to facilitate this have been developed, one of the simplest* being a small block of wood of convenient size and sufficient length to take care of the longest increment core expected. The block is grooved along one edge sufficiently, to the depth of about one quarter of an inch. A scale graduated in

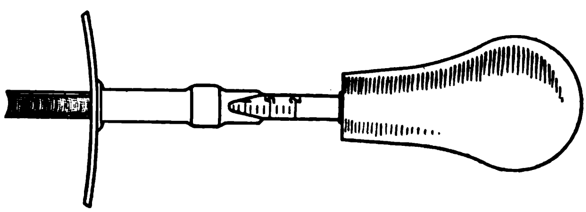


FIG. 13. — The Swedish Bark Measurer.

inches and tenths of an inch is marked along one edge of the groove. Trees bored can be marked with a timber scribe.

The Swedish Bark Measurer.† — A wooden handle carries a steel shank about 5½ inches long which ends in a gouge chisel edge. The steel shank has a movable brass sleeve fitted at its anterior end with a 3 by ½ inch steel plate curved to approximate the curve of the bole of a tree. When the instrument is thrust against the tree stem the chisel edge bores through the bark until the solid

* Described by R. R. Fenska, Jour. of Forestry, May, June, 1925, pp. 540–542. A more elaborate type, involving the use of clamping screws and a lens, has been described by Duncan Dunning, Jour. of Forestry, Feb. 1925, p. 183.

† This description and the drawing of Fig. 13 were made from an instrument loaned by Prof. T. S. Hansen of Cloquet, Minn., to whom due acknowledgment is made and appreciation extended.

wood is reached, while the sleeve, held by the plate, slips back on the shank for a distance equal to the depth of the bark. An engraved scale on the face of the shank permits a direct reading in inches and tenths of an inch. See Fig. 13. The use of this instrument permits a direct determination of the inside bark measurement of standing trees through the subtraction of twice the bark thickness measurement from the diameter as measured outside of the bark. This instrument was devised in Sweden, where it enjoys considerable popularity and use. It is a comparatively recent innovation in America.

CHAPTER III

UNITS OF VOLUME

26. Measurement and Volume. — The demand for any material implies its use, and use implies its measurement in any of its forms from raw material to finished product. Measurement is necessary in all transactions involving the purchase or sale of a given commodity. It is unfortunate that with wood as a commodity a wide variety of use and a lack of uniformity within the lumbering industry itself, in plants, in specifications, and in methods have developed a rather extended list of units by which forest products may be measured. The most important are:

Piece timbers
Cords
Cubic feet
Board feet

PIECE TIMBERS

27. Piece Timber and Volume. — The unit of volume in Piece Timbers is the individual piece which, meeting the definite specifications* of the buyer, can be cut directly from the log or tree. It is measured by tallying as an individual item. Finished piece products are classified as round, hewn, or sawed stock. The use to which the product is to be put sometimes determines its methods of manufacture. In some cases, as in mine timbers, two or even all three classes may be used on the same job. Although round, hewn, or sawed products could be measured by any other unit desired, such as cubic feet, cords, or board feet, the measuring is seldom done by them.

Round Timbers, especially where short lengths and small

* Shingle bolts, stave bolts, spoke stock, handle stock, tooth pick wood, matchwood bolts, spoolwood, etc. are generally cut, and often split, directly from the felled tree or from logs, to meet definite size specifications laid down by the manufacturers of such products. By definition they may also be classed as " piece timbers."

diameters can be used, represent the closest degree of utilization. They are usually intended for purposes where an inherent ability in the wood to endure moisture conditions in air and soil is combined with strength and resistance to weight and stresses of compression and shear. The bark is removed to lessen weight in transportation and to increase length of life.

Precise specifications both for dimensions of diameter and length as standardized, either by custom or agreement, regulate production and manufacture. Both maximum and minimum lengths that are acceptable are quoted.

In regard to diameter, only the minimum is quoted, because additional diameter dimension invariably means additional strength. Specifications will also cover a minimum of taper between the larger and smaller ends as well as precise instructions as to the kind and minimum amount of defect which can be present without degrading or discarding the piece. Almost any species of wood will be acceptable but will be classed as to native desirability. Those needing artificial preservative treatment to prevent decay will be separately graded.

Sawn or hewn timbers are used where one or more flat faces to be exposed to the weather are desired. Square timbers so used, though reducing the weight in shipment, undoubtedly increase the degrees of waste in utilization through the discarding of four slabs in removing the bark and rounded edges.

The more common classes of piece timbers are telephone poles, piling, posts and fencing, railroad ties and mine timbers.

THE CORD

28. The Standard Cord. — The cord is a unit of custom and convenience rather than a unit of volume. It is a unit of the space occupied by a pile of wood of specified dimensions rather than the contents of the sticks making up the pile. Its specifications are subject to considerable variation in different parts of the country, but unless otherwise stated, it shall be understood to be a pile of wood 4 feet high, 8 feet long, and 4 feet wide, occupying 128 cubic feet of space. This is known as the standard or stacked cord.

The cord as a unit of volume is adapted especially to transactions involving the purchase, sale, and utilization of bulk products whose ultimate use is in the rough, such as pulpwood, fuelwood, and acid wood. The individual pieces are known as billets, bolts, or sticks. The billets contained within the cord may vary in diameter, shape, form, and species, but will be uniform in length, a length equal to the specified width of the pile or stack making up the standard cord — 4 feet.

29. Long and Short Cords. — Cordwood may be cut to a longer length than 4 feet. Fuelwood, for example, is often cut 5 feet long because such length of sticks lends itself better to cross piling on a wagon. When this is done, a pile 4 feet high and 6½ feet long containing 130 cubic feet is considered as a standard cord,* and a pile 4 feet high and 8 feet long containing 160 cubic feet is considered as a *long cord.*

Such a long cord is approximately equivalent to 1¼ standard cords. Unless the sticks are cut in longer lengths than 4 feet either by agreement or custom, long cords are generally reduced to their value in standard cords. Bolts cut 8 feet long, for example, are measured in the pile as *double cords.*

When wood is cut less than 4 feet† in length and piled in stacks 4 feet high and 8 feet long, it is spoken of as a *short cord.* The standard cord length of the stick is the unit of purchase by the retail fuelwood dealers. Their unit of delivery to meet the requirements of the ultimate consumer is the 4 foot stick cut 4, 3, or 2 times into 12 inch, 16 inch and 24 inch lengths. A pile of these lengths 8 feet long, 4 feet high, and 1 tier deep is known as a *stove cord,* that is, a cord of stove wood of such and such length of sticks. Obviously, the stove cord never equals the standard cord. It will take as many tiers or ranks of the short wood as there have been cuts in the standard length to approximate the value of the standard cord and the stove cord is priced accordingly.

Dealers often sell as a cord short length fuelwood thrown *loosely* into a wagon box 4 feet by 8 feet by 4 feet. As pointed out by Hawley,‡ such measure practically never approximates the volume of the standard cord.

* In Quebec the cord dimensions are 8½ by 4 by 4½ feet, comprising 144 cubic feet. Acid wood is often cut in 4⅙ foot lengths and a pile 8 by 4⅙ by 4 feet or 133⅓ cubic feet is accepted as a cord. The standard cord is, however, recognized by the Quebec Forest Service as the basis for paying stumpage dues.

† Handle stock, for example, is cut in lengths varying from 12 to 40 and over 60 inches, and stove stock in lengths from 16 to 48 inches. These, however, are generally measured in terms of standard cords.

‡ Measuring Cordwood in Short Lengths, by R. C. Hawley, Jour. of Forestry, Vol. XVII, 1919, p. 312.

30. The Measurement of Cordwood. — The measurement of cordwood may be done either with the use of a tape or with a measuring stick. The latter is the more common method. It is required that the stack of piled wood shall show full measure, and, in piling, it is customary to pile a few inches (2–4) above an exact 4 feet of height to compensate for the settling of the stack.

When the wood is piled with the length of the sticks across the slope of a hill, the height of the piles should be measured perpendicular to the general slope of the ground.

Sometimes in measuring piles of wood, of which the length, or rather, depth, when piled, is standard, that is, 4 feet, only the height and length of the piles are measured, getting the superficial or square surface of the side of the piles. This is especially done with 12 inch wood. It is summed up in the expression "*surface feet.*"

31. The Solid Contents of the Stacked Cord. — Although the stacked cord occupies 128 cubic feet of space, it does not contain 128 cubic feet of wood. A certain proportion of the stack will be occupied by the air spaces between the individual sticks. Under the most ideal conditions a standard stacked cord would never contain more than 116 cubic feet of solid wood, and under ordinary conditions rarely contains more than 100 cubic feet. Even this figure presupposes a regularity of form and diameter in the individual sticks and a care in piling seldom met with in practice. The solid contents of the stacked cord vary with the following factors:

1. *The Species.* — A cord of softwood contains about 3 per cent more solid wood than a cord of hardwood, because of the greater smoothness and straightness of the sticks. Within these two groups there are several variations according to individual species.

2. *The Presence and Thickness of the Bark.* — Thick barked wood, such as hemlock, will contain less solid wood per cord than a thin barked species like balsam fir. Rossed* or barked wood (pulpwood) contains more solid wood material per cord than a cord of the same species unbarked.

3. *The Form of the Sticks.* — If the sticks are smooth and straight, there will be a greater amount of solid wood in the cord

* The process of removing the bark from pulpwood previous to grinding and cooking is known as "rossing."

than if there are present stubs or branches, crooks, knots, burls, forks, or any other irregularities of form which increase the air space in the stacks. Rossed wood is always much more regular in form than unbarked wood.

4. *Whether the Wood is Split or Not.* — On first glance one would think that since splitting increases the number of sticks required to complete the cord space, it increases the volume of the solid wood. Such is not the case, for splitting also increases the number of air spaces, and thus decreases the solid volume required to fill the cord. Split wood cannot be stacked as closely and as snugly as round sticks. The flat surfaces on the split wood tend to increase the number of open spaces between the individual sticks. If one had a cord of round wood and went through the operation of splitting each and every stick, when the cord stack was made up again, it would be found that there would be a number of pieces remaining which could not be included within the pile of the exact cord, thus indicating that there is less solid wood.

5. *The Length of the Sticks.* — The shorter the stick, the closer it can be piled, hence the smaller volume of air spaces and the greater number of sticks to fill the standard cord, that is, the greater volume of solid wood. Conversely, the longer the stick, the greater chance for irregularities of form, such as crook, or stubs, or knobs, which make close piling difficult, increase the number and size of the air spaces, and thus reduce the solid contents of the stack.

6. *Portion of the Tree Used.* — Stem wood or body wood in cordwood lengths taken from the main stem of the tree will show a greater volume of solid wood per cord than a pile made up of limb wood or branches.

7. *The Diameter of the Sticks.* — By actual experiment it can be demonstrated that for a given length of cordwood there is more solid wood in a cord made up of sticks of larger diameter than in one made up of sticks of smaller diameters. The aggregate of the volume of the larger number of air spaces will be found to be greater with the smaller sized sticks, hence there is less solid wood. This is true even with short lengths and when the sticks are even and straight.

8. *Seasoning.* — In piling cordwood for firewood, it is customary to make the stack higher than the standard cord height to compensate for settling and also for shrinkage in drying or season-

ing. Hardwoods shrink more than softwoods, due to a greater water content in their cells and intercellular spaces. Hardwoods shrink from 9 to 13 per cent, and softwoods 10 per cent in drying.

9. *The Method of Stacking.* — This is a matter in the technique of making piles where it is customary to put stakes in the ground at either end of the stacks. Piles with only one stake at either end have more solid wood per cord than piles made with two stakes driven into the ground at either end of the pile to hold it in place. This is due to the outcurving or outlapping of crooked sticks, which is greater with one stake than with two.

10. *The Method of Piling.* — This is an exceedingly important factor. Loosely piled wood will contain less solid volume than well piled wood. Often in piling by contract unscrupulous jobbers will endeavor to introduce into each pile as many crooked sticks and as much brushwood as possible, thus decreasing the amount of solid wood per cord and also, incidentally, the amount of work, necessary to make a cord. Another trick is to leave small twigs and branches on the sticks or to casually throw a piece of brush into the pile, all of which prevent close piling, increase the air space content and decrease the volume of solid wood.

No one of these factors will explain the variations in the volume of the stacked cord. Although each one plays its separate part, all must be taken together into consideration. In the inspection of cordwood for purchase, equal attention should be paid each of these factors. Probably it will be found for any particular job, that one greatly outweighs the others in the frequency with which it is found.

32. Variable Values of the Solid Cord. — No particular rule can be laid down for the amount of solid wood per cord. Various industries that buy wood by the cord, but whose main interest is solely in the amount of solid material present, have made many studies along this line. The results of a number of such studies are generalized as in Table II, p. 40.

33. Cords by Weight. — It has been proposed that a better measurement of solid volume with fuelwood could be obtained by weight rather than by stacked volume. The merits of the proposal are recognized, especially in view of the increasing difficulty of determining absolute volume with increasing irregularity of form and shape and decreasing size. Two factors operate against its speedy adoption: one the great differences in weight of different

TABLE II

SOLID CONTENTS OF THE STACKED CORD

Character of the individual sticks comprising the stacked cord	Amount of solid wood in the stacked cord in cubic feet	Aggregate of air spaces in the cord in cubic feet	Percentage of stacked cord in solid wood
First class split wood from round pieces 4 ft. long, 12 inches or more in diameter.	102.4	25.6	80
Second class split wood from round pieces averaging 6 inches in diameter and 4 feet in length.	96	32	75
First class wood unsplit, 6 to 15 inches in diameter general run of logs cut and piled in 12 foot lengths.	90	38	70.4
Heavy round branch wood cut in 4 foot lengths and averaging 4 inches in diameter.	87	41	68
Inferior branch wood cut in 4 foot lengths and averaging 4 inches in diameter.	77	51	60
Superior fagot wood in 4 foot lengths and running 2 to 4 inches in diameter.	50	78	40
Inferior fagot wood in 4 foot lengths averaging 1 to 2 inches in diameter.	25	103	19

species, volume for volume; and the other the variation within the same species between green and dry weight. In the best interests of the fuel trade, some standard of seasoning or moisture percentage would have to be adopted and honestly maintained. In some parts of the arid southwest, bone-dry juniper, pinon and mesquite, root, stem and branch wood, are sold by weight. This is mainly due to the fact that excessive crookedness and form lend themselves to no other accurate volume determination. The main argument for selling fuelwood by weight perhaps is that such practice would place it on exactly the same basis as coal or any other fuel.

34. Cord Measure and Hemlock Bark. — Hemlock bark for tanning is usually bought and sold by the cord. Instead of measuring the bark, that is, accepting 128 cubic feet as a cord of bark, some concerns have bought it by weight, establishing a standard of 2240 pounds as equivalent to a cord of bark. This is erroneous and productive of a variable standard, there being a lesser product in the number of cords for a given amount of green bark over a similar bulk of the bark seasoned. It is therefore to the interest of the purchaser to buy his bark well seasoned and of the seller to get rid of his bark as green as possible.

More modern practice is to measure the bark piled in the car or yard.

In eastern Canada, it is generally estimated* that there is 1 cord of bark for every 2000 board feet of lumber for saw material, or 1 cord of bark to every 4 cords of pulpwood.

THE CUBIC FOOT

35. The Cubic Foot as a Unit of Volume. — Inasmuch as trees and logs are solid bodies with measureable dimensions, they lend themselves readily to a determination of their cubic volume by use of the several formulas for cubing geometric solids. Since lengths are commonly taken in feet and diameters in inches, care must be taken to reduce one of them to values of the other in order that cubes of the product may be in proper terms. The cubic foot† is the most common unit so used. One thing to note is that cubic measure purposes to give the *entire* amount of wood material within the tree or log and disregards any loss or waste in the processes of manufacture. But with the exception of some costly

* The amount of bark derivable from a given acre of hemlock varies greatly, dependent upon the size and age of the trees. The older and bigger the trees, the thicker and hence greater yield of the bark. E. H. Frothingham of the U. S. Forest Service made a study of this in connection with his report and technical study of the eastern hemlock. (Forest Service Bulletin 152.) Reference to this publication will give more exact information than can be given here.

† In Europe the cubic meter is commonly used.

imported woods, it is seldom used in commercial transactions in this country. It is, however, used extensively by the forester in calculations of growth and yield, since he finds this unit most adaptable to his computations. The nearest approach to any extended commercial use of the cubic foot has been the suggestion from one section of the pulp industry that production figures of cost and yield should not be based on the cord, as is the prevailing custom at present, but on the yield per hundred solid cubic feet of pulpwood. This would establish a standard which its sponsors suggest naming as the " cunit."*

THE BOARD FOOT

36. The Board Foot Unit. — The Board Foot is the most commonly used unit of measure in American forestry and lumber practice. It is applied to measure the volume of sawed lumber and also to express the probable yield of lumber capable of being sawed from standing trees and from logs. Theoretically, it is a board 12 inches wide, 1 inch thick, and 1 foot (12 inches) long. Thus, a board 1 inch thick, 12 inches wide, and 12 feet long will contain 12 board feet, and a 16 foot board of the same width and thickness will contain 16 board feet. From this we derive the formula that the number of board feet contained within a board is equal to the product of the thickness in inches by the width in inches by the length in feet, divided by 12:

$$\dagger\ \text{Volume in Board Feet} = \frac{T(\text{inches}) \times W(\text{inches}) \times L(\text{feet})}{12}$$

For 1 inch boards, since the thickness is always one or unity, there would be as many board feet as there are square or superficial feet on one face of such board. This fact has given rise to a somewhat loose interchange of the terms " board feet " and

* What is a Cord? by R. S. Kellogg. Jour. of Forestry, Vol. XXIII, Nos. 7–8, July–August 1925, p. 608–610.

† The solid contents of a piece of lumber are equal to $T(\text{inches}) \times W(\text{inches}) \times (L(\text{feet}) \times 12)$ which gives its volume in cubic inches. Since the ratio of board feet to cubic inches is 1: 144, this value must be divided by 144 to read

$$\frac{T(\text{inches}) \times W(\text{inches}) \times L(\text{feet}) \times 12}{144}$$

which, by solution becomes $V = \dfrac{T(\text{inches}) \times W(\text{inches}) \times L(\text{feet})}{12}$.

"superficial feet" as meaning one and the same thing. They only mean one and the same thing when applied to boards cut *one inch thick.*

Lumber is not cut to but one standard width, nor to but one standard thickness, any more than it is cut to but one standard length. All widths, thicknesses, and lengths come out of the same mill from the same log according to the sawyer's ability to get the best yield from the timber. Lumber when cut to the thickness of 2 inches is generally spoken of as "*Dimension Stock*," and when cut to thicknesses greater than 2 inches it is spoken of as "Timbers." Both dimension stock and timbers are measured in terms of board feet by exactly the same formula:

$$V = \frac{T(\text{inches}) \times W(\text{inches}) \times L(\text{feet})}{12}$$

Thus a 2 by 8 whose length is 16 feet* would contain $21\frac{1}{3}$ board feet or, more properly, 21 board feet, the fractional value of a board foot being customarily dropped; and a timber 12 inches by 14 inches by 32 feet long would be calculated:

$$V = \frac{T(\text{inches}) \times W(\text{inches}) \times L(\text{feet})}{12} = \frac{12 \times 14 \times 32}{12} = 448 \text{ b.f.}$$

For the purpose of simplifying the calculation of the board foot contents of a run of lumber which may show considerable variety of dimension, graders or inspectors generally equip themselves with a special measuring stick called a "board side rule" or, more briefly, a board rule. This is a flat stick fitted with a brass catch head on which are engraved board foot volumes corresponding to the dimensions of length, width, and thickness.

The commercial transactions involving the purchase and sale of lumber are generally in large quantities. This has given rise to the custom of speaking of lumber in terms of thousands of feet, and the Roman numeral "M" is used as the abbreviation. The cost of lumber, for example, is quoted at so much per "M," and standing timber is customarily bought or sold as so many thousands of feet board measure, abbreviated to "M per b.m."

* Although timbers may be cut to any width and thickness and length dimensions, it is the more common custom in the lumber business to saw to only even width, length, thickness. With boards and dimension stock, it is the usual custom to saw even widths and even lengths. This is undoubtedly a cause of serious loss or waste, both in the woods and in the mills.

37. The Limitations of Use of Board Foot Measure. — The peculiar thing about the board foot unit is that when applied to logs, it endeavors to state volume in terms of the ultimate product. This is not done in any of the other units of volume so far discussed. Theoretically, the ultimate product in lumbering and sawmilling is boards, boards **one** inch thick.

As late as 1805 standing timber and logs were measured in terms of cubic feet, but their main product seems to have been inch boards, so that the custom was undoubtedly instituted first of estimating and later of measuring the *productive possibilities* of logs and trees in terms of the boards, or rather the number of board feet, they were capable of yielding. By 1850 this custom was firmly established in the lumber industry; and, hallowed by tradition and entrenched by custom, it remains so to this day.

The main objection to its use when applied to standing timber or to logs is that it never gives the *entire volume* of the wood material contained within the tree or log. Obviously this is so, because, stating volume in terms of the ultimate product, it must necessarily disregard all material wasted in the process of manufacture, slabs, edgings, trimmings, and sawdust.

38. Variation in Board Foot Contents of Logs. — The number of boards or the amount of lumber measured in board feet which can actually be cut from a tree, (or the logs from the tree), depends in the first place upon the relative dimensions of diameter or length — the larger the size, the greater the volume in board feet. But within logs of exactly the same diameter dimensions and log length there can be great variation in the total number of board feet actually cut due to several factors.

1. *The Relative Freedom from Defects and Character of the Log Itself.* — Logs are seldom perfect in condition, but show defects of rot, large knots, worm holes, fork and crook. Boards or lumber to be merchantable must be sound, free from defect, or any other factor which reduces their structural value. Hence such defects, when they occur, cannot be utilized and must be cut out and thrown away as waste, thus reducing the possible yield.

2. *The Thickness of the Saw or Saws Used in the Manufacturing Process.* — At each cut of the saw an amount of wood material equal in width to the thickness of the saw is wasted as sawdust. The actual width or path cut out at each cut of the saw is known as the **saw kerf**, or more briefly, the **kerf**. The thicker the saw, the

wider is the kerf, the greater the loss or waste of otherwise usable material. The employment, for example, of a circular saw with $\frac{5}{16}$ inch kerf as the head saw in a mill represents approximately the loss of a one inch board in sawdust for every three actually cut, whereas a rotary band saw with a $\frac{1}{8}$ inch kerf means but the loss of one board in sawdust for every eight boards cut.

3. *The Degree of Waste from Slabs and Edgings.* — In addition to being sound and free from defect, manufactured boards must be square and free from bark edge or waney edge or, more briefly, **wane**. Part of the bark which surrounds the log is removed in the manufacturing process known as slabbing, which, reduced to simplest terms, means simply the acquiring of two parallel flat surfaces on opposite sides of the log by as many cuts of the saw. The log could be four-square, that is, rid of all bark by four slabs, and then sawed through and through, but this is a slow and wasteful method. The more common custom is to slab but twice on the headsaw, removing the bark edges in the edger when the wide boards are sawed into standard widths. The waste in slabbing and edging varies with the kind of lumber to be sawed. There is less slab waste when inch boards are being sawed than with dimension stock or large timbers.

The heaviness of slabbing depends on a number of factors, the most important of which is the relative straightness of the log or its freedom from crook or sweep. Logs are seldom straight. A crook or sweep up to 2 inches in 16 feet has come to be accepted as normal crook. Ordinarily, no log will be accepted as merchantable in which crook exceeds 5 inches in 16 feet. With heavy crook there is always the possibility of a certain amount of recovery in short lengths taken from the heaviest part of the slab.

4. *The Thickness of the Lumber Sawed.* — It is evident that there is greater loss in sawdust when inch stock is cut than where either dimension stuff, 2 inch planks or large timbers are sawed, due to the necessity of making more cuts.

5. *The Width of the Narrowest Board Sawed.* — This board must be of an even inch width. The narrower the board which can be regarded as minimum width, the less the amount of material thrown out as slabs and edgings. That there is less waste in slabbing with narrow boards than with wide ones can be seen by referring to Fig. 14 which shows two logs of equal diameter and length. In one a 6 inch board represents the narrow-

est width accepted, whereas from the other 4 inch widths are taken.

6. *The Skill of the Sawyer.* — From the standpoint of production the sawyer is the most important man in the mill. He must be a man of wide experience, accurate perception, and quick judgment. Each log as it comes to him presents a new set of problems demanding instantaneous solution, the result of which means the number of boards and the number of board feet he will get from the log. Two men of equal skill might attack the same log differently and yet obtain the same result. But two men of varied skill, no matter how they tackle the problem, would get

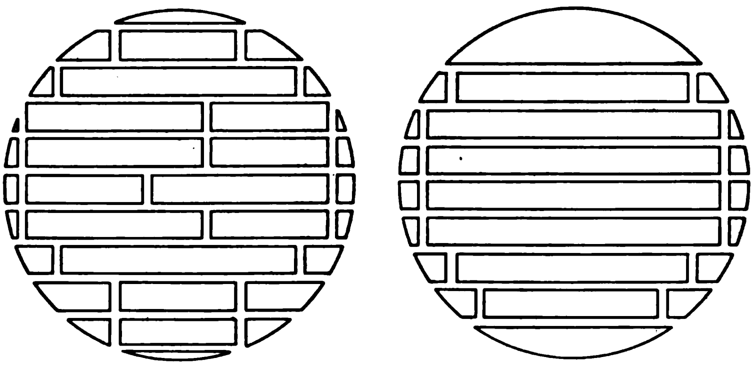


FIG. 14. — In the 12 Inch 12 Foot Logs Illustrated, the One on the Left was Cut to a Minimum 4 Inch Board Width and Sawed Out 68 Board Feet, While the One on the Right was Cut to a Minimum 6 Inch Board Width and Sawed Out 62 Board Feet, a Difference of 8.8 Per Cent.

widely varied results. That of the less skillful man would represent not only a much smaller board foot yield, but also a high degree of waste.

7. *The Efficiency of the Milling Machinery.* — High grade mills, when they are of the band saw type, will show throughout the highest possible efficiency in type and maintenance of machining. The band mills always show the better yields in board feet, due to their ability to use thinner saws and to waste less wood material in sawdust. At the other end of the scale is the small portable mill with a thick circular headsaw, a wobbly carriage, poor skill in manipulation, and a high degree of waste. Boards when cut will not be of even thickness, edgings will show much waste through poor trimming on the headsaw, and the board foot

yield will be proportionally small. Between these two extremes are all types of mills having a relative degree of efficiency.

8. *The Taper of the Tree or Log.* — Full bodied logs with little taper will yield the volume capable of being sawed from the cylinders represented by their top diameter and length, butt logs with large or excessive taper* offer opportunity for manufacturing and utilizing short length boards whose normal position in the log falls outside of this cylinder. Consequently, rapidly tapering logs of equal top diameter and similar length will yield the more board foot contents.

9. *Shrinkage.* — Green wood just sawed from the log contains a large amount of water. This will be lost through evaporation in the seasoning process, and a certain amount of shrinkage from the sawed-out dimensions is to be expected. It is, however, a minor variable and may be allowed for by adjusting the saw milling machinery to cut slightly oversize.

39. Lumber Grades. — The purpose of sawing trees and logs into boards and lumber is to meet the public demand for wood material and to sell these products at the highest price obtainable in the best market available.

This price is not the same for every board foot or for every thousand board feet sawed out of a given run of logs. Some portions of a tree are capable of yielding a very much more desirable class or **grade** of lumber, beautifully clean, clear, and free from the slightest defect; other portions of the tree, particularly in the upper sections, yield lumber full of knots, seams, pitch streaks, etc. Naturally, the more desirable grades command the higher prices. The main determinant of lumber grades is the presence or absence of certain defects. The standard defects recognized in grading lumber are:

1. *Knots,* of arbitrarily predetermined sizes, firm or loose, sound or unsound, and knot holes.

2. *Rot or decay,* indicated by a definite softness or dodiness in the wood, or by whitish spots similar to small pin holes.

3. *Heart Stain,* a marked and general discoloration of the heartwood.

4. *Pitch Streaks,* seams or pockets; well defined openings and accumulations, in the wood, of pitch, either liquid or solid.

* In modern, up-to-date mills, the practice of "taper sawing" materially cuts down the loss due to this factor and reduces the consequent waste.

5. *Sap Stain,* particularly blue stain.

6. *Worm holes,* caused by certain boring insects.

7. *Wane,* bark edge or rounded edge retaining some portion of the bark.

8. *Shake,* a separation of one annual ring from another.

9. *Checks,* or cracks caused by seasoning or some one of manufacturing processes.

10. *Manufacturing Imperfections,* noted principally on the face of a board consisting of chipped grain, torn grain or loose grain.

11. *Improper Sizes,* slightly oversize to allow for shrinkage in seasoning may be allowed, but undersize always degrades lumber.

The foregoing are the principal defects recognized in the grading of lumber on the Pacific Coast. Any one of them or combination of them which may reduce the utility and desirability of lumber will serve to reduce its grade or value.

In order to standardize the consideration and application of the various defects found in lumber, regional groups of lumber manufacturers, all handling the same types or species of timber, have established standard lumber grades for their products with standardized rules or definitions of procedure for defining such grades.

TABLE III

CONVERTING FACTORS FOR STANDING TREES*

(Cubic Measure to Board Measure (Values Curved))

D.B.H. in Inches	1 Cubic foot is equivalent to Number of Board Feet	D.B.H. in Inches	1 Cubic foot is equivalent to Number of Board Feet
6	4.50	17	6.20
7	5.00	18	6.25
8	5.25	19	6.30
9	5.50	20	6.35
10	5.60	22	6.40
11	5.70	24	6.45
12	5.80	26	6.50
13	5.90	27	6.55
14	6.00	28	6.60
15	6.10	29	6.65
16	6.15	30	6.75

* Frothingham, U. S. Dept. Agr. Bull. 152.

40. Converting Factors. — Converting factors are ratios or multiplying factors for converting values into terms of another unit, as, for example, from cubic to board feet. Table IV * shows a list of standard converting factors which have been adopted by the U. S. Forest Service.

TABLE IV

STANDARD CONVERTING FACTORS

Product	Assumed Dimensions	Equivalent in Board Feet†
	Feet	
Standard cord	4 × 4 × 8	500
Long cord	4 × 5 × 8	625
Cord, shingle bolts	4 × 4 × 8	600
Cord, fuelwood bolts average 5 inches diameter	4 × 4 × 8	333⅓
	Inches × Feet	
Railroad tie	7 × 9 × 8	35
Railroad tie	7 × 8 × 8	30
Railroad tie	6 × 6 × 8	20
Pole or piling	8 top × 45	200
Pole or piling	8 top × 35	100
Pole or piling	7 top × 60	280
Pole or piling	7 top × 50	200
Pole or piling	7 top × 40	100
Pole or piling	7 top × 30	60
Cubic foot		6
Derrick pole	7 top × 30	60
Derrick set (11 pieces)		480
Fence post	6 × 7	7
Fence brace	4 × 6	2
Fence pole	4 × 20	10
Converter pole	4 × 20	10
Prop	6 × 10	10
Lagging (6 pieces)	3 × 6	10

* Taken from Standard Instructions of Scaling National Forest Timber, 1928.

† These figures are general only. The actual number of board feet in any piece of timber of specified dimensions varies with the log rule used and the relative taper. For any cord the equivalent board foot value varies with the log rule, the size of the sticks, and the manner of piling.

CHAPTER IV

LOG RULES

41. Board Foot Volume in Logs. — It can be readily appreciated that under any given set of conditions the measuring of the contents of logs in terms of any unit previous to manufacture is only an approximation of the yield which may possibly be sawed from them. The actual results will vary from the estimate directly as the physical conditions surrounding the sawing operation vary from those upon which the estimate was based. The volume contained in any given log is determined directly from a log rule.

A log rule or *log table* is in the last analysis a *standard* of measurement. Briefly stated, it is a tabular or tabulated form which shows the volume contained in logs of specified diameters and lengths. The unit in which this volume is stated may be cords, cubic feet, standards, or board feet as desired. Generally log rules are in terms of board feet. An important thing to remember about a log rule is that it is accepted as a standard for any species through custom and agreement rather than on account of its absolute accuracy.

A log rule as conventionally presented is based on diameter dimensions and length. The log lengths are generally in terms of *even foot* values — 8, 10, 12, etc. The diameter dimension is almost always taken inside the bark at the top* end of the log. All diameter values are taken from the smallest size of log from which it is practicable to saw boards or dimension stock, up to the largest met with in actual practice. The diameter values are generally stated down the left-hand column, while the log length values are listed across the top. This form is not necessarily standard and the relative positions of the two dimension values may be reversed.

42. Relative Accuracy of Log Rule Volumes. — The amount of lumber actually sawed out of a log of given diameter and length rarely tallies and often is hardly more than approximated by the

* When not taken at the top it is taken at the middle, that is, at a point midway between the smaller and larger ends of the log. Whichever dimension is taken, the log is then usually considered as a cylinder of such and such dimensions.

values given in the log rule tally. When the amount of lumber actually sawed from a log or a run of logs exceeds the volume figures previously scaled in the log scaling, that is, when the mill tally exceeds the log tally, the surplus of the former over the latter is known as the *overrun*.

When the log tally exceeds the mill tally, that is, when the volume measured in the log proves to be greater than what the mill actually saws out, the deficit between the mill scale and the log scale is known as *underrun*.

Mill overrun is a varying factor from mill to mill, from species to species of tree, and from one section of the country to another. In reality, it is an aggregate of several factors all working together to add to the mill scale material not considered in the log scale. It is made up by:

1. Less waste in sawdust from using a saw with a kerf less than the standard for the log rule used.
2. Less waste in sawdust from fewer cuts through cutting two inch planks, dimension stock, and other large timbers.
3. Less waste in slab through sawing out narrower width boards than those included in the computations of the log rule used.
4. An increase in yield and hence less net waste through utilization of short lengths taken from the taper of the logs outside of the cylinder considered by the log rule.
5. Any local or trade practice of cutting of scant widths and thicknesses thus increasing the potential yield. Cutting and selling $\frac{7}{8}$ inch stock as inch lumber is quoted as an example.
6. Utilization of cheap grades, perhaps to meet some local or special demand, which admit of considerable amounts of rotten heart, sap or wormholes, not commonly acceptable in the trade.

43. The Mathematical Basis of Log Rules. — Theoretically, at the outset, logs are considered as cylinders, the factors of taper and normal crook being for the time disregarded. The diameter of such a cylinder is assumed to be that of the top diameter of the log. By application of the principles of solid geometry the volume of the cylinder computed from

$$V = \frac{\pi D^2}{4} \times L$$
$$= 0.7854\ D^2 \times L$$

where D = the top diameter of the log in inches.
L = its length in feet.

But L is in feet whereas D is in inches. Further, in order to convert to terms of board feet the whole formula must be divided by 144, the ratio of cubic inches to board feet, hence,

$$V = \frac{0.7854\ D^2 \times (L \times 12)}{144}$$

$$= 0.7854\ D^2 \times \frac{L}{12}^*$$

This formula states the theoretical board foot volume of logs when no allowance has been made for sawdust, slabs, edgings, and other losses, either incidental to the normal processes of manufacture or arising from the normal non-conformity of logs to cylindrical form. It follows that in order to use this formula as the basis of any computation of actual board yields from logs, certain deductions or corrections must be made in proportion to the value of the anticipated losses. In all formulas or log rules which have any claims to accuracy this has been done. Unfortunately there has not been an agreement of opinion as to the method to be followed, nor the amounts to be deducted in making such corrections.

The factors which must be considered and from which loss is incurred are:

1. *Sawdust.* — Each cut of the saw through the log removes a measurable volume in sawdust. This loss is directly proportional to the cross-sectional area as expressed by D^2.

2. *Slabs and Edgings.* — The most important factor influencing this loss is the standard of utilization as expressed in the width of the narrowest board sawed, and accepted as merchantable. The wider this board, the wider and deeper the slab. The loss in edging is closely connected with the loss from slabs, since the greater the number of boards produced, the greater the number of edgings. The actual amount of loss in slabs and edgings depends absolutely upon the size of the log and is in direct proportion to its " bark area."

3. *Crook.* — Logs are seldom perfectly straight. A certain average departure from straight form should be recognized as " normal crook." The main effect of crook is to increase the loss in slabs and edgings.

* With 12 foot logs the formula becomes $V = 0.7854\ D^2$.

4. *Thickness of Lumber Sawed.* — When a log is sawed into two inch planking or into larger dimensioned stock, less material is wasted in sawdust due to fewer cuts of the saw, and hence a larger net yield, measured in board feet, results. Almost all log rules are constructed so as to state their yield in terms of one inch boards. Unless this factor is recognized and due allowance is made, discrepancies may appear which will throw a cloud of suspicion over the results of an otherwise carefully constructed and accurate log rule.

5. *Taper.* — When logs are scaled as cylinders, the diameters of which are those at the top ends of the logs, the factor of taper, or increase in diameter from the top toward the butt ends, is totally disregarded. The greater the degree of taper and the longer the log length, the greater the opportunity to increase the total board foot yield of the log from short boards taken out of the heavy slabs outside of the log cylinder. Very few log rules give this factor any consideration.

6. *Shrinkage.* — Since logs are usually scaled green, definite allowance should be made for the subsequent shrinkage in width and thickness. Sawmills may overcome this by setting their saws to cut oversized stock ($\frac{1}{8}$ to $\frac{3}{8}$ inches in the width, and $\frac{1}{16}$ to $\frac{3}{32}$ inches in the thickness). Unless similar adjustment is made in log rule computation, error will result.

The great variations in opinion and method of making deductions for these various factors have led to a multiplicity of log rules until some fifty different rules have been used in this country in the lumbering industry at one time and another. The greater number of these are now obsolete, due either to their inferior accuracy or to their inability to oust those entrenched by long use and public custom. Only the more important of them will be discussed.

44. Methods of Constructing Log Rules. — There are five ways in which log rules may be constructed:

1. By mathematical formula.
2. By constructing diagrams drawn to scale.
3. By empirical mill studies.
4. By basing volume on arbitrary standards.
5. By combining or adapting existing log rules.

FORMULA RULES

45. The Doyle Rule. — The Doyle rule is one of the oldest of the formula rules. It has enjoyed a wide use throughout the eastern and southern United States and has been adopted as the official statute log rule of the state of Arkansas and of the Canadian province of Ontario. Until 1914 it was the official log rule of the state of Louisiana. It has not been used in the West to any great extent. Its formula is:

$$\text{B.M.} = \left(\frac{D - 4}{4}\right)^2 \times L$$

where B.M. = volume in board feet.
D = diameter in inches.
L = Length in feet.

Stated directly, this rule reads: " Deduct 4 inches from the top diameter of the log inside the bark as an allowance for slab and edging. Square one quarter of the remainder and multiply by the length in feet. The result is the contents in board feet."

The rule is probably the least accurate log rule in common use. Its fundamental errors lie in deducting too great a proportion for slabbing and not enough for sawdust. Taper is entirely disregarded. The result of this is that the rule shows great inconsistency with the amount that can actually be sawed out. For logs 6 inches in top diameter the mill tally overruns the log tally by 400 to 480 per cent. At 9 inches top, the overrun is approximately 100 per cent; at 12 inches, 50 per cent; and at 20 inches, 20 per cent. The overrun steadily diminishes until at 30 to 36 inches top the rule practically holds to the mill scale. Above this value there is a gradually increasing underrun.

The wide prevalence of its use was probably* due to its easy adaptation by rule of thumb, especially in application to 16 foot logs, the standard log length used by timber cruisers in their estimating. For 16 foot logs they simply dropped 4 inches from the estimated top diameter and squared the remainder, or,

$$\text{B.M.} = (D - 4)^2$$

* In an early edition of its publication it was misnamed and called the "Scribner Rule," a log rule which was then enjoying considerable use and popularity. This may also have been a contributing factor.

46. The International Rule. — This rule, which was first published by Dr. Judson F. Clarke in 1906, represents a definite endeavor on the part of its author to construct a log rule which would meet modern conditions of milling and manufacture. In addition to considering deductions for slabs, edging, and sawdust, allowance was also made for shrinkage and for normal crook. Dr. Clarke started with the premise that in modern band mills the proper saw kerf would be $\frac{1}{8}$ of an inch. He then deduced that, with such a kerf and with $\frac{1}{16}$ inch allowed for shrinkage in seasoning, the residual volume available in boards would be $0.842 \times 0.7854\ D^2$ or $0.66\ D^2$ when D was the top diameter of the log. From this he deducted for slabs and edgings and for loss due to normal crook a plank equivalent to $2.12\ D$. The formula then reads:

$$\text{B.M.} = (0.66\ D^2 - 2.12\ D) \times \frac{L}{12}$$

In order to compensate for taper, the log was divided into 4 foot sections, a taper increase of $\frac{1}{2}$ inch in diameter being allowed for each 4 feet of length. The volume for each 4 foot length is then:

$$\text{B.M.} = (0.66\ D^2 - 2.12\ D) \times \frac{4}{12}$$
$$= 0.22\ D^2 - 0.71\ D$$

This was found to set a higher standard of utilization than could be met under practical conditions. In 1917 Dr. Clarke modified his formula to allow for $\frac{1}{4}$ inch saw kerf. The original values of the $\frac{1}{8}$ inch rule were discounted by 9.5 per cent and then rounded off to the nearest 5 or 10 board feet. For 4 foot sections the formula now reads:

$$\text{B.M.} = (0.22\ D^2 - 0.71\ D) \times 0.905$$

In this form the International Rule* has been adopted by the Federal Land Bank of Springfield, Massachusetts, as the official rule by which timber shall be scaled in all appraisals undertaken as a part of its business.

* A bill authorizing this rule as the official log rule in cases of dispute in the state of New York was passed by the 1928 session of the Legislature and only failed of enactment through veto of the Governor for legally technical reasons. It was reintroduced and passed by the 1930 Legislature. Signed by the Governor it became effective July 1, 1930. As stated in the law the use of the International Rule is permissive rather than compulsory.

47. The British Columbia Rule. — This rule is the official log rule of the Canadian province of British Columbia. It is one of the few examples on record of legislative adoption for commercial scaling of a log rule developed on sound mathematical principles. When first developed it was based on diagrams but was subsequently expressed as a formula stating, for logs up to 40 feet in length deduct 1½ inches from the diameter of the small end inside of the bark; square the result and multiply by 0.7854. From this product deduct three-elevenths and multiply the result by the length of the log divided by 12, or,

$$\text{B.M.} = (1 - 3/11) \times 0.7854\,(D - 1.5)^2 \times \frac{L}{12}$$

$$= 0.727\,\frac{\pi\,(D - 1.5)^2}{4} \times \frac{L}{12}$$

For logs over 40 feet a taper of one inch for each half log length was allowed. The minimum width of board was 3 inches. The small allowance for taper constitutes the main weakness in the rule. It was said to be a concession to the low standards of utilization prevailing in the province at the time (1909).

DIAGRAM RULES

48. Log Rules Based on Diagrams. — This method is based on the idea of drawing rectangles to scale with due allowance for saw kerf, which represent the ends of one inch boards within circles representing the top diameters of logs of different inside bark dimensions. The board foot volumes for logs of different lengths are then computed, totaled and tabulated by diameter and log length classes. Even when carefully constructed a considerable variation within a given set of diagrams will be found due to factors inherent in the methods of diagram construction. The more important of these are:

(*a*) Whether a saw kerf or the rectangle representing an inch board is placed on the center line of the circle. With some diameter values one method gives better results; with other diameters, the reverse is true. The controlling factor is the relation existing between the sum of the board thicknesses plus the saw kerf and the diameter of the log.

(*b*) Whether the log is sawed through and through with subsequent removal of wane in the edger, or whether the log is slabbed on four sides and some arbitrary method of sawing around, approximating quarter sawing, is followed with utilization of narrow boards diagramed within the slabs.

(*c*) The width of the narrowest board sawed and acceptable under an arbitrary standard of utilization.

(*d*) The acceptance or rejection of fractional inch values in scaling board widths. If the boards are to be read and scaled to the nearest full inch only (dropping all fractional values) a considerable loss in volume will result.

Diagram log rules seek to state product in 1 inch boards of cylinders of specified diameters and lengths, after arbitrarily adopting standardized saw kerf, width of slab, and probably a minimum width of board. Taper is consistently disregarded and any waste for normal crook and irregular form cannot be diagramed or computed.

Most diagram rules show a considerable amount of irregularity in the increase of board foot values from dimension class to dimension class both for diameters and for lengths, no attempt having been made to round off such irregularities by graphic methods.

A certain amount of variation will also be found due to the custom of log rule constructors of increasing the width and thickness of the slab on large sized logs, a practice which does not conform to actual sawing methods. The more important of the diagram rules are now described.

49. The Scribner Log Rule. — This is one of the oldest log rules in terms of the board foot unit. It was published previous to 1846 and has enjoyed a wide and extended use in almost every timber-using state in the Union. It is the statute rule by legislative enactment of the states of Minnesota, Wisconsin, West Virginia, Oregon, Idaho, and Nevada, and is also the official log rule on all timber sales under the administration of the Forestry Branch of the Canadian Department of the Interior.

It is based on full sized diagrams for 1 inch boards with $\frac{1}{4}$ inch saw kerf. The original minimum width of board is not known and the rule, by later modification, incurred considerable error, by increasing the allowance for slab in large sized logs. Log diameters are taken inside the bark at the small end to *full* diameters, fractional values being disregarded. Taper is not considered, nor is crook. No attempt is made to even off the increases from dimension class to dimension class, and the log rule shows irregularity. For values below 28 inches the general tendency of the rule is to be consistent,* but with values above that figure an increasingly large overrun is experienced.

50. The Scribner Decimal C Log Rule. — This is the official log rule of the United States Forest Service on all of its timber sales. It was evolved from the old Scribner rule by the device of dropping off the last cipher of the tabular value and rounding off to the nearest 10. Thus a log scaling 173 feet on the Scribner scale would be tabulated as 17 and scaled as 170 board feet whereas one with 217 board feet would be read as 22 and scaled as 220 board feet. Any errors for individual logs are thus compensated in scaling a large run of logs.

A further development from the old Scribner rule was the extension of the rule in both directions to apply to logs smaller than 12 inches and larger than 44 inches top diameter, and for lengths shorter than 10 feet and longer than 24 feet. This modification was undertaken by E. A. Ziegler by methods based on graphic projection, and first published in 1910. In its present form, it gives the board foot volume for all logs from 6 inches to 102 inches top diameter, and for lengths from 6 to 40 feet, thus being available for use in scaling a large variety of timber.

51. The Spaulding Log Rule. — This is sometimes known as "The California Rule" and is the official statute rule of the state of California. It was first constructed by N. W. Spaulding in 1868 to apply to logs 10 to 96 inches in diameter and for lengths from 12 to 24 feet. The diagrams as scaled allowed a rather generous saw kerf of $\frac{11}{32}$ of an inch, a convention necessary for the type of saws and sawmill at the time. Taper was disregarded and further error was incurred by varying the slab width on large sized logs. Long logs were to be scaled by doubling the values of the table, but no lengths longer than 40 feet were to be so scaled. Due mainly to its generous allowance for saw kerf a rather larger overrun is experienced in modern band mills. But on the whole the rule is consistent and is generally regarded as a well constructed rule.

52. The Maine Log Rule. — This is sometimes known as the "Holland Rule" or the "Bangor Rule." Although its use is principally restricted to northern New England and the state of

* This is true when applied to logs 16 feet or less. For long length logs there is an increasing overrun.

Maine, it is one of the most accurate and consistent log rules in common use. It was constructed from diagrams calling for 1 inch boards and ¼ inch kerf with a minimum board width of 6 inches. No variation was made for slab width with increasing size, thus avoiding all fundamental defects inherent in the other diagram log rules heretofore discussed. Taper is disregarded and its application to long logs is best accomplished by dividing them into smaller length sections, preferably of 16 feet.

53. The Quebec Log Rule. — This rule was constructed from two sets of diagrams for logs from 6 to 40 inches. One set was for 1 inch boards and the other for 3 inch planks or deals. The average values of these two sets for corresponding dimensions were subjected to certain arbitrary deductions, the maximum of which did not exceed 17 board feet, and were incorporated in the statute log rule of the province. Diameter values are based on the mean diameter of the two ends of the log. Taper is neglected.

Modern developments in the province of Quebec are away from a board foot rule. The Quebec Forest Service is at present engaged on a study of the measurements of some 4000 logs with the purpose of developing a log rule based on the cubic foot unit.

54. The New Brunswick Log Rule. — This log rule, which is the official statute rule of the province, is of rather obscure origin, but seems to have been derived from diagrams. When originally constructed it did not allow for values of logs smaller than 10 inch top diameter. Subsequent legislative enactment fixed values down to 5 inches top on a more or less arbitrary basis. Taper is not considered. The yield is in 1 inch boards. Logs longer than 26 feet are scaled in two lengths.

MILL TALLY LOG RULES

55. Log Rules Based on Mill Tallies. — A log rule based on the actual measurement of a run of logs through a mill has the advantage of stating what actually has been done rather than of making an approximation of possibilities. Unfortunately its application is restricted to the region, the species, and the manufacturing conditions of saw kerf, board width and thickness, width of slab, etc., prevailing at the time of the original study. Vary these conditions in any particular and marked discrepancies of the mill scale from the log rule will at once be revealed.

The Massachusetts Log Rule. — This is the only log rule constructed on a basis of mill tallies that has enjoyed any acceptance and use. It is based on the measured boards actually sawed out from some 1200 logs. The yield is in terms of 1 inch boards, round edged and square at the ends. The boards were measured on their *average* face thus most fully utilizing the factor of taper and at the same time reducing the factor of slab to a minimum. In actual practice the rule works best with second growth eastern white pine under conditions of close utilization.

STANDARD LOG RULES

56. Log Rules Based on Standards. — The earliest measurement of timber in this country of which we have any record was in terms of cubic measure. Due to the fact that the shape of a log roughly approximated the shape of a cylinder, the custom seems to have grown of accepting a log of specified dimensions as unity instead of the simpler unit of the cubic foot. This had the effect of establishing an arbitrary standard in place of a fundamental one. The volumes of measured logs varied with those of the standard as the squares of their respective diameters and lengths and could be expressed in a log rule in terms of multiples and decimals of the standard. The fact that board foot measure is never expressed directly, coupled with the fact that the ratio between board foot volumes and solid contents increases with diameter, thus offering no set conversion factor, and their rather local application, provide the main weaknesses for this type of log rule.

The Glens Falls Standard. — This is sometimes known as "Dimicks Standard," sometimes as the "Adirondack Market." It seems to have been devised by one Norman Fox who lumbered on the Sacandaga River and Upper Hudson River watersheds about 1814 to 1821. The log which he established as a standard was one 19 inches on the top end and 13 feet long. Why these dimensions were chosen is not definitely known.* The conventional custom in using the Adirondack Market was to allow 5 standards as the equivalent of 1000 board feet of sawed lumber and 3 standards were allowed as the equivalent of a stacked cord.

* In the 1846 edition of the Scribner Rule a log 13 feet long and 19 inches at the top end was given a volume of 196 board feet or approximately 200 board feet.

The Saranac Standard. — Another standard in wide use in New York State was known as the Saranac Standard. It was used especially in the northern section of the state in the region adjacent to the Saranac River drainage. It was based on a log 12 feet in length and 22 inches at the top end. By convention and custom 4 Saranac Standards were allowed as the equivalent of 1000 feet board measure.

The Quebec Standard. — This was based on a log 12 feet long and 20 inches top diameter. Its origin is ascribed to the same Norman Fox who is responsible for the Adirondack Standard. Sometime subsequent to 1821 Fox moved to Lower Canada and went into the lumber business there. Five of these standards were allowed to the 1000 board feet. It is supposed that the extra volume achieved in selecting a somewhat larger top diameter was developed by Fox to take care of certain discrepancies arising from neglect of taper and normal crook in the older standard.

The Blodgett Log Rule. — This is the official log rule of the state of New Hampshire and is based on a log 16 inches in diameter and 1 foot long, which is called the Blodgett foot. It was originally purposed to accept 100 Blodgetts as being equal to 1000 board feet. But this could not be met in actual practice and by agreement 115 Blodgetts are now accepted as being the equivalent of 1000 board feet. Peculiarly among log rules this rule calls for the measurement of diameter not at the top or small end, but at the *middle point* of the log.

ADAPTED LOG RULES

57. Combination Log Rules. — Sometimes in order to meet certain local conditions two existing log rules will be combined. Another method is to take an existing log rule and to correct or adapt it to local conditions by means of mill tally values obtained in sawing. Sometimes the corrected or combined rule is given a new name, sometimes not.

The Doyle Scribner Log Rule. — In an effort to devise a standard of measurement which will compensate for loss in defective and over mature timber, recourse has been sought in a log rule which will state lower values than can customarily be sawed out of sound timber. For small sized timber the Doyle rule meets this condition, but with large sized logs it is not considered safe. The Scribner rule, though not considered desirable for small sized defective logs, gives an increasing overrun with large sized material. By combining these two rules, the low values of the Doyle rule are used up to 28 inches top diameter, and above that figure the recognizedly low values of the Scribner scale.

The Scribner Doyle Log Rule. — Exactly the opposite process has been adopted in the Scribner Doyle which has been the official log rule of the state of Louisiana since 1914. The Scribner Doyle offers no advantage to the straight Scribner scale with logs less than 28 inches top diameter.

CHAPTER V

MEASUREMENT OF VOLUME — SCALING

58. Purpose. — The measurement of logs for the purpose of ascertaining their volume contents is known as scaling. By *volume contents* as herein used is understood only such material as will make sound lumber, boards, planks, dimension stock, pulpwood, fuelwood, etc., desirable for purchase or sale. Timber is scaled in strict accordance with the amount of defect present in the log, since defect is the prime factor in reducing merchantable values, and not on the basis of any particular grades of lumber which it might produce. Scaling is a determination of quantity rather than quality values.

A man who makes a business of measuring the contents of logs is known as a **timber scaler** or, more briefly, a scaler. Scaling is not an art which can be picked up within a few weeks, or a few months, but requires both intelligence and experience. In several of the Canadian provinces where log rules and scaling procedure are matters of statute law, from three to five years' apprenticeship with a registered scaler is required before a young man is allowed to take the government examinations for his official scaler's certificate.*

Scaling timber is a very important procedure in the process of manufacturing lumber. It is on the basis of the measurements and report of the scaler that the jobber receives payment for the logs he has cut and delivered, and that the mill man computes the costs of his logs and his lumber production.

The scaler determines the net or sound volume of the logs.

* Comment by Mr. U. S. Swartz, Logging Engineer, U. S. Forest Service, Ogden, Utah. " In the old days when I learned to scale, the effect of fungus defect on timber was learned by observation. Today the forest student has the benefit of research studies which show the effect of the different fungus defects on different timbers and timber species. Many Junior Foresters are as good scalers after one year's scaling as they would have been after five years' experience twenty years ago. Depending upon the interest taken in the work, from one to two years' experience will make scalers out of most of the forestry students who pass the Junior Forester examination."

He must make definite decisions on all factors which affect their ultimate yield. Interior defects in timber which reduce the scale of a log are common, especially so in over-mature trees. Scalers have the right to reject any log or logs which they consider unsuitable. Too strict scaling, however, results in a high degree of waste in the woods, the loggers refusing to saw up a tree which, on being felled, shows any sign of defect. Slack scaling results in a high degree of waste and loss at the mill, since money has been paid for lumber material in the log which will not saw out. The efficient scaler ever strives for a mean between the two.

Few logs are perfect, and sometimes logs are not given their full value for their dimensions of diameter and length in order to compensate for obvious imperfection. Such deductions are made according to the scaler's best judgment and knowledge of defect based on his experience in the woods and in sawmills where he has seen similar logs sawed up.

59. Standards of Merchantability. — The purpose of felling trees and of lumbering operations is to meet the demands of one step in the process of producing manufactured articles from wood. Logging is the first necessary step in this process. It delivers its final product to the sawmilling or manufacturing plants in the form of logs. Necessarily not *all* logs or pieces which can be produced from a tree are acceptable. Some may not be considered *merchantable*.* An unmerchantable log is one whose yield in terms of the final product will not pay the labor charges of logging and milling. This may be due to one or more of three factors:†

1. The log may be of such small dimensions of diameter and

* There seems to be a lack of general information within the lumbering industry itself as to what constitutes a merchantable tree. A recent publication of the U. S. Forest Service shows rather conclusively that many small trees are cut and small logs handled in logging operations which are actually unmerchantable. See, Small Trees Wasteful to Cut For Saw Timber, by W. W. Ashe, U. S. Dept. of Agr. Leaflet 55, Washington, D. C., Jan. 1930.

† It is pointed out that these conditions of unmerchantability are not constant, but vary according to the economic conditions of the lumber market, and that with increasing values for finished lumber timber now considered unmerchantable because of inaccessibility, high logging costs and expensive transportation costs will gradually become merchantable. The receipts from the sale of the product must also include a fair margin of return for the operator over and above the actual charges for labor. Thus logs, or timber, where the sale value even though exceeding the costs of production makes no allowance for profit can hardly be considered merchantable.

length that even under the most efficient methods of sawing the actual amount of lumber produced does not pay for the labor of handling.

2. There may be so much defect present in the log that the net yield in sound lumber will not pay the labor charges.

3. The distance to be transported may be so great that the actual amount of money in labor, freight or towage charges necessary for the process of transportation of large sized timber from timber standing or from the stump to finished lumber in the mill, even under the most efficient management, may be so great as to eat up all anticipated returns from the sale of the finished lumber.

It is very important that within any given timber sale there should be a full and clear understanding on the following points:

(*a*) The minimum* length to which logs may be cut and be considered merchantable.

(*b*) The minimum diameter to which logs may be cut at the small end, which is particularly important with logs taken from the upper sections of the tree.

(*c*) The minimum amount of sound lumber per log which will be acceptable after defective material has been deducted or *culled out*. On some timber sale areas no log will be accepted if its culled scale is not equal to 50 per cent or more of its gross scale. On other timber sale areas as little as 25 per cent of the gross scale will be accepted as a merchantable log.

The minimum amount of sound material which will be acceptable in the log after deductions for defect have been made, should be, and nearly always is, based on the species and the relative sale price of the product. Species with a higher lumber value require smaller proportions of sound material in the log to be classed as merchantable. For example, a log of western white pine with an average lumber value of $35 per M, f.o.b. mill, would be accepted if it contained only 25 per cent sound material, whereas a log of western white fir with a mill run price of $20 per M would have to contain 50 per cent sound material before it would be considered merchantable.

60. Measurement of Length. — Logs may be scaled, where convenience offers the best opportunity: — at the stump, while they

* The actual length will be greater than the accepted minimum length by the excess of the "trimming length." See Section 60.

are being skidded, on rollways, on wagons, at the landing on the river bank or logging railroad, on railroad cars, or at the mill just previous to being sawed. Usually logs are cut to standard or uniform length in accordance with the terms of the timber sale. It is important that the scaler should check lengths of logs in order to see that these terms are being adhered to. It is a custom in the lumber industry to require a certain excess of the log length known as the **trimming length.*** The purpose of this extra few inches is to act as a cushion or absorber of all the grime, grit, and the shocks of transportation between the stump and the mill. The final passage of the all but finished lumber across the grading table in the mill trims off this excess length to a standard board length. In National Forest practice, a trimming length of 3 to 4 inches is allowed on small logs under ordinary logging conditions. But where the danger of splintering or of brooming is excessive, and on all large sized logs, 6 inches is allowed.

61. Measurement of Diameter. — With but few minor exceptions, all commercial log rules call for the diameter measurement to be taken in inches at the top or smaller end of the log *inside* of the bark. Diameters, as a rule, are rounded off to the nearest full inch above or below the actual measured diameter. Logs showing a top diameter of an exact half inch are scaled as of the next lower inch.

Extra care must be taken to ascertain the small end of the logs when scaling rollways or skidways. While logs are piled in rollways with one end of the pile even faced, there is no uniformity in the location of the small or big end of the logs at one end of the pile or the other. This would be a physical impossibility from the technical standpoint of piling.

The diameter measurement taken is to be the average diameter. Logs are seldom truly circular in cross-section; and, in scaling, the mean of at least two measurements should be determined. In Fig. 15 the measurement taken would be the average of the major axis of 22 inches, and the minor axis of 19 inches. The mean diameter is thus 20.5 inches. Dropping the half to the next lower inch, the log would be scaled as 20 inches.

62. Numbering and Stamping. — On large timber sales some progressive system of serially numbering the logs, the rollways

* In Canada, trimming length is sometimes called "overrun." This is a misuse of the latter term as commonly employed.

and the scale tally sheets must be devised in order that the continuity of the log scale for the entire operation can be maintained. The purpose of numbering logs is to assist the scaler in making a correct tally. The scale of each merchantable log is entered in the scale sheet opposite its number in the column provided for the species. Numbering logs also serves as a check in case of rescale,

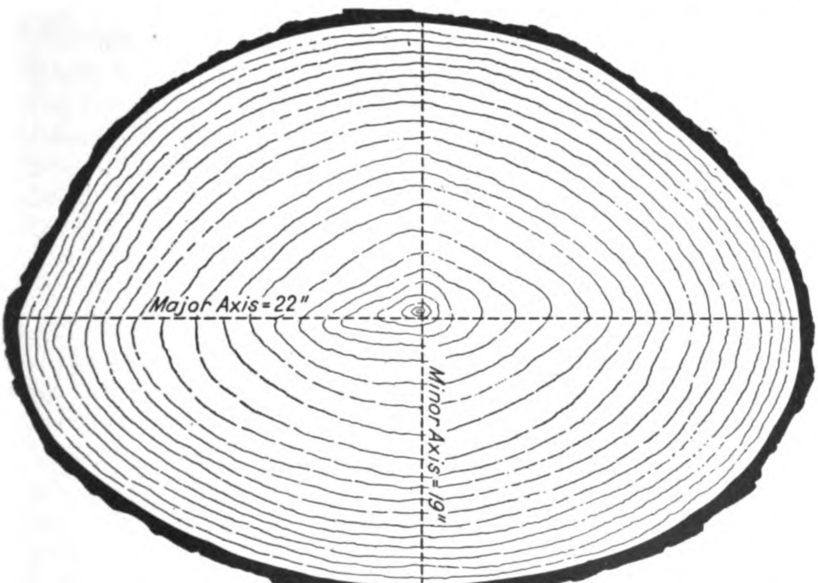


Fig. 15. — The Desired Diameter in Scaling Logs is the Mean of the Major and the Minor Axes as Measured Inside the Bark at the Top End of the Log.

and offers to the examining official an accurate identification of every log. Such rescale is known as a *check scale.**

It is the usual custom to number each log as it is scaled. Cull logs are numbered as well as merchantable logs. A common practice in scaling logs piled in rollways is to number all logs in the rollway first and scale them afterwards. Numbering is done at only one end of the log and preferably at the end not stamped.

* Check scaling is frequently done as regular routine in the U. S. Forest Service, as a check on the accuracy and efficiency of the men employed as scalers on timber sales. On sales involving an annual cut of a million board feet or more per year, there will be at least one check scale per season.

Stamping logs on the end with a heavy marking hammer has somewhat the same force as branding cattle. The marking hammer is embossed with the log mark of the owner, and the stamp will act as a check mark in identifying logs in a sorting boom. On U. S. National Forest timber sales the stamp is the official sign that the log has been scaled.

63. The Log Rule Used. — The specific log rule employed on any logging operation is a matter of agreement between the seller and the purchaser and should be clearly stated in the articles of the timber sale. It is to the sellers' interest to seek a log rule which will approximately measure what the timber will cut out. It is to the buyers' interest to insist on a log rule with a big overrun, thus getting a larger cut of lumber than the scale for which he has paid. Generally, the owner of the stumpage has the position of advantage and can protect his own interest.

The official log rule of the U. S. Forest Service is the Scribner Decimal C rule. It is regarded as a log rule which is ordinarily fair. This rule will be used in the subsequent discussion of methods of scaling logs and culling out defect. These methods also will follow closely the procedure current in U. S. Forest Service scaling practice.

64. Defects Commonly Recognized in Scaling. — If all logs were normally perfect, straight, and sound, the problems of scaling would be simple. Logs are never perfect and are seldom consistent in their imperfections. In manufacturing logs into lumber, the main desire of the sawyer is to get from a run of logs the maximum number of boards normally sound, clean and clear. Any characteristics, singly or together, which constitute an undesirable condition are known as defect. Normal defect, of course, may be present in the logs before they are opened. Sometimes, however, a form of defect known as "operating defect" may be caused by carelessness or willful waste in the processes of manufacture. Defect in logs includes any undesirable conditions which will reduce the volume of the finished lumber. Defect does not include the necessary losses in lumber manufacture. The most common unsound conditions rated as defects in logs are: rot, check, shake, seams, unsound sap, unsound heart, cat face and worm holes. Crook and fork are two undesirable conditions which increase rather than directly cause waste. On some of the U. S. Forest Service timber sales, masses of black or red pitch commonly found in old fire-scarred logs of western yellow pine, and grouped clusters of large knots in the upper sections of old, large, overmature trees are also regarded as defects affecting the scale.

The effect of rot and other defects upon logs varies from species to species, and within a given species from region to region. Consequently it is hardly practicable or possible to lay down hard and fast rules for making deductions which are to be applied at all times and in all places with absolute inflexibility. The effect of a conk on western white pine in Idaho, for example, is very different from the effect on red fir on the Pacific coast. Heart rot in western yellow pine in Arizona offers an entirely different problem than for the same species in California. In the final analysis, what chiefly enables a scaler to do a good job at scaling is the factor of experience in any given region backed by a knowledge of the timber and the characteristics of its defects.

It is only when such defects are absolutely visible in the log that deductions are made in its gross scale. As a general rule, hidden defect, or defect such as "peckiness" in incense cedar, bald cypress, etc., which does not become visible until the log is opened up in the mill, is not considered by the scaler in his measurement and scale.

Neither does the scaler give any weight to the estimated efficiency of the mill, its methods of sawing nor the amount of mill overrun to be expected.

65. Classification of Defects. — The defects which are commonly recognized by timber scalers and for which compensation is allowed by decreasing the gross scale in proportion to its amount may be briefly classified as follows:

(1) *Interior Defects.* — Rots, dodcy heart, etc. which cause waste in lumber sawed from the interior sections of the log such as:

Heart rot.
Butt rot.
Heart shake.
Pitch ring.
Heart check.
Pitch seam.
Split.

(2) *Side Defects.* — Causes of waste in lumber sawed from the outer sections of logs, such as:

Unsound sap.
Wind or sun check.
Worm holes.
Lightning scars.
Frost check.
Exterior pitch seams.
Cat face.

(3) *Crook Defects.* — Crook or Sweep causes extra waste in sawing according to its severity.

(4) *Crotch Defect.* — Crotch or Fork causes loss in sawing according to its position and character.

66. The Standard Rule for Deduction of Defect. — Different scalers approach the problem of culling their defective timber in different ways, but all are seeking the same result. Any one method carefully considered and accurately applied will probably be as good as another. What is desired by the student is knowledge of a simple, accurate method capable of application under any condition and with any type of timber.

The method of culling or deducting for defect hereinafter described is based on the principle of diagramming, or blocking out, on the end of the log, the rectangle, or square, which will consistently contain the entire defect and represent its cross-sectional area, of determining the contents of this timber in board feet in terms not of the timber itself *but of the inch boards which would usually be sawed from such a timber;* and finally, of deducting this value from the gross scale of the log. This method is logically and mathematically accurate, and commends itself to wide use throughout the industry on account of its practicality.

This method, which is known as " the standard rule " method, is probably the best general rule that is available for culling defect although in actual practice it is not at all times readily workable, particularly in cases where speedy work is necessary, such as scaling ahead of a hoist or working on a conveyor as the logs pass by. In such cases it is the usual practice of the scaler to resort largely to various rules of thumb* or memorized deductions for

* In U. S. National Forest Service scaling practice, rules of thumb may be used in specific cases by scalers of requisite judgment and experience under direct and written approval of the District Forester. Three rules of thumb which may be so used for deducting for center rot are:

1. Obtain the average diameter of the rot. Add to this average diameter

certain defects of specified sizes which give practically the same results as are secured by application of the standard rule. The accurate use of these rules of thumb is acquired by the scaler as a result of wide experience and a trained memory. They should not be applied by the untrained man or by one with but little experience until he has had full opportunity to compare such short-cuts with results obtained by use of the standard rule.

In applying the standard rule care must be taken to ascertain and use no other than the exact saw kerf with which the log rule was constructed. In the construction of the Scribner Decimal C log rule the diagrams employed are those of 1 inch boards with a ¼ inch saw kerf, which means that from every four 1 inch boards actually sawed out from within the log cylinder the equivalent of one more 1 inch board is wasted in sawdust. This means that if the log was at once quickly slabbed on all four sides, in the reduction of the residual timber to its component boards 20 per cent of its volume will be lost in sawdust. Thus, if **W**, **T** and **L** are respectively the width in inches, the thickness in inches, and the length in feet of the residual timber after slabbing, and if **V** is the volume of its component 1 inch boards in board feet, then

$$V = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{12} \times \frac{80}{100}$$

$$= \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

(raised or lowered to the nearest 10 to conform with the Scribner Decimal C rule)

½ its value if it is 9 inches or less, ⅓ if from 10 to 19 inches, ¼ if it exceeds 19 inches. Scale a log of this diameter and log length and deduct from the gross scale of the log.

2. With logs 14 and 16 feet long the deduction for circular rot of 8 inches or less can be found by squaring the diameter of the defect and rounding off to the nearest 10.

3. For center defects not exceeding 17 inches in diameter allow twice the scale of a log having the length and diameter of the defect. This rule of thumb should never be used for defects greater than 17 inches in diameter.

Another rule of thumb for scaling center defects has been proposed by Prof. F. G. Clarke, — " Square the diameter of the defect and add to or subtract from the product one-fifteenth of the product for every foot that the defect is longer or shorter than 15 feet." Jour. of Forestry. Vol. XXVIII No. 1. Jan. 1930. p. 95.

The procedure is simple in application and absolute in conclusion. The scaler first scales the log as if it were sound. He then estimates or measures the dimensions of the square or rectangle containing the defect which must be thrown out as waste due to defect. Care must usually be taken to allow a slight excess, generally not more than one inch over and above the exact dimensions of the defect, to permit inclusion of the necessary loss of otherwise sound material incidental to the elimination of the defective material. The scaler then determines the *length of the defect* and applies the formula. Reducing to the nearest 10 to conform to the log rule, he subtracts its value from the gross scale of the log to get its net or culled scale.

Where the defect under scrutiny appears at both ends of the log, which incidentally determines its length, the scaler will use the larger dimensions of the defect in determining his cull if the logs are not greater than 16 feet in length. But with logs longer than 16 feet the usual custom is to take not the greatest, but the mean dimensions of its two ends.

This method of applying the standard rule for center defects extending the entire length of logs not greater than 16 feet long varies in the different U. S. Forest Service Districts and regions. Its method of application depends, to a great extent, upon the minimum length of board accepted by the trade. In the Ogden and Missoula districts, where it is a common practice to manufacture 6 foot boards, particularly with a valuable species like western white pine, and in all other regions where similar standards of close utilization prevail, the usual custom is to add one inch to the *mean dimensions* of the defect, regardless of log length, and apply the standard rule.

When logs are cut in long lengths but are scaled as two or more shorter logs and defect shows at both ends of the long timbers, the scaler shall determine the diameter of the defect at each point of log section on the basis of its proportional taper between the top and the bottom of the long log.

When the defect occurs at but one end of the log the scaler must decide the length of the defect. Careful inspection of the outside of the log for surface indications is of great assistance, but the scaler must have knowledge and experience as to what to expect in the way of punk scars, punky or rotten knots according to the kind and degree of the defect present. Butt rot which tapers rather rapidly to a point will probably show no surface indication of its length, and the scaler must fall back on his store of past experience both in woods and mill with timber of the same species and age showing the same degree or kind of rot. With other rots, other conditions will prevail. Circumstances vary considerably.

67. The Log Cylinder. — In the Scribner Decimal C rule, as with almost every other log rule, the usable portion is assumed to be contained within a cylinder whose diameter is equal to the top d.i.b. of the log, and whose length is equal to the length of the log as measured. This cylinder is known as the *log cylinder.** Obviously the factor of the taper is ignored and consequently any defect contained in any portion of the log outside of the log cylinder will not be taken into consideration in deducting for cull.

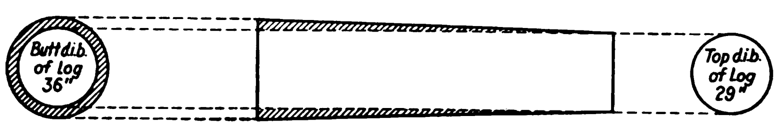


Fig. 16. — The Log Cylinder.

It should be pointed out that the Scribner rule as diagramed makes allowance for slab; consequently in considering marginal or surface defects deduction is allowed for only that portion which is within both the slab and the log cylinder. The usual allowance for slabs with the Scribner rule according to Forest Service practice is 1 inch on the radius within the log cylinder.

INTERIOR DEFECTS

These are sometimes called End Defects because they are most easily apparent from one or both ends of the log.

68. Heart Rot, Circular Rot, Center Rot. — A log is 16 feet long, 25 inches in diameter inside the bark at the top end, and 29 inches d.i.b. at the butt end. See Fig. 17. A center rot extending the length of the log shows at the butt end an average diameter of 13 inches, tapering to 6 inches at the top end. What is the net or culled scale of the log?

1. Scale the log as if it were sound. According to the Scribner

* In the U. S. Forest Service Scaling Manual the " log cylinder " is called the " *right cylinder.*"

Decimal C rule, a log 16 feet in length and measuring 25 inches at the top end has a volume of 460 board feet.

2. Determine the average dimensions of the larger end of the defect. A circle of 13 inches diameter or a square 13 × 13 inches will fully contain it. Add* 1 inch to the dimensions of this square

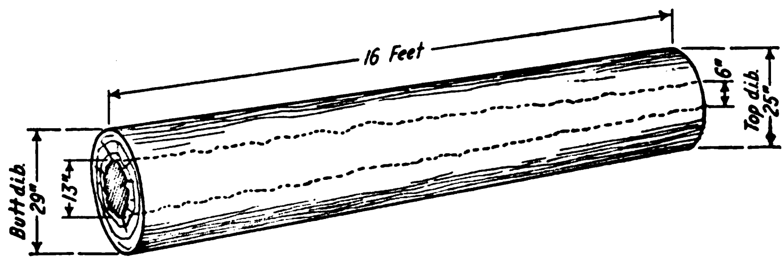


FIG. 17. — Heart Rot.

for waste in the surrounding sound material incidental to the processes of manufacture and apply the rule:

$$\text{Cull in b.f.} = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

$$= \frac{(13 + 1) \times (13 + 1) \times 16\,\dagger}{15}$$

$$= \frac{14 \times 14 \times 16}{15}$$

$$= 209.$$

3. Round off the 209 to the nearest 10 and deduct from the gross scale of the log:

* There seems to be a difference of opinion among some practical scalers as to the propriety of adding the one inch margin to the dimensions of all center defects, regardless of size, for the purpose of absorbing necessary manufacturing loss. When the dimensions of the defect exceed 12 to 14 inches, it has been found from actual mill tallies that the mill scale exceeds the log scale by an amount directly proportional to the ratio of the area of the square of the defect and the area of its inscribed circle. Consequently, it is felt by some scalers that this factor will take care of the necessary deductions without any increase in the dimensions of the defect. However, the standard practice of the Forest Service, as laid down in its manual, is to add this one inch margin as illustrated above.

† Henceforth in applying this rule it will be understood that *W* and *T* are usually equal to 1 inch in excess of the actual measurements.

$$\begin{aligned}\text{Net scale} &= \text{gross scale} - \text{defect}\\ &= 460 - 210 \text{ board feet}\\ &= 250 \text{ board feet}\end{aligned}$$

Suppose, however, that the dimensions of the defect had been 15 inches at the larger end and 11 inches at the mill end.

$$\text{Then Cull} = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

$$= \frac{16 \times 16 \times 16}{15} = 273 \text{ board feet,}$$

which, rounded off, $= 270$ board feet

And $460 - 270 = 190$ board feet as the net scale of the log.

Under a 50 per cent merchantability standard this log would be discarded as unmerchantable although under a 30 per cent merchantability standard it would still be accepted.

However, if the dimensions of this defect had been 17 inches at the larger end and 13 at the smaller, the scale of the defect would have figured as equal to $\frac{18 \times 18 \times 16}{15} = 340$ board feet. And the net scale of $460 - 340$, or 120 board feet would not have met even a 30 per cent merchantability standard. In spite of the fact that the log contained more than 100 board feet, it would have been culled absolutely and discarded.

The foregoing procedure for scaling this type of defect has been illustrated with 16 foot logs. When the log length exceeds 16 feet, the average, not the major, dimension of the defect is the basis of the deduction.

Suppose, in the first case illustrated, that we assume a log length of 18 feet and the same dimensions of diameter for log and defect, namely, a top diameter in the log of 25 inches, and a defect 6 inches at the top end and 13 inches at the butt. Proceed as before:

1. Scale the log as sound, 520 board feet.
2. Measure the defect at each end of the log to determine the average of its greater and lesser dimensions, $\frac{13 + 6}{2} = 9\frac{1}{2}$ inches rounded off to 9 inches. Add 1 inch margin.
3. Determine the contents of the defect by the standard rule:

$$\text{Cull} = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

$$= \frac{10 \times 10 \times 18}{15}$$

$$= 120 \text{ board feet.}$$

4. The gross scale minus the defect equals 520 − 120, equals 400 board feet.

69. Butt Rot, Ground Rot, Stump Rot. — This type of rot occurs mainly at the lower end of butt logs and, rapidly tapering to a point, extends only a short distance of the length. The first duty of the scaler on the basis of his knowledge and past experience is to determine the length of the defect, or, the depth to which it has penetrated the log. Suppose, as shown in Fig. 18, a 21 inch log

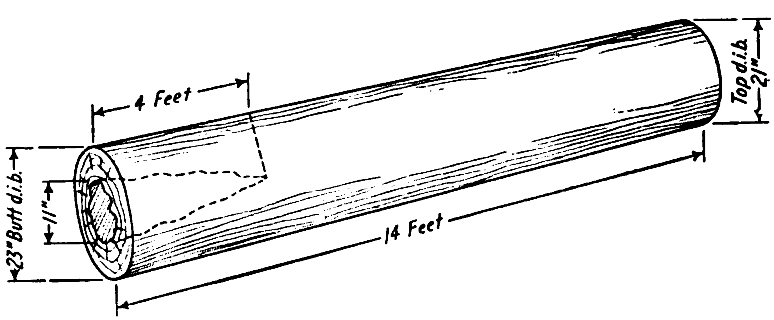


Fig. 18. — Butt Rot.

14 feet long shows a butt rot 11 inches in diameter at the lower end. To determine the net scale, the scaler proceeds as follows:

As before, he first scales the log as sound. He then examines the butt rot defect and determines its dimensions on the face of the butt end of the log and also the depth to which it penetrates the log, thus determining its length. In the case illustrated, this is 4 feet. To the dimensions of width and breadth of the square or rectangle which will just contain the defect, he adds 1 inch margin and applies the rule. He then deducts to get the net scale, thus:

1. The gross scale of a 21 inch 14 foot log according to the Scribner Decimal C rule is 270 board feet.

2. The volume of the defect is equal to

$$\frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

$$= \frac{12 \times 12 \times 4}{15}$$

$$= 38$$

which raised to the nearest 10 equals 40 board feet.

3. The gross scale − the scale of the defect = 270 − 40
= 230 board feet.

An alternative method would be to cut a 2 foot section from the defective end of the log and scale it as a 21 inch 12 foot log with a volume of 230 board feet net scale. In this particular log, since the defect, only 11 inches in diameter, extends 4 feet at the most, it would be unfair to cut off the whole 4 feet of its length because of the large amount of sound material surrounding the rot. Or, in other words, only ½ of the 4 foot section will actually be a loss.

However, had the extent of the defect been larger, involving, say, a diameter dimension of 17 inches, then 17 + 1 = 18 inches.

$$\text{Volume of defect} = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

$$= \frac{18 \times 18 \times 4}{15} = 86,$$

rounded off to 90, board feet

and $270 - 90 = 180$ board feet.

It will be seen that the standard method* yields less scale for log than had we eliminated the waste entirely by cutting off 4 feet from its length.† This is due to the fact that when the dimensions of the rectangle approach the diameter of the log cylinder, due to the construction of the Scribner rule, the boards contained in the rectangle will exceed in volume those diagramed for the rule.

70. Pitch Ring, Ring Shake, Pitch Shake. — These are types of defects which occur in and follow the annual ring. Two distinct

* There might be a question in the mind of the scaler as to which method to use under conditions such as have been outlined. Fortunately, the Forest Service in its practice has set a standard which can well be followed. This rule states that where the deduction obtained by the standard rule exceeds that attained by reducing the log length, the *latter* shall be used.

† The scale of a 21 inch 10 foot log, Scribner Decimal C rule is equal to 190 board feet.

problems are presented to the scaler depending upon the character and extent of this defect.

I. *Where the defect is of such extent and character that practically no merchantable material is contained within the pitch ring.* Suppose, as illustrated in Fig. 19, we have a 16 foot log 23 inches in top diameter, and a ring shake 14 inches at the butt end extends clear through the log and is of such character that no usable material is contained within the defect. The method of scaling

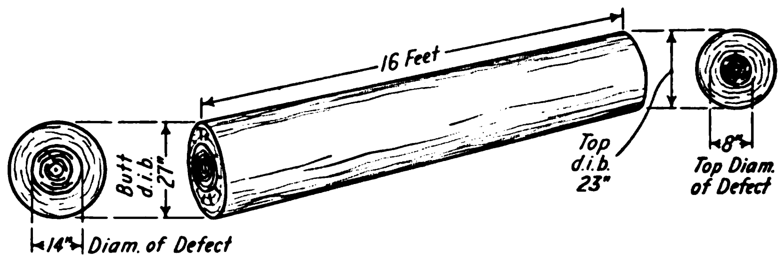


FIG. 19. — Pitch Ring or Pitch Shake.

for such a log follows exactly the same methods as for scaling out heart rot.

1. Scale the log as if it were sound, a 23 inch 16 foot log scales 380 board feet.

2. Scale the defect by standard rule 14 + 1 = 15 inches.

$$\text{Volume of defect} = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$

$$= \frac{15 \times 15 \times 16}{15}$$

$$= 240 \text{ board feet.}$$

3. Gross scale − volume of defect

$$= 380 - 240 = 140 \text{ board feet}$$

and if 30 per cent merchantability standards prevail the log will be accepted.

II. *Where the defect is confined to a restricted zone and contains within it a certain diameter of salvageable material.* Consider the same log as before; but suppose that the ring shake is confined to a narrow zone or band 3 inches wide containing within it a sound core 8 inches in diameter of obviously usable material. The scaler proceeds as follows:

1. He scales the log as sound, as before = 380 board feet.

2. He determines the volume of the rectangle containing the whole defect, as before = 340 board feet.

3. He then considers the core of sound material within the board of defective wood. This he treats as if it were a separate log, a log within a log, as it were. Its diameter is 8 inches, its length 16 feet. An 8 inch 16 foot log, according to the Scribner Decimal C rule, contains 30 board feet. This is the volume of the usable material contained within the defect.

4. This volume of material is then deducted from the volume scaled from the rectangle under point 2 to get the net defect.

5. The net scale of the log is then equal to:

$$\begin{aligned} \text{Gross scale} - \text{Net scale} &= 380 - (240 - 30) \\ &= 380 - 210 \\ &= 170 \text{ board feet.} \end{aligned}$$

When the ring shake does not extend the full length of the log the scaler must estimate its length and proceed in his deduction with methods closely approximating those pertaining to deducting for butt rot.

71. Pitch Seam, Split, Heart Shake. — This is a type of defect which follows the medullary ray for the most part, rather than the annual ring.

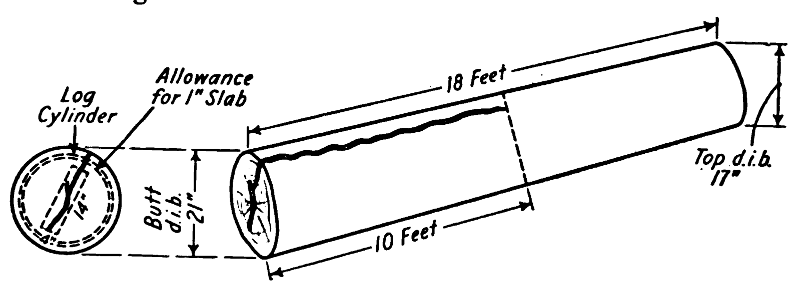


Fig. 20. — Pitch Seam.

Consider a log of 17 inch top d.i.b. with a length of 18 feet. This log has a butt diameter of 21 inches and on the butt face shows a heart shake more or less filled with pitch, as is illustrated in Fig. 20. This defect from surface indications extends for a distance of 10 feet up the log.

From an examination of the defect, its location, extent and character the scaler determines that although the pitch seam extends to and is continued on the surface of the log, the amount of lumber really affected can be contained within a rectangle 4 inches wide and 14 inches in depth diagramed *within the projection of the log cylinder on the butt end of the log.* The log is then scaled and the standard rule is applied. In applying the standard rule, however, it is to be noted that with this type of defect it *will not be necessary to add an inch* to the measured dimensions of the diagram about the defect to compensate for waste incidental to sawing it out.

1. The gross scale = 210 board feet.

2. Volume of the defect $= \dfrac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$

$$= \frac{14 \times 4 \times 10}{15}$$

= 37, rounded off to 40 board feet.

3. Net scale = 210 − 40 = 170 board feet.

One of the big difficulties in scaling for pitch seam is that it usually spirals or twists, following the grain of the wood, and, passing entirely through the log, comes out on an angle with its transverse axis in an entirely different plane. This causes an increasing loss in lumber due to non-utilization of short lengths and an increased size of diagram with a greater cull volume to compensate for its occurrence.

SIDE DEFECTS

Side defects are found mainly in the superficial surfaces of the log. Their effect in influencing cull is neither so serious nor so severe as in the case of interior defects. Often they may be so shallow as to penetrate not even the slab of the log cylinder. In such cases they can be totally disregarded. Where they do penetrate to any distance within the log cylinder, it must be remembered that it is only those portions which lie *inside* its periphery which are of any concern to the scaler.

72. Lightning Scar. — Trees are often hit by lightning which follows down the tree ripping the bark and gullying a more or less deep gouge for the whole length of its contact. The seriousness of this type of injury depends upon its severity and its consequent effect in lessening the amount of usable material in the log. Suppose a 27 inch 18 foot yellow pine log with a severe lightning scar running its entire length. The severity of the scar is further evidenced by its depth, to almost the center of the log.

The simplest method of handling this condition is to divide the log up into sections or sectors, giving the scaler the opportunity to estimate what proportion of the log is damaged by the injury. As diagramed in Fig. 21, one-sixth of the log will be affected. Consequently, one-sixth of the gross scale of the log will be deducted as compensation for the defect:

1. Scale the log as sound, 620 board feet.
2. Volume of the defect = $\frac{1}{6}$ of 620 board feet = 103, rounded off to 100 board feet.
3. Net scale = Gross scale − Scale of defect
   = 620 − 100 = 520 board feet.

Lightning scars are almost invariably sources of infection for insects and fungi. Hence due care must be taken in estimating

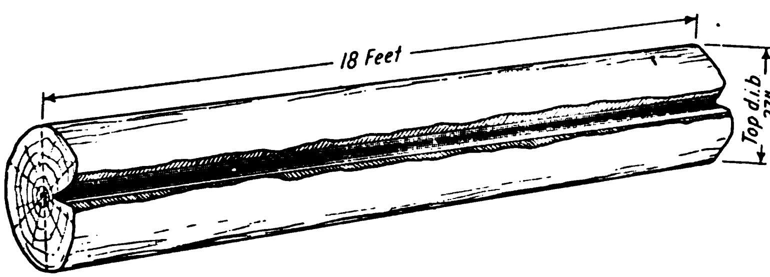


Fig. 21. — Lightning Scar.

the sectors bounding the injury to include a sufficient amount of sound material on either side of the scar to compensate for any loss due to unsound, fungus-infected or insect-bored wood adjacent to it.

Lightning rarely follows in a straight line, as shown in the case just illustrated, but twists or spirals around the tree following its surface grain. When lightning has spiraled completely around a log the net scale can be determined by scaling the log as a cylinder inside the depth of the scar. When it has not entirely spiraled the tree or log, but has only passed around a fraction of the distance, then the deduction should be in direct proportion of the amount of lumber affected by the injury.

Suppose in the log illustrated as before in Fig. 21, that a lightning streak 6 inches deep passes one-quarter of the distance round the log in traversing its length from top to butt. If it had passed all the way round, we would have scaled the core inside of the scar; namely, a 15 inch log 18 feet long yielding 160 board feet, as the scale of the log. But obviously, three-quarters of the wood material in the log is unaffected by the injury and is available for manufacture; or, stating it another way, one-quarter of the difference between the gross scale and the scale of the core. To get the net scale:

1. Scale the log as sound; a 27 inch 18 foot log contains 620 board feet.

2. Scale as a log the inner core whose diameter shall be equal to the original diameter of the log minus twice the depth of the injury: $27 - (2 \times 6) = 15$ inches. A 15 inch log 18 feet long contains 160 board feet.

3. The volume of the defect is equal to one-quarter of the difference between the gross scale and the scale of the core, that is, $\frac{1}{4}$ (620 − 160) or 115 rounded off to 110 board feet.

4. The net scale then is equal to the gross scale minus the scale of the defect, or, 620 − 110 = 510 board feet.

73. Frost Check, Surface Seam, Surface Check. — This type of injury is fairly common with northeastern timber and with

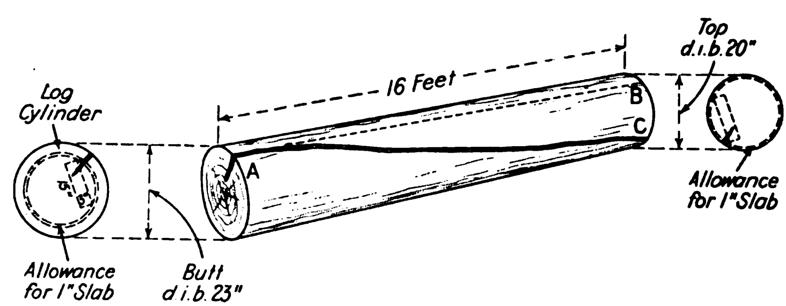


FIG. 22. — Surface Seam.

timber from regions where severity of climate produces lesions of the bark and splitting of the wood structure. These are apt to fill up with pitch and are points of attack for fungus and insect enemies. This injury, though similar, is never so extensive or deep seated as lightning injury. In scaling it is treated in much the same manner. Usually it is shallower and narrower, and hence a smaller sector need be discounted. In cases where the seam is straight probably the culling of a 2 inch plank equal in length to the log and in width to the depth of the seam within the slab of the log cylinder will suffice. Suppose in the log illustrated in Fig. 22, the seam, instead of spiraling from *A* to *C*, ran straight, as from *A* to *B*. Suppose this seam penetrates the log to a depth of 8 inches measured from the surface. This log has a top diameter of 20 inches and a length of 16 feet. It scales 280 board feet.

On the butt end of the log, where the seam penetrates to a depth of 8 inches, the diameter is 23 inches. The projection of the diameter of the log cylinder (the top d.i.b.) cuts off 3 inches of the penetration of the seam. The slab on the log cylinder cuts off one more inch, so that, were one 2 inch plank to be culled as compensation, its minimum width would be 4 inches. Hence, a plank 2 × 4 inches × 16 feet containing 10 board feet gives a net scale of 270 board feet.

Where the seam spirals from *A* to *C*, as is illustrated in Fig. 22, the method followed for lightning scar will obtain the net scale of the log.

Another method which will give substantially the same result is to diagram within the log cylinder at the butt end of the log the rectangle which will wholly contain the defect, being careful to diagram its dimension of depth as the *mean distance of penetration within the log cylinder as measured both at the top and at the butt ends of the log.*

In Fig. 22, as is shown, the defect volume is equal to $\frac{3 \times 15 \times 16}{15}$ $= 48$, rounded off to 50 board feet. This subtracted from 280 gives 230 board feet as the net scale of the log.

74. Cat Face. — Cat face is the name given to a fire scar, occuring usually on the lower end of butt logs. It is always a ready source for fungus and insect infestation. Suppose a log 16 feet long, 33 inches at the top end, with a heavy cat face as illustrated in Fig. 23. This cat face follows up the log from the butt end for a distance of approximately 10 feet. But inasmuch as it tapers to a point, the last foot of its length falls outside of the log cylinder and a length of 9 feet will suffice to catch its length within the log. By inspection the scaler determines that the depth of the defect *within* the log cylinder is 11 inches and that its *average* width is 21 inches. To scale the log proceed as follows:

1. Scale the log as sound, 780 board feet.
2. Scale the defect by applying the standard rule:

$$V = \frac{W(\text{inches}) \times T(\text{inches}) \times L(\text{feet})}{15}$$
$$= \frac{11 \times 21 \times 9}{15}$$
$$= 138, \text{ rounded off to 140 board feet.}$$

3. Net scale = gross scale − volume of defect
= 780 − 140
= 640 board feet.

Another way of getting the same result is by inspection as to what proportion of the butt section inside of the log cylinder is affected by the defect. In the case illustrated in Fig. 23 we will

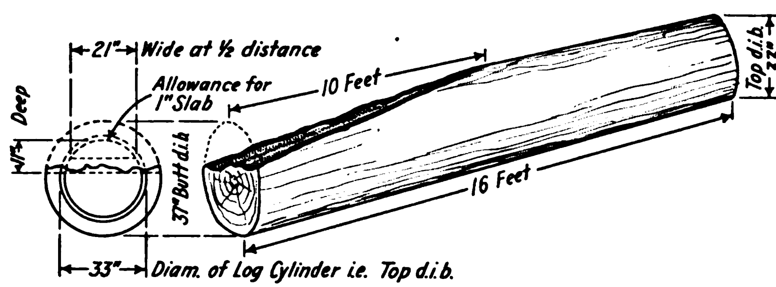


FIG. 23. — Cat Face.

assume that it is one-third. A length of 9 feet will take up all waste inside of the log cylinder. So what is actually affected by the defect is a log which is 9 feet long and which is 33 inches in diameter at the top end. A 33 inch 9 foot log according to the Scribner Decimal C rule has a volume of 440 board feet, and the estimated proportion of this log actually eliminated as waste by the cat face is ⅓ or 146, rounded off to 150 board feet, which, deducted from 780, the gross scale of the log, gives 630 board feet.

Another way in which the preceding method may be considered is to assume that since the defect affects one-third of the volume of the portion of the log culled it will equally affect one-third of its length. Hence, if one-third of this length is eliminated from the length of the gross log, such elimination will compensate for the defect. One-third of 9 feet is 3 feet. 3 feet taken from 16 feet equals 13 feet. A 13 foot log 33 inches at the top end scales 640 board feet, the net scale to be allowed for the cat face log.

75. Sun Check, Wind Check, Weather Check, Shrinkage Check. — Defects of this character occuring in green timber after being cut and placed on the skidways are not considered in scaling and rate as a loss which the operator must stand. This type of defect is only considered in standing, dead or down timber containing enough sound material to fall within the merchantable

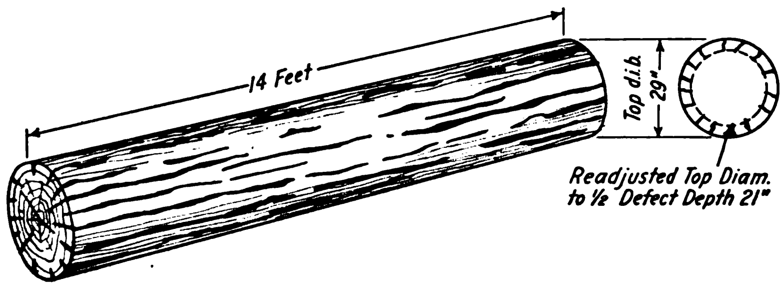


FIG. 24. — Sun Check.

clause of the contract. In Fig. 24, a 14 foot 29 inch log shows sun checks penetrating to a depth of 8 inches. The common method of culling this defect is that of lowering the diameter value by an amount equal to the depth of the checks themselves, achieved by refixing the log cylinder at a point half their depth, and scaling the log as such. The reason for not scaling clear inside the checks is due mainly to the fact that weather checks appear much worse on the ends, and the waste actually ensuing is not so great as end conditions indicate. The 29 inch 14 foot log with the 8 inch checks would be scaled as a 21 inch* 14 foot log with a net scale of 260 board feet.

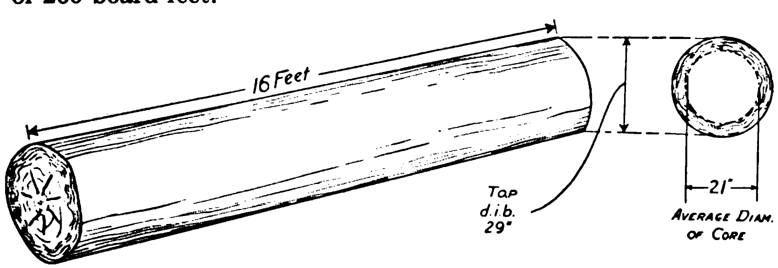


FIG. 25. — Unsound Sap.

76. Unsound Sap. — Unsound sap usually occurs as a shell around a solid core. The method of determining the volume of

* The top d.i.b. $= 29'' - \left(2 \times \frac{8''}{2}\right) = 21''$, and not $29'' - (2 \times 8'') = 13.''$

solid merchantable material merely requires the scaling of the diameter dimensions at a point well within the unsound ring. Usually one inch should be deducted from the diameter of the core of solid material to compensate for the waste incidental to the manufacturing processes. Suppose, as in Fig. 25, a 29 inch 16 foot log with a 4 inch shell of unsound sap. The diameter of the solid core measures 21 inches. Drop one inch for necessary waste and scale the log as a 20 inch, 16 foot log with a volume of 280 board feet.

Unsound sap is a fungus infection and is a wood-destroying disease. Another fungus disease commonly met with which is not wood destroying is Blue Stain. Its main effect is to destroy the quality values of wood for some special uses but has no effect on its structural character. No deduction normally is made for Blue Stain in scaling logs.

77. Worm holes. — Worm holes are often brought about by forest fires of such severity and area that the timber cannot be salvaged before insect infestation sets in. Unless salvaged within

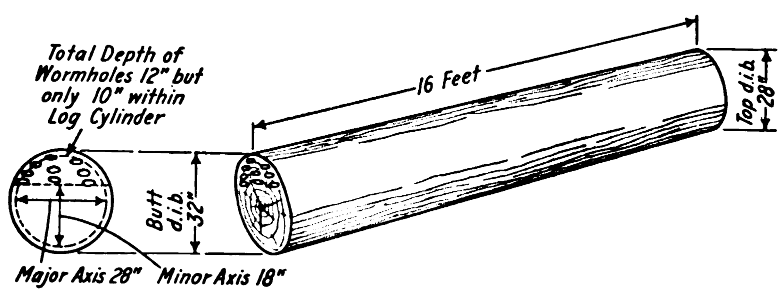


FIG. 26. — Worm Holes.

a year and a half after the burn, it may become so badly bored as to be absolutely useless. Worm holes sometimes follow unsound sap brought about by fungus infection. Consider a 28 inch 16 foot log. This log has a butt diameter of 32 inches showing a 4 inch taper in 16 feet as illustrated in Fig. 26. At the butt end and extending up the log to within 5 feet of the top, a boring insect infestation occurs. Inasmuch as there is no market for 5 foot boards, the entire length of 16 feet will have to be considered in dealing with the defect.

To find the net scale of the log, it is necessary to find the *mean* diameter of the uninjured portion within the log cylinder. From the figure we see that though the worm hole area on the butt face of the log is 12 inches deep, on the projected face of the log cylinder it is but 10 inches. Thus we have a minor axis of 18 inches and a major axis of 28 inches. The mean diameter then is $\frac{28 + 18}{2}$ = 23 inches. Then the scale of the log 23 inches top d.i.b. and 16 feet long or 380 board feet gives the scale of the log.

With insect infested areas of short length, the method of scaling out the defect would be exactly the same as that used for cat face.

78. Crook or Sweep. — The severity of defect from sweep or crook depends upon the degree of its curvature outside of the log cylinder. In the illustration in Fig. 27 the sweep is in but one

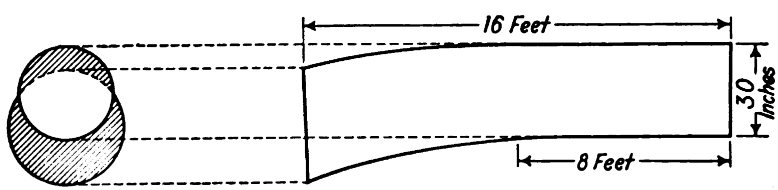


FIG. 27. — Crook.

plane and is comparatively simple to handle. In cases, however, where the sweep is in two planes, the proportion of the log cylinder contained within the log becomes decreasingly less and the crook may be so excessive as to reduce the amount of available wood material below the merchantability standard.

The most common method of culling for sweep is to reduce the length of the log in a measure directly proportional to the loss through the occurrence of the crook. Consider, in the illustration shown in Fig. 27, a 30 inch 16 foot log with a normal scale of 660 board feet. Suppose there is a sweep as illustrated. Obviously half of the 16 foot log is not affected at all. The defect is contained within half of the other half of the log. To compensate for waste, half of such length is discarded, and the log is scaled as a 30 inch 12 foot log with a scale of 490 board feet. Or, stating it another way, the scale of the entire log is equal to the scale of a 30 inch 8 foot log plus half of the scale of a 30 inch 8 foot log or $330 + \frac{330}{2} = 495$, reduced to 490 board feet.

79. Crotch or Fork. — Whether or not a log shall be cut off below a crotch, or whether the crotch is to be included within the log length requires accurate and comprehensive knowledge on the part of the log maker. It depends on two factors, one of which is the severity or depth of the crotch and the other whether the extra lumber included within the log length will pay for the extra labor required in sawing it out. The proper place to obtain the scaling diameter of a crotched log is just below the swelling of the fork. However, since such diameter can only be obtained accurately by the use of calipers or diameter tape, and if so obtained must be reduced to terms of its true value inside of the bark, it is customary

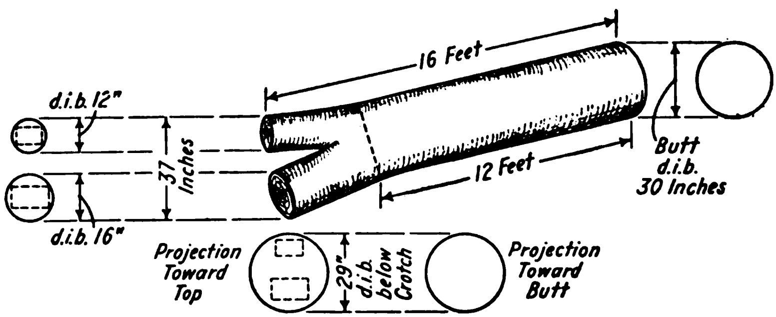


Fig. 28. — Crotch or Fork.

for the scaler to measure the butt diameter and make allowance for taper to the point of the scaling diameter.

If the log length is cut very close to the point of departure of the crotch so that little or none of the end surface is broken or appreciably lowered in potential volume value by a heavy excess of bark or by a deep split, no deduction will be made and the log will be scaled as if of the before determined diameter and the full log length.

If, however, there is a pronounced crotch with two forks cut off at some distance from their point of departure from the main stem, care must be taken in the determination of the proper scaling dimensions. Consider, as is illustrated in Fig. 28, a 16 foot log with a deep and pronounced crotch at the upper end of the log. The depth of the crotch is some 3 feet and the fork is rather pronounced. The extreme flare measured from the outside limits of the two forks is 37 inches and the mean transverse diameter of the individual forks averages 14 inches. Obviously, the averages of these top dimensions, 37 and 14 inches, will not give a true scaling diameter. The proper scaling diameter is measured at a point far enough below the crotch to avoid any abnormality or distortion. In the log illustrated, a value free from distortion due to swelling of 29 inches is determined. The full scale of a 29 inch 16 foot log is 610 feet. This total amount of usable wood material cannot possibly be sawed from the log illustrated.

To get the net scale of the log, the scaler adds to the scale of a 29 inch 12 foot log — 460 board feet; the scale of a 12 inch 4 foot log — 20 board feet; and the scale of a 16 inch 4 foot log — 40 board feet, for a total scale of 520 board feet.

Exactly the same result can be gotten by diagramming on the end diameters of the two forks the timbers which it is possible to saw out.

80. Hidden Defect. — In some species there are certain defects which do not reveal their extent or effect on the scale until the logs are opened up in the mill. Peckiness in incense cedar and bald cypress has been mentioned in point. The usual method for allowing for such defect is to deduct a flat percentage from the scale, the amount of which shall be in direct relation to the prevalence of the defect as disclosed in the sawing processes.

81. Scaling for Cubic Volume. — In scaling for cubic measure two diameter dimensions must be taken on each stick, — namely, the average d.i.b. at each end. If Huber's cubing formula rather than Smalian's* is to be used, care must be taken when measuring the diameter at the *middle* point of the stick or log not only to determine the *mean* diameter, but to reduce to terms of *inside of the bark.* Length is taken in feet, and from these dimensions volume is computed directly in terms of cubic feet. Deductions for defect follow the principles and the methods already laid down; namely, that the net scale is always equal to the gross scale minus the scale of the defect. It is to be noted, however, that in cubic volume *no allowance will be made for saw kerf;* consequently, all defects will be scaled at their full value. Deduction for crotch and for crook are not usually made in scaling for cubic volume.

82. Scaling for Cord Volume. — Fuelwood, pulpwood, shingle bolts, cooperage bolts, acid wood, and tan bark are commonly cut and sold by the cord. The unit of measurement in such cases is

* See Section 87.

the *sound cord,* that is, a cord of sound wood, rather than the standard cord which may contain a greater or less amount of defective material. Close inspection of every stick is the main requisite in scaling for cordwood, and according to the terms of the sale, sticks containing more than a certain percentage of defective material will be discarded. This requires the throwing in of additional sound sticks of similar volume or equivalent dimensions to make up to full value of the measured cord.

Care must be taken to see that the length of the sticks complies with the specifications of the timber sale. If 4 foot sticks are specified, then a stack 8 feet long, 4 feet high, and 4 feet wide containing 128 cubic feet will be accepted as the unit. If 5 foot sticks are specified, then a stack 8 feet long, 4 feet high, and 5 feet wide with a volume of 160 cubic feet will be the unit.

The necessary measurements of length, width, and height may be determined with a tape or measuring pole. When the stacks are piled on a slope, the length measurement will be taken on a plane parallel to the direction of slope and the height measurement of several places at right angles to this plane.

U. S. Forest Service procedure requires that each stack shall be serially numbered at both the top and the bottom, and at least 12 pieces in each pile shall be stamped.

83. Scaling for Piece Timber. — Usually the specifications covering the sale and purchase of piece timber are so carefully drawn that scaling and inspection mean one and the same thing. The main point is to see that requirements of diameter and length have been rigidly adhered to and that there is no evidence of rot or other undesired defect. Within certain well and usually narrowly defined limits, failure to meet the specifications means that the entire timber will be discarded, at least for the class for which it is offered, if not entirely.

It is often desirable to establish the volume of standard railroad ties, mine props, telephone posts and piling in terms of board measure, in order, for example, to meet or compare results with statistics tabulated in that particular unit. The usual practice is to accept standard converting factors based on average conditions rather than go through a laborious computation of the exact board foot contents of every piece sold.

84. Stump Scaling. — The most accurate scale is obviously the one made directly on the log. In some cases of timber trespass where logs have been removed from the sale area without their being measured or paid for, it may become necessary to assess the damage on the basis of a scale taken from the stumps and the tops of the trees removed.

The direction in which the tree has been felled can be determined from the undercut. The used length of the portion of the tree taken can be determined by the distance measured from the point where the lower end of the top is located, not to the stump, *but to the point on the ground where the butt struck* after being severed from the stump, which, especially in Pacific Coast timber, may be several feet from the stump. This distance may then be divided by the standard log length to get the number of logs taken from that tree.

Diameters are obtained on the basis of the taper between the d.i.b. on the top and the d.i.b. on the stump, computed and correlated to their proper position on the true stem according to the log lengths just previously determined.

Any deductions from this scale for probable defect should be on a flat percentage basis in accordance with the best information available concerning the average amount of defect usually present in logs of that particular class of timber, species, and age.

If, following the trespass, the tops have been burned or removed, making the obtaining of dimensions of length a practical impossibility, the usual custom is to measure standing trees of similar stump dimensions and obtain the required figures by deduction and approximation.

85. The Scaler's Tally. — In its simplest and oldest form it was a record of field measurements of the logs by species, length and top diameter. On arrival at the camp, the scaler looked in a log rule for the volume for each diameter and log length, and multiplied by the corresponding number of logs tallied. Under this system defect was culled by a lowering of dimensions, a method based neither on actual conditions nor on mathematical accuracy. In addition, this method permitted no exact check scale, or rescale, of any given portion, or on all of the total run of logs.

Under more modern methods, scale books and the pages thereof, log piles and logs, are all numbered serially, care being taken to enter volumes in the proper scale book for the numbers corresponding to those on the logs. The scaler's record should be clear, concise and complete. Its ideals of both unity and continuity are best expressed in the fact that with it, and by it, granted that no logs have been removed from the logging area, any other forester or scaler should be able to go back, identify and check the scale. As such, it is the continuous evidence of the scaler's skill and efficiency.

Various forms of scale records are used in different parts of the country. The most comprehensive form of scale tally should include separate columns for log number, log length in feet, top diameter inside the bark in inches, gross scale deductions, and net

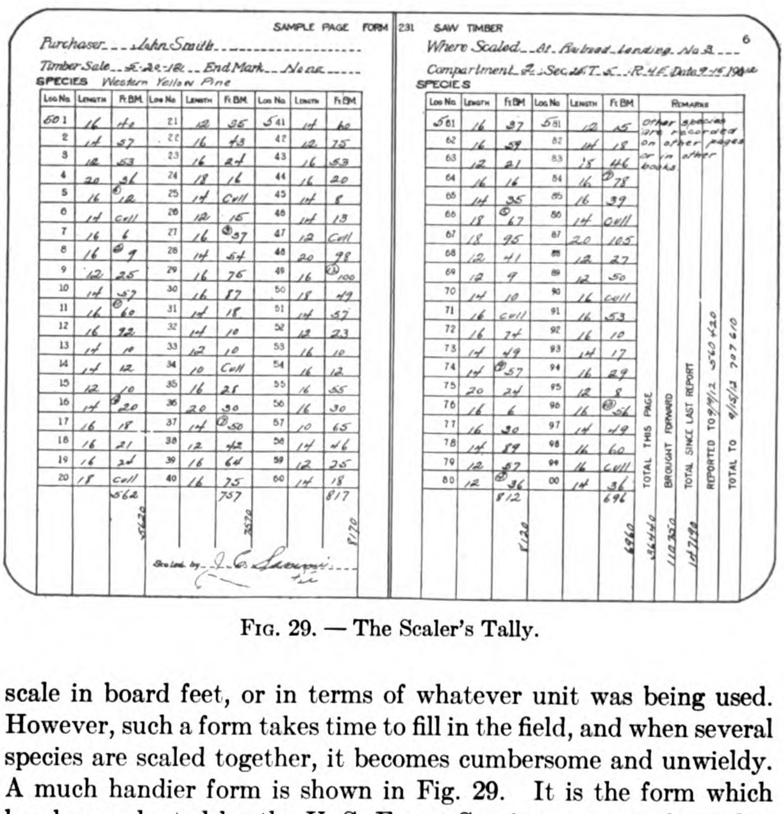


FIG. 29. — The Scaler's Tally.

scale in board feet, or in terms of whatever unit was being used. However, such a form takes time to fill in the field, and when several species are scaled together, it becomes cumbersome and unwieldy. A much handier form is shown in Fig. 29. It is the form which has been adopted by the U. S. Forest Service on its timber sales. It includes only columns for log number, log length, and volume by species. The usual custom is to record only the net volume of each log in the space corresponding to its species and log number, but for purposes of guide and check, the cull deduction is entered in the upper left-hand corner of the space in smaller type, surrounded by a circle, thus indicating a deduction of so many board feet. Logs wholly defective, or whose net volume is so small as not to meet the terms of merchantability of the sale, are entered in the same way except that the word " cull " replaces the value of the net scale.

Care must be taken to fill in all headings of each scale sheet or page of the scale book particularly in regard to the identity of the timber sale area, the number and location of the skidways, and the serial numbers of the logs measured. The name of the scaler and the date on which the scale is made should also be clearly set forth. Printing should be clear and legible, particularly for the numerical values of the scale itself. The volumes of the timber and the number of pieces should be carefully totaled and carried forward to the next page or sheet, not only for purposes of providing continuity of the record, but so that the exact status of the scale and the progress of the timber sale may be known at any time.

CHAPTER VI

THE DETERMINATION OF VOLUME IN STANDING TREES

86. The Purpose of Measuring Standing Trees. — The tree stands as the ultimate source of all materials made of wood. Just as logs can be scaled in terms of the product unit prior to the operations of milling and manufacture, so the tree can be measured likewise in terms of the product prior to the operation of logging. This process is known as *estimating*. It is so called because the volume of standing trees can never be obtained directly, but is accomplished through surmise or estimate based on the more or less careful measurement of dimensions of diameter and length. When the process of estimating considers the contents of the entire main stem of the tree, stump, logs, and top, it is said to derive a statement of *total volume*. But when the estimate considers only those portions of the tree removed from the woods for manufacture and use (the logs), it is said to state the *merchantable volume*.

87. The Volume of Single Trees. — The main stem of a tree is a solid, more or less cone-like in form. If the surface lines of this solid were straight and showed a regular progressive diminution in diameter, or *taper*, from base to tip, it would be considered as a *cone* and its contents could be found by using standard mathematical formulas for cubing the solid contents of such a solid. A cone is conceived as being formed by the revolution of a right-angled triangle about a central axis.

In nature, however, trees never assume such regular form. Sometimes the surface lines curve outward from the base in their tapering progress toward the tip, and develop a notable fulness in the mid portions of the tree. Such a solid is known as an *Appolonian Paraboloid.* It is conceived as being formed by the revolution of a parabola about a central axis.

Again, there are cases where the surface lines curve inward from the flare of a spreading root system, and show a very convex taper especially in the lower sections of the trees. Such a solid is known as a *Neilian Paraboloid,* or *Neiloid.* It is conceived as being formed by the revolution of a parabola of third degree about a central axis.

If it could be accurately determined that the form of an individual tree approximated either a cone, or a paraboloid, or a neiloid, its volume could be at once computed from standard mathematical formulas. However, it is practically impossible to determine the exact and true form of the standing tree before its felling offers better opportunity for exact measurement. The

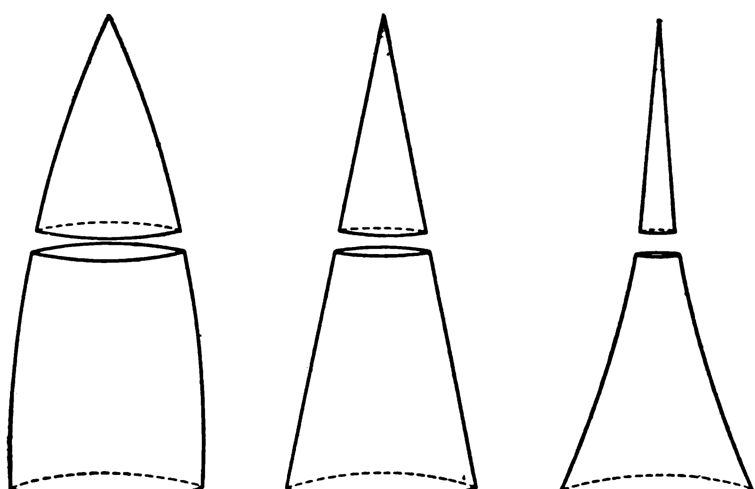


FIG. 30. — The Solid Forms of Trees Generally Fall within the Lines of a Cone, Paraboloid or Neiloid. When the Tree is Cut into One or More Logs, Each Log Becomes a Frustum of One or Other of These Solids.

use of the formula for cubing the entire contents of standing trees will only give approximations of the correct volume.

There are three formulas which may be used for cubing the contents of standing trees regardless of their relative approximations of form. All require, in addition to a measurement of total height and basal diameter, a determination of one or more other diameter dimensions with a subsequent reduction of such values to the corresponding sectional areas in square feet.

Schiffel's Formula. — This formula was derived from the prismoidal formula for cubing the contents of solids. This is sometimes known as Newton's formula (see page 98). This formula holds good for the cylinder, paraboloid, cone, or neiloid, and consequently also for logs of regular form derived from these solids. It requires a measurement of length and also diameter measurement at both ends and at the middle of the section. By considering the value at the top end of the tree as zero, Schiffel* derived the formula as follows:

$$V = (B + 4\,b\tfrac{1}{2} + b)\frac{H}{6} \text{ (Newton's formula)}$$

$$= (B + 4\,b\tfrac{1}{2} + O)\frac{H}{6}$$

$$= (1/6\,B + 2/3\,b\tfrac{1}{2})H$$

$$= (0.16\,B + 0.66\,b\tfrac{1}{2})H$$

where H = the total height of the tree in feet.
B = the sectional area in square feet at the base of the tree or section.
b = the sectional area in square feet at the top of the tree or section.
$b\frac{1}{2}$ = the sectional area in square feet at a point half way between B and b.

Pressler's Formula. — This is a very cleverly devised formula applicable to those trees whose forms approximate that of a cone or paraboloid. It first requires a basal diameter measurement at a point well above the influence of butt swelling; it then requires a determination of height at the point where the diameter of the tree is exactly half of that of the basal diameter. Then

$$V = (2/3\,B \times H\tfrac{1}{2}d)$$

where B = the sectional area in square feet corresponding to the measured basal diameter.
$H\frac{1}{2}d$ = the height above the ground at the point where the diameter is equal to one half of the basal diameter.
V = volume in cubic feet.

Hossfeld's Formula. — To apply this formula the basal diameter on the top of the probable stump is measured. The distance is then

* Further reference to the more extended use of Schiffel's formula is made in Section 142. As more commonly written, Schiffel's formula is $V = H\,(0.16\,B + 0.66\,b)$.

measured from the top of the stump to the top of the tree and at a point $\frac{1}{3}$ of this distance from the base, the diameter is determined by dendrometer or is estimated. Then

$$V = \frac{H}{4}(3\, b\tfrac{1}{3} + B)$$

where $b\frac{1}{3}$ = sectional area in square feet corresponding to the diameter at $\frac{1}{3}$ height above stump.
H = total height in feet.
V = volume in cubic feet.
B = basal sectional area.

The Volume of a Tree as the Sum of Its Component Parts. — Until a tree has been felled and cut into logs or sections, and these sections carefully measured, there is absolutely no way to determine with accuracy the form of the main stem, nor the best method of cubing its volume. The greater number of measurements possible insure a greater exactness because they avoid error arising out of the eccentricities of the growth of trees. For purposes of accuracy, it is more desirable to consider the volumes of trees as the sums of their component parts, stump, logs, and top.

If any cone-like solid is decapitated by the removal of a greater or lesser length of its upper section or tip, the truncated portion remaining is known as a *frustum*. (See Fig. 30.) Thus, there can be cone frustums, paraboloid frustums, and neiloid frustums. Any log from any given tree is thus a frustum, the exact form of which depends upon the surface lines approximated by the tree or portion of the tree from which it was cut. Various formulas are available for computing the cubic contents of solid volumes; a tabulated list of them is presented here.

STANDARD MATHEMATICAL FORMULAS FOR CUBING SOLIDS

1. Cylinder	$V = B \times H$
2. Cone	$V = \frac{B \times H}{3}$
3. Appolonian paraboloid	$V = \frac{B \times H}{2}$
4. Neiloid	$V = \frac{B \times H}{4}$
5. Frustum of a cone	$V = (B + b + \sqrt{B.b})\frac{H}{3}$

6. Frustum of a paraboloid a. $V = \frac{B + b}{2} \times H$ = SMALIAN'S FORMULA

b. $V = b\frac{1}{2} \times H$ = HUBER'S FORMULA

7. Frustum of a neiloid $V = (B + 4\, b\frac{1}{2} + b)\frac{H}{6}$ = NEWTON'S FORMULA

In these formulas H = dimension of height or length in feet.
B = Sectional area on face of lower section in square feet.
b = Sectional area on face of upper section in square feet.
$b\frac{1}{2}$ = Sectional area at $\frac{1}{2}$ H in square feet.
V = Volume in cubic feet.

In all of the above cases where volume is determined, sectional areas must be based on *inside bark diameter dimensions.*

The general consensus of opinion seems to be that Huber's formula is somewhat the more accurate, but falls short of general use and adoption due to the difficulties of determining its required diameter dimension with consistent exactness and ease. When the form of the log approximates that of the frustum of a paraboloid the application of either Huber's or Smalian's formula will measure the cubic contents with equal accuracy. If the surface lines of the log are more convex than the standard paraboloid frustum, then Smalian's formula underestimates while Huber's formula will show too large a result. With concave or convex lines of less than paraboloid form, Smalian will show an excess and Huber too small a value. In either case the error by Huber's formula is approximately half that incurred by the use of Smalian's formula and is mathematically opposite in value.

To determine the cubic volume of a tree from a measurement of its component parts, it is suggested that the treatment should be as follows:

(*a*) Cube the stump as a cylinder, $V = B \times H$

(*b*) Cube each log as the frustum of a paraboloid, $V = \frac{B+b}{2}H$

or $V = b\frac{1}{2}H$

(*c*) Cube the top as a paraboloid, $V = \frac{B \times H}{2}$

None of these assumptions may be absolutely correct, but they will be close enough to give approximately accurate results until more precise data can be obtained.

88. Volume Computation of Merchantable Contents with Logs of Equal Length. — As a rule the forester is most concerned with the amount of wood material in a tree which can be utilized. The stump and top represent unavoidable waste. Hence, the main consideration is centered in the merchantable portion of the tree which is commonly sectioned into logs for convenience in handling. These logs may, or may not, be of equal length. When all logs in the merchantable portion of the stem are cut to exactly the same length, computation of the total merchantable volume can be considerably shortened by use of the following formulas.

A. When the diameter dimensions are measured at the ends of the several logs, following Smalian. — Suppose a tree is of determinate merchantable length and let it be cut into five logs of equal length (L). Suppose that the sectional area at the butt end of the first log is known as B and that the sectional areas at the top ends of the five logs is respectively known as b^1, b^2, b^3, b^4, and b^5. The volume of this tree can be computed as follows:

$$V = \frac{B + b^1}{2} L + \frac{b^1 + b^2}{2} L + \frac{b^2 + b^3}{2} L + \frac{b^3 + b^4}{2} L + \frac{b^4 + b^5}{2} L$$

$$= \left(\frac{B + b^1}{2} + \frac{b^1 + b^2}{2} + \frac{b^2 + b^3}{2} + \frac{b^3 + b^4}{2} + \frac{b^4 + b^5}{2}\right) L$$

$$= (B + 2\,b^1 + 2\,b^2 + 2\,b^3 + 2\,b^4 + b^5)\frac{L}{2}$$

B. When the diameter dimensions (i.b.) are measured at mid-point of the several logs, following Huber.

$$V = (b^{1\frac{1}{2}} + b^{2\frac{1}{2}} + b^{3\frac{1}{2}} + b^{4\frac{1}{2}} + b^{5\frac{1}{2}})\,L$$

where $b^{1\frac{1}{2}}$, $b^{2\frac{1}{2}}$, $b^{3\frac{1}{2}}$, $b^{4\frac{1}{2}}$, $b^{5\frac{1}{2}}$, are respectively the corresponding areas in square feet at the mid-points of the several logs and L is the constant of equal length in feet.

89. Rules of Thumb for Determining Approximate Volumes of Standing Trees. — A rule of thumb represents an attempt to develop a simple method of procedure stated in such terse, concise terms as may be easily remembered. Applied to volume determination of standing trees, it seeks an easy method of quickly computing the volume of any tree without recourse or reference to a table. Though never accurate, rules of thumb are approximations in which the error involved is considered a minor disadvantage which is balanced by the simplicity of its application. The endeavor is to develop a rule which can be applied to the dimensions readily measured or estimated by the eye. The following are offered as examples.

A. Cubic Foot Values.

1. FERNOW'S RULE.

For trees with a form factor* value approximately of 0.5 and averaging 80 to 100 feet in height, the cubic foot volume of the entire tree is equal to the basal radius squared.

$$V = r^2$$

Thus, in a 90 foot tree with a basal diameter of 18 inches, the cubic volume would be equivalent to 81 cubic feet.

2. SCHENCK'S RULE.

For "*average*" trees, the total volume in cubic feet is equal to one fifth the basal diameter squared.

$$\begin{aligned} V &= \frac{D^2}{5} \\ &= \frac{(19)^2}{5} \\ &= \frac{361}{5} \\ &= 72 \text{ cubic feet} \end{aligned}$$

B. Board Foot Values.

1. As based on the Doyle Rule. The basal diameter (i.b.) on the stump is estimated. The number of standard 16 foot logs is estimated and an estimate is made of the top d.i.b. of the top log. The mean of the top d.i.b. and the basal d.i.b. is assumed to be the diameter inside the bark on the top of the average 16 foot log in the tree. The volume of this log according to the Doyle rule multiplied by the number of standard logs gives the merchantable volume of the tree in board feet:

$$V = \{(D - 4)^2 \times N\}$$

Thus in a tree with a basal diameter of 18 inches containing 5 logs, the topmost of which has a top d.i.b. of 10 inches; the d.i.b. of the average log would be equal to 14 inches and the board foot volume of the tree would be:

* For definition of form factor, see Section 141.

$$V = \{(14 - 4)^2 \times 5\}$$
$$= 100 \times 5$$
$$= 500 \text{ board feet.}$$

2. Subtract 60 from the square of the estimated* diameter at the middle of the merchantable length of the tree. Multiply the result by 0.8 and the product will be the volume of the average log in board feet. Multiply this result by the number of 16 foot logs in the tree to get the total merchantable volume of the tree.

$$V - \{(\text{Mean } D^2 - 60) \times 0.8\} \times N$$

Thus in a 4 log tree with a diameter at the middle of the merchantable length of 12 inches,

$$V = \{(12^2 - 60) \times 0.8\} \times 4$$
$$= (144 - 60)\ 0.8 \times 4$$
$$= (84 \times 0.8) \times 4$$
$$= 67 \times 4$$
$$= 268 \text{ board feet}$$

3. A rule used in the U. S. Forest Service by J. W. Girard, formerly lumberman in the Northern District with headquarters at Missoula, Mont., added 6 to the D.B.H. of the tree. This value was then divided by 2 and was used as the top d.i.b. of the average log in the tree. This log was then scaled and the volume was multiplied by the number of standard 16 foot logs to get the volume of the tree.

$$V = \left\{\frac{D + 6}{2} \text{in board feet}\right\} \times N$$

Thus in a 4 log tree with a D.B.H. of 16 inches,

$$V = \left\{\frac{16 + 6}{2}\right\} \text{ in board feet multiplied by 4.}$$

And an 11 inch 16 foot log according to the Scribner Decimal C rule has a volume of 70 board feet. The volume of the tree equals 4 times 70 or 280 board feet.

This rule holds good for western white pine and spruce cut to a 6 inch top and for western larch cut to an 8 inch top. When Douglas fir is cut to an 8 inch top, add 4 inches instead of 6 inches, and with lodgepole pine cut to a 6 inch top add 5 inches.

* This rule does not consider a tree with a middle diameter of less than 8 inches as one containing measureable board foot volume.

For western yellow pine under 20 inches D.B.H. add 6 inches; from 20–25 inches D.B.H. add 8 inches; for 26 inches D.B.H. and over add 10 inches.

4. A rule used by T. S. Woolsey, Jr., formerly of the U. S. Forest Service, while working with western yellow pine in Arizona and New Mexico was to add $1\frac{1}{2}$ times the number of 16 foot logs to the measured D.B.H. and divide by 2. This gave the d.i.b. of the average 16 foot log. This log was scaled and multiplied by the number of logs to get the volume of the tree. Thus in a tree with a D.B.H. of 20 inches, running 5 logs to the tree,

$$V = 20 + (5 \times 1\tfrac{1}{2} \text{ or } 7\tfrac{1}{2}) = \frac{27\frac{1}{2}}{2} = 13\tfrac{3}{4} \text{ or } 14.$$

A 16 foot 14 inch log according to the Scribner Decimal C rule has a volume of 110 board feet. Multiplying 110 by 5 gives 550 board feet as the volume of the tree.

CHAPTER VII

THE DETERMINATION OF VOLUME IN FORESTS

90. Purpose and Application. — The actual volume contained in a forest can never be exactly known until the trees which compose it are felled, sawed up, and the final volumes totaled. This procedure is impracticable when large areas are considered, especially when the information is desired in anticipation of sale or utilization or as a basis for predicting growth.

The determination of the volume of stands and forests involves two distinct but complementary problems, namely:

(*a*) The measurement of the area of the stand or forest.

(*b*) The measurement of the volumes of the trees which are growing on this area.

These two processes are so complementary to one another that a high degree of accuracy or intensity of refinement in one activity avails little unless corresponding care and precision is taken with the other. Careful measurement of trees and exact determination of volume will prove to be so much wasted effort if applied to a stand whose area has been determined by careless surveying or pure guess. Conversely, error and carelessness incurred in the measurement of the volumes of the trees will not be compensated by an accurate determination of the area. Carelessness in both activities only intensifies the inaccuracy of the estimate.

91. Total vs. Partial Estimates. — When the volume of a stand of timber is determined by the actual measurement of every tree found present on the area, such estimate is known as the *Total Estimate*. This is the most exact method of estimating. Variation in type, form, character, crown class, height, diameter, and all the rest, can be disregarded by the estimator since each individual tree is considered on its own merits and is estimated accordingly. The limitations of the method are the expense involved, and the physical impracticability of its application to large areas.

When the volume of the timber, particularly on large areas of forest land, is determined from the intensive examination of selected samples within the tract judged to be representative of the whole, it is known as *Partial Estimate*. Consistent accuracy on the portion of the area actually measured is demanded as the only means of attaining accuracy for the whole.

When definite information is desired concerning the volume of the timber standing on extensive forested areas, the usual practice is to make a partial estimate on such sample areas as are included within:

(*a*) Random plots.
(*b*) Line plots.
(*c*) Strips.

92. Random Plots. — These are representative areas chosen for measurement by the cruiser for the purpose of determining the value and volume of timber for a much larger stand of similar prevailing character. Such plots are best used for checks on ocular judgment. The main objection to the use of these random arbitrary plots for the compilation of stand volume is that their location is too apt to be influenced by the personal preference of the cruiser, whose judgment ever tends to their selection under conditions either too good compared to, or too much below, the average run of timber on the tract.

93. Line Plots. — The random plot method is systematized by tallying all the timber on a continuous line or series of plots of uniform area placed arbitrarily along compass lines at specified distances apart. The main consideration is to be given more to the distribution of the plots than to their particular size. It is, in reality, a modification of the strip method, in which continuity is sacrificed to speed. The time-consuming delays of counting or tallying timber are restricted to smaller intervals separated at predetermined distances along the lines of travel. The plots may be square, rectangular, or circular, the last being the more common.

The exact amount of area actually covered by the plots depends directly upon three factors:

1. The relative size of the plot.
2. The spacing or the distances between centers when measured on the same cruise line.
3. The spacing as influenced by the distances between adjoining and parallel cruise lines.

These factors can be best understood by reference to the accompanying Table V.

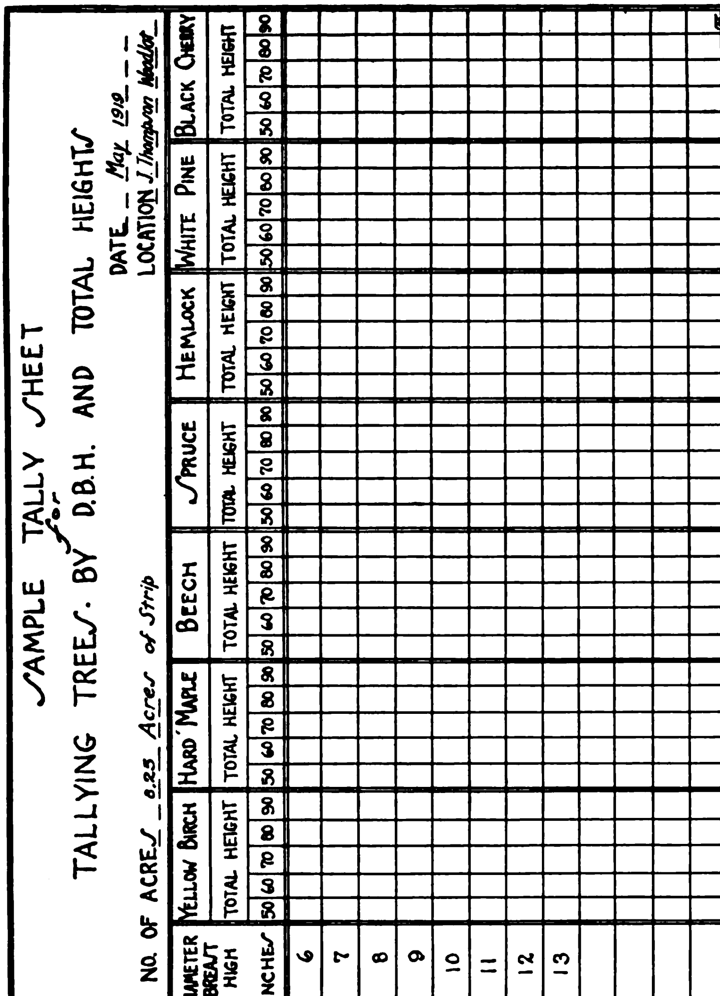


FIG. 31. — A Form of Timber Tally Which Classifies Trees by D.B.H. and by Total Height.

TABLE V

PERCENTAGE OF AREA COVERED BY LINE PLOTS ACCORDING TO SIZE AND SPACING

Distances in Chains as Measured from Center to Center on the same Cruise Line	Size in Acres	Plot Dimensions: Circular — Radius in Feet	Plot Dimensions: Square — Length of one Side in Feet	Per Cent of Area Actually Covered when the Interval between Adjacent and Parallel Cruise Lines as Measured in Chains is: 40	20	10	5
10	1/4	59	104 1/4	5/8 of 1	1 1/4	2 1/2	5
	1/2	83	147 3/8	1 1/4	2 1/2	5	10
	1	118	208 1/2	2 1/2	5	10	20
5	1/4	59	104 1/4	1 1/4	2 1/2	5	10
	1/2	83	147 3/8	2 1/2	5	10	20
	1	118	208 1/2	5	10	20	40
4	1/4	59	104 1/4	1 9/16	3 1/8	6 1/4	12 1/2
	1/2	83	147 3/8	3 1/8	6 1/4	12 1/2	25
	1	118	208 1/2	6 1/4	12 1/2	25	50
2 1/2	1/4	59	104 1/4	2 1/2	5	10	20
	1/2	83	147 3/8	5	10	20	40
	1	118	208 1/2	10	20	40	80

From the above definite conclusions can be drawn, namely:

1. That with maintenance of spacing and cruise line interval the percentage of the area covered increases directly with the increase in the size of the measured plots.

2. That with a given size of plot, the percentage of the area covered decreases as the distance between plot centers on the same cruise line is increased.

3. That with a given size of plot and a given spacing, the percentage of the area covered decreases directly as the interval between cruise lines is increased.

The circular line plot method of estimating when run with care and understanding is one of the best available. It is used very extensively by the U. S. Forest Service in Districts 1 and 4. One argument which has been advanced against its general adoption is that it is less accurate than the strip method inasmuch as the plots often fall in over-dense or in understocked areas, and that a representative sample of the stand is not secured. This point undoubtedly might hold true for small areas of " spotty " timber and where small sized plots are spaced at intervals of 5 chains or more on the same cruise line. But when one considers the fact that with ¼ acre plots spaced at intervals of 2 chains, there is a distance of only 14 feet between the outer edges of adjacent plots, this argument scarcely stands.

The choice of the exact size of the plot and the interval between adjacent plots on the same cruise line, though depending pimarily upon the per cent intensity of estimate desired, is influenced greatly by the character and distribution of the stand. In open stands of western yellow pine ½ acre plots taken at intervals of 2½ chains may be preferred. In brushy country, such as the western white pine type of northern Idaho ¼ acre plots taken at intervals of 2 chains have proved satisfactory. On the Ouachita National Forest in Arkansas and on several of the national forests of the Montana-Idaho district, ⅙ acre plots spaced at intervals of 2 chains have shown better results than had been obtained from the strip method in the same type of timber. The spacing of line plots should always be predetermined and arbitrary; in no case should the selection of plots be left to the judgment of the cruiser.

Tallying on line plots may be done by diameters and height classes or diameters and log lengths according to the type of volume table available. Care must be taken to see that the plot number and plot description coincide with the forest map. A method of tallying used with success by J. W. Girard, logging engineer of the U. S. Forest Service in District 7, formerly in District 1, is to count the number of trees per plot and derive their average D.B.H., estimate the number of logs per tree and then to estimate the number of logs per thousand. The tally sheets are lined up to show the proper description. One column is provided for the plot number, and as many columns for species as is required. The entry for a given plot under western yellow pine, for example, of 6/5/10 means that there are 6 yellow pine trees averaging 20 inches D.B.H. running 5 logs to the tree and 10 logs to the thousand, indicating 3000 board feet per quarter acre plot or

12,000 board feet per acre. It is claimed that this is the most rapid method of cruising known where an actual tally of the timber is made.

The advantages of the ¼ acre line plot method regardless of the system of tallying are:

1. The method can be used very effectively by a one-man crew.
2. The position of the estimator on his line of travel designates the center of the plot and provides an easy point from which to measure or pace out to the line trees which establish the boundaries.
3. The cruiser usually selects as his beginning a tree which is easily recognizable and by counting or measuring round the circle in one direction the danger of counting individuals twice, or missing trees, is reduced to a minimum, or is practically eliminated.
4. The estimator is standing still at the center of his plot while estimating diameters and heights and for this reason better results are obtained.
5. When the cruiser checks his judgment of dimensions with occasional instrumental measurement and bases his determination of volume on standard volume tables, added confidence is given to his estimate.

94. Strips. — Lines of predetermined width (½ chain, 1 chain, 2 chains, etc.), or strips, are run parallel to one another at arbitrary distances apart. Within the limits of the strip, all trees falling within specified dimensions are measured and tallied according to diameters, species, height, or any other specification which may have seemed desirable. All strips are definitely tied at each end to points of established control, either primary or secondary base lines, and are placed to the best advantage when run in a direction that is perpendicular to the main stream drainage so as to run at right angles to the major topography, thus intersecting all types and conditions of timber between the valley bottoms and watershed divides. The actual manner in which the strips are run is of little importance, but great emphasis must be placed on the percentage of the area covered. This percentage is governed by the width of the strips themselves and by the interval between them. Fundamentally, however, the percentage covered depends on:

(*a*) The time available to make and complete the estimate.

(*b*) The value and volume of the timber itself as approximated by guess and the desire of the owner for having a close estimate.

(*c*) The average permissible cost per square mile or acre. This is the real criterion for the percentage chosen in covering the area. In short, the timber estimator must cut his garment to suit

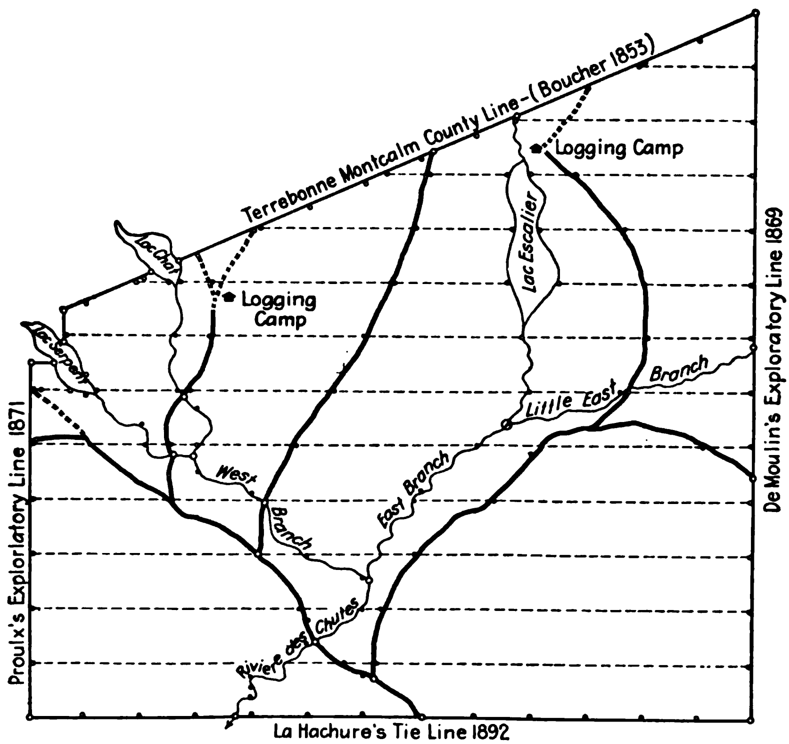


FIG. 32. — System of Running Strips for Timber Estimating in Unsurveyed Country. County and Other Official Survey Lines Form Primary Control. Secondary Control is Established by Survey of Roads, Trails and Driveable Streams. One Chain Strips are Run at Intervals of 20 Chains for a 5 Per Cent Estimate.

his cloth, or, in other words, gage the closeness of his estimate by the money available.

The width of the strip and the interval between two parallel strips are the fundamental determinatives of the intensity or percentage of the estimate. When the interval between strips is constant and the width of the strip is doubled, it can easily be seen that the percentage estimated is doubled. It may be a matter of question whether a wide or a narrow strip should be used in a given case. In open stands of western yellow pine, for example, the estimating of the timber on a strip two chains wide would not require twice as much time as the estimating of the timber on a one chain strip and the results from the wide strip would be much more accurate. Even if it did require twice as much time the increased accuracy from the wider strip would favor its use. In dense stands of timber, particularly small sized trees, and in stands of timber with thick impeding underbrush, as in the Puget Sound region, the extra effort and cost* of the wider strip would greatly exceed the proportional increase in accuracy and would not be justified. Probably in this case if increased intensity is desired, efforts should be bent rather toward reducing the distance of the interval between the strips.

95. Comparative Accuracy Between Line Plots and Strips. — When both are carefully measured, there is little difference between the comparative accuracy of strips and line plots. Where strip widths are measured at intervals of 66 feet, probably the difference between strip volumes per acre and carefully determined line plot volumes *on the same area* will not exceed one-half of one per cent. Where strip widths are checked only by pacing, this deviation may increase as much as 5 per cent. When the 66 foot width of strip is simply estimated with no measurement as a check, the error incurred may be as great as 21 per cent. This inaccuracy is not a constant, but shows a wide range of variation with different individuals.

The general tendency of cruisers who use the estimated width of a 66 foot strip seems to be to overestimate in scattered open stands of timber, and to underestimate in dense stands. This error is probably due to the inability of the cruiser to maintain his strip boundary as a straight line. In open stands line trees (trees on the line) always present difficulty and are apt to sway personal judgment; and the tendency seems to be to include many trees concerning which there may be some doubt, thus producing an overestimate. In very dense stands the opposite is often the case. The cruiser fails to appreciate the full width of his strip and as a result there is a considerable underestimate.†

* In the Pacific Coast forests it is estimated that the average cost for a 10 per cent cruise should be 1 cent per thousand of timber standing for western yellow pine in California and for Douglas fir in Oregon and Washington.

† Cited from correspondence with J. W. Girard.

The common practice with circular line plots is either to estimate the dimensions of the circular plot by the eye, or to pace off four cardinal radii. The square line plot lends itself better to accurate measurement. Distances should be taped, not paced, and corners established by compass or angle mirror. As soon as plot dimensions are left to ocular estimate inaccuracies similar to those cited above for strip width will be found.

The task of carefully measuring a series of square plots on cruise lines, though it attains maximum accuracy, very materially slows up the work and adds to the expense. This factor generally precludes its extended use in timber cruising. However, when carefully and accurately located the method lends itself to further use as a sample plot of semi-permanent character and thus serves other purposes than merely securing an estimate of the present stand of timber.

96. Advantages of Strips. — Of the three methods suggested, that of continuous strips of one or more chains in width, running in parallel direction and spaced at equal arbitrary distances, may be preferred for the following reasons:

1. These strips when tied to permanent base lines completely gridiron the tract and form the best basis for obtaining the data for a reliable map as the basis of accurate stand area.

2. Strips which start and stop on a point of permanent location can be considered sample areas of semi-permanent character, for the determination in later measurements of the rate of growth and increment.

3. The estimate is always under the control of the cruiser, since the amount of ground covered is always known.

4. Cruisers are forced to traverse all kinds of timber types and measure the trees on all sorts of country, good and poor, including open areas, swamps, cliffs, etc., thus getting a better representative estimate of the tract.

5. The occurrence, extent, and proportion of the different types, barren and non-timbered areas, water surfaces and burns, can be obtained with reliable accuracy.

6. A simple and progressive record can be kept of the tally sheets.

7. The field work in this system is a continuous progression with a minimum of interruption and delay.

The great advantage of the method is its definite endeavor to reduce all guessing to a minimum. It also lends itself to use by inexperienced or non-technical men. Extra care must be taken in the supervision of such type of labor, due to a tendency to hold the calipers too low and thus record trees as representing larger dimensions than actually exist, and also to the fact that untrained men so often fail to recognize defective material, particularly in the upper portion of the tree. The results with non-technically trained men are much better when a gross estimate rather than when a net estimate is made.

97. Factors Governing the Width of Strips. — The factors governing decision as to the width of the strip are:

(*a*) The relative experience of the crew in their work.
(*b*) The method of calipering and tallying the data in the field.
(*c*) The degree of brush as affecting range of vision.
(*d*) The size and density of timber.

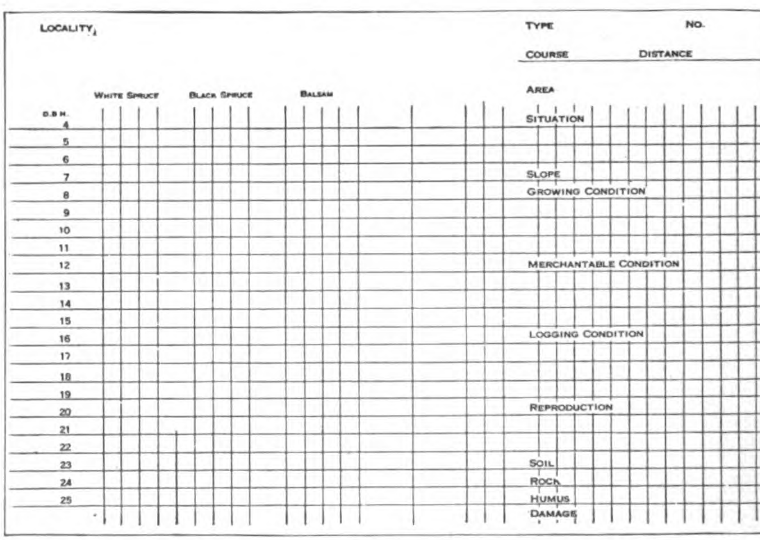


FIG. 33. — A Concise Form of Tally Sheet Where the Estimate Gives Major Consideration to Two or Three Associated Species as in a Pulpwood Estimate.

Usually the 1 chain strip is preferred. In open country, with a scattered stand and with little underbrush, the strip may be widened to 2 chains or more without any great diminution in speed. The degree of underbrush and the density of timber is always a criterion. The relative experience of the crew is important.

The width of the strip also varies inversely with increase in the number of trees. As trees increase in number in a stand, the diameters tend to decrease. Conversely, the larger the diameter, the fewer the trees per acre and the wider the strip. With a freedom from obstructing underbrush, increasing openness of stand and greater irregularity of stocking, strip or partial estimate methods may give way to a count or tally of trees on the entire area. This will usually be accomplished by working up the estimate in units of 40 acres, one quarter of a quarter section, a standard subdivision in the rectangular system of surveying, and a common term in land subdivision in the western forests.

The width of strip adopted depends also upon the methods of tallying the number of measurements required for each tree. The greater the amount of detail required, the narrower the strip permissible in order to secure adequate speed. The smaller the number of men included in the cruising party, the more restricted is the number of measurements, the more is the dependence on ocular estimates with an increasing of the width of the strip.

98. Factors Governing the Interval Between Cruise Lines. — The interval between strips, or the spacing of the cruise lines, is dependent upon several factors:

1. The value, relative volume and density of the timber to be examined. The more valuable the timber, the greater its volume and the more intense the estimate as reflected by a closer spacing.

2. The kind, density and effect of the underbrush or timber. Freedom from underbrush permits not only easier travel but also a greater length of vision. Strips can be spaced at wider intervals, especially where a portion, or all, of the dimensions are estimated by the eye. Dense underbrush, on the contrary, will tend to narrow the strip interval.

3. The degree of relief and the steepness of the slopes. The main effect of slope often is to develop more pronounced variations in timber types, while an absence of relief may be conducive to type uniformity. In order to seek an exact correlation of area and volume, the cruiser must take measures to acquire the exact delineation of types by mapping them. The immediate effect of pronounced variation in timber type is to narrow the cruise line interval.

4. Whether a topographic map is to be plotted in connection with the survey, and the value of the precise contour interval adopted. The closer the contour interval, the closer spaced the cruise lines must be.

5. The degree of accuracy required in the estimate.

6. The method of measuring and tallying the timber. The more precise these methods, the narrower the strip and the closer the spacing to get a desired degree of accuracy.

7. The fundamental purpose of the estimate in itself; the more precise the purpose, the closer the spacing.

99. Surveying and Estimating. — In the estimating of large forest areas, there is a definite application of the technique of the engineer to a forest problem. It is known as a forest survey. Even the roughest cruise of a stand of timber requires a definite knowledge of location. It will imply the making of a map, perhaps only a rough sketch, locating, for example, the main bodies of timber and the general course of streams. From this, there are all kinds of gradations up to the most accurate and most elaborate of type and topographic maps.

The only conditions under which a map may not be required are those where either a 100 per cent estimate is made, or timber is cruised by small compact units such as forties (40 acre squares). Yet even in this method, surveying is definitely suggested in the implication, first, that a survey establishing the forties has been made prior to the cruise, and, second, that the lines indicating their existence on the ground must be recognized and actually identified by the cruiser in the progress of his work.

A map adds little or nothing to the accuracy of the measuring of the dimensions of the timber on the area actually covered. It does, however, greatly increase the efficiency of the estimate and the accuracy of the application of the volume measured.

100. Forest Maps. — The information gathered in a forest survey is compiled and presented in two forms, the estimate and the forest map. One is not only supplementary to, but also complementary to, the other. Taken together, they are to be regarded as the inventory of the forest property, setting forth its resources concisely, completely and clearly. Maps made in such a way that all of the pertinent information is clearly shown, and kept up to date by annual revision, offer an easy and convenient method of taking boards of directors and executives of large timber owning corporations over the ground, thus enabling them to visualize and appreciate its development and to obtain a comprehensive view of its operation. Only too often is it the case that the men who are financially interested in such enterprises are quite out of touch with the actual field operation, and have no adequate knowledge of the problems to be met in its development and management. Due to this lack of appreciation, there are many forest properties abandoned and wasted which might have been maintained on a basis of continuous forest production, which is the goal of all forestry.

Frequently, the knowledge and information most pertinent to efficient management of a forest property is contained within the brain of one key man. His removal from the concern, through resignation or death, places a heavy burden on the new manager which is only overcome through the gradual acquisition of knowledge over a long term of years. A forest map and estimate endeavor to collect and coördinate this information and compile it in such form that it is not only available but easily digestible, so that a few months of intensive study will place a new manager in that position of knowledge and information of his property only possible under the older order after years of experience.

Forest maps are of three classes:

Type maps.
Topographic maps.
Drainage maps.

Often these are combined, the types, designated by different colors, being superimposed on a topographic or drainage map. The combined form has the advantage of being more compact, and of more easily revealing the relations of the timber to the topography.

Types recognized in forest mapping are generally cover types based on the *cover then on the ground.* Type distinctions should always be drawn on broad, simple lines capable of easy and consistent identification by the field men. It is best to have a few types which can be mapped in with consistent accuracy. A large number of types only leads to the confusion of the field men and becomes a source of chaos in mapping.

When timber types are recognized, all tree tallies must be rigidly separated on the basis of type. This means that just as soon as the cruise line intersects a change of type a new tally sheet must be used. It is important that the field notes, the timber tally, and the map must be in absolute agreement as to timber type at any given point.

The purpose of a topographic map in forest mapping is to show the relation of the timber to the ground on which it is standing. It is usually a contour map of conventional form with a contour interval of 20, 25, 50 or 100 feet, depending on the intensity of the work as reflected by the relative closeness of spacing of the cruise lines and the dependability of the vertical control. Another important point which must be considered is the relative degree of relief and the steepness of the slopes. With a flat terrain, a contour interval of 100 feet would be practically of very little value whereas on a 40 to 60 per cent slope it would probably show all the detail necessary. Although all of these factors are important, probably the last will outweigh all others in considering the contour interval to be adopted in any specific case. In the California forests with valuable stands of timber and probable railroad logging in a rough mountainous country, they figure on a 25 foot contour interval with a 10 per cent cruise and a 50 foot contour interval with a 5 per cent cruise. This agrees with a rule more or less general with commercial cruising concerns working in the northeast, to the effect that, where a topographic map is desired, the contour interval in feet should be equivalent in value to $2\frac{1}{2}$ times the distance between cruise lines as measured in chains.

Topography may also be indicated by a series of hachures. The hachure map when skillfully and accurately done presents a very convincing picture of the actual topography, but the difficulties and expense of its construction seem to forbid its general adoption.

A form of forest map used in several of the districts of the U. S. Forest Service is the drainage map. The location and position of the main ridges are shown by some conventional sign such as a series of herringbones or hachures. The ramifications of the drainage are shown in some detail, including not only the main streams but secondary creeks and feeders. The direction of the slopes are shown by a series of arrows, the gradient, usually as a per cent, being indicated by a numeral at the base of the arrow.

101. Methods of Timber Estimating. — The precise methods to be employed in obtaining the estimate of a stand of timber depends primarily upon the value of the timber in question, and secondarily upon the size of the area within which it is contained. Obviously, the more valuable the timber the more intensive the estimate, and the more precise the methods used. Conversely, the larger the area the less precise the application and closeness of the estimate. Under all conditions of estimating where the entire area is not examined the main problem of distribution is to so locate samples as to obtain a fair and reliable representation. The larger the area the greater the opportunity of securing an adequate sample by their judicious scattering.

Methods Applicable to Small Areas. — Small tracts must be estimated much more intensively than large tracts of timber. This is mainly due to the impossibility of getting a representative sample from small areas, but in great part may be induced by the value of the timber standing on the area itself. As a rule, small sized areas of valuable timber ought to be cruised by a total estimate. It is rather difficult to state as to what size area determines classification within this category, but where the value of the timber justifies it, no tract of 50 acres or less should be cruised by less than a 100 per cent estimate.

Methods applicable to small areas are described as follows:

1. Each tree is regarded as a unit, and the cruiser examines every individual tree, mentally dividing it into its several component logs. The top diameter and length of each of these logs is estimated and reference to the current log rule gives their volume in board feet. Deductions from this scale for each log for defect are made as may be indicated, according to the best judgment of the cruiser. The total volume of all of the logs gives the volume for the tree. The total volume of all the trees examined on the area gives the volume for the tract.

2. All of the trees on the area are examined by the cruiser and tallied by D.B.H.o.b. A sufficient number of heights on D.B.H. are taken to construct a reliable height curve. A standard volume table (Section 151) constructed on the basis of D.B.H. and corresponding heights is reduced to the status of a local volume table (Section 156). The number of trees tallied within each diameter class is multiplied by the corresponding tree volume to determine their volume. The total volume of all the diameter classes gives the volume of the area.

3. The cruiser carefully examines every tree on the area and makes a count of all that are merchantable. At the same time he

makes a decision as to the D.B.H., height and appearance of the *average tree.* This average tree may be fictitious, but it is more apt to be actual and present, and is definitely picked out and identified by the cruiser. This average tree is very carefully examined and its volume estimated. The volume multiplied by the number of trees counted gives the total volume for the area.

4. All of the trees on the area are carefully examined by the cruiser and are mentally sectioned into their several component logs. Logs are counted but not tallied. While counting, the cruiser makes a decision as to the dimension of the average log on the area, its length and top d.i.b. The volume of this log according to the local log rule multiplied by the total number of logs counted gives the volume of the area.

5. All of the trees on the area are examined. Logs are again counted, but not tallied as before, but, this time, the mental processes of the cruiser work, not toward a decision of the average log, but toward the average number of logs required to make 1000 board feet of lumber, or the *log run.* The number of logs counted divided by the log run gives the volume of the area.

6. The stand is completely covered by the estimator so that he *sees* it entirely. He may make a mental estimate of its area. Area may also be obtained by pacing or by separate measurements. Irrespective of which method of obtaining area is used, the cruiser definitely endeavors to make a determination and decision as to the average volume per acre *from ocular inspection only.* This acre volume multiplied by the number of acres gives the volume of the area.

Many variations of these six methods of determining volume are used by cruisers in estimating the volume of stands. They are, as a rule, best applicable to pure stands, or to small stands with a rather restricted diameter range, and containing but two or three commercial species of similar character and value. The introduction of a wide range of diameters enlarges the opportunity of error in determining the tree or log of average size. The ability to determine the average tree or the average log or the average stand per acre is a faculty that is developed to a wonderful degree in some individuals. It is based on wide experience and training, but is derived mainly from an intense personal capacity in the man himself to do such work. Whether he determines his decisions of the average from a single glance or from logical deduction it is rather to be regarded as a gift than an art. In the hands of a skilled and able man its results are surprisingly close and accurate but in the hands of the unskilled it is never more than an approximation.

Methods Applicable to Large Areas. — Large areas are seldom if ever cruised entirely. The volume is usually determined by some form of partial estimate. Apart from the physical impracticability of entirely covering extensive tracts of forest land within a given time, sufficient accuracy can be attained by equal care on a much smaller sample. The technical forester seeks to offset his lack of experience or personal capacity by the mechanical accuracy of the line strip or line plot systems. Which of these he shall use depends to a great extent on personal choice and training, some men leaning to one method and some to the other. Cruise lines are generally run with staff compass and chain, at intervals established by the desired cruising per cent, between base lines of primary, secondary or even tertiary control arbitrarily established at predetermined distances apart. Under certain conditions the staff compass may give way to a hand compass, and the chain to pacing, but with less precise instruments and methods it is not probable that the same degree of accuracy can be attained without the expense of extra time and effort.

The size of a cruising unit varies from one or two, to three or four men, depending upon the type and character of the timber, the method of recording the data, and the degree of topographic detail and silvicultural information desired. Diameters may be measured with calipers, diameter tape, Biltmore stick, or estimated by eye. Heights may be measured by hypsometer or estimated by eye with occasional hysometer check. The data taken may simply be a record of the dimensions of the timber actually measured or estimated, or it may include observations of growth and reproduction with topographic data of sufficient intensity to make a detailed contour map. Each particular job presents its own set of problems which can only be answered on the ground according to the conditions actually encountered. Hence the following examples of methods of partial estimate are not to be taken as absolute prescriptions capable of covering every case, but are merely cited as examples of what has been done under specific conditions for a specific purpose, and are to be accepted as but a general guide, which may be adapted to meet local conditions as circumstances may warrant.

Case I. A logging project in a California National Forest* is being examined in anticipation of a timber sale. Engineers from the district office establish points of primary control by triangulation and define the area. Since topography is of paramount importance in logging in this region, they also establish such points of secondary control within the area as will be of maximum assistance to the cruising party. A cruising unit consists of two men and the size of the cruising party will consist of as many cruising units as can most efficiently carry out the work. Two, four, six, or even eight units may make up the party depending upon the size of the operation.

The first task of the cruising party consists of establishing tertiary control. Tertiary control in this region consists of running the standard township and section lines over the logging unit according to the U. S. system of Government rectangular survey. Tertiary control must be definitely tied to all points of established primary and secondary control. Cruise lines 1 chain wide are run up and down the slopes at distances 10 chains apart, and are tied to the tertiary control at 2 mile intervals.

The cruise or estimate is made by separate "forties." A cruising party consists of a two man unit. One man acts as compassman, front chainman, topographer and mapper. Absolute elevations are carried forward by means of topographic Abney and slope chain and the map is built up as the work goes along. The other man acts as rear chainman and estimates and tallies† the timber. All trees on the strip above a given diameter

* The exact procedure followed by the U. S. Forest Service for cruising National Forest timber varies in the several regions according to the local problems of logging, the character of the topography and the relative volume and value of the timber. The method described above holds for the California district during the period 1923–1929.

† Comment by J. W. Girard. "I believe that better results can be obtained by having the compassman tally the timber and the estimator do the mapping. This makes it possible for the estimator to concentrate on the width of the strip, diameters and heights, because he does not have to pay any attention to the tally sheet. While the compassman is going ahead to his next set-up the estimator does the mapping, classifies the site and counts the reproduction and the saplings in cases where this is required. I usually classify the site at every 2 chain point and count the reproduction and the saplings on a circular plot of 11.7 feet radius, which is 1/100 of an acre."

TIMBER TALLY SHEET

LOCALITY Woodlot Jas. Wilson. Canastota, N. Y.

12.3 ACRES.

BEECH

DATE June 6, 1920.

TALLEYMAN Allen

DIAM. BREAST HIGH, INCHES	1 LOG TREES						2 LOG TREES						3 LOG TREES					
	CUT			LEAVE			CUT			LEAVE			CUT			LEAVE		
	TALLEY	NO.	VOL.	TALLEY	NO.	VOL.	TALLEY	NO.	VOL.	TALLEY	NO	VOL.	TALLEY	NO.	VOL.	TALLEY	NO.	VOL.
6	.	1	10	:.	3	36	:.	3	45	ꟷ:	7	115						
7	:	2	24	I:	5	65	⊠	10	200	::	4	80						
8				::	4	64	I:	6	150	.	1	25						
9	.	1	20	:	2	36	I:	6	180	:	2	60	.	1	77			
10				.	1	22	.	1	47				:	2	174	::	4	348
11	.	1	35				:.	3	156				::	4	400			
12				.	1	29	::	4	208	:	2	182	I:	6	720	:.	3	360
13	:	2	70	.	1	35	:	2	132	:	2	220	:	2	280	:	2	280
14				:	2	84	:.	3	222	::	4	520	I:	5	700			
15							::	4	480				⊠.	11	1870			
16										::	4	600	::	4	800	:	2	460
17							.	1	180				:.	3	690	:.	3	810
18													::	4	710	:	2	620
19													:.	3	910	::	4	1400
20													::	5	1950			

FIG. 34. — A Compact Form of Tally Sheet for Small Areas Where Only a Portion of the Standing Trees are to be Cut.

are measured with a Biltmore stick. Merchantable lengths to a 10 inch top are estimated by eye or measured by hypsometer. The major species are tallied separately.

The topographic map is constructed on a 25 foot contour interval, although in the Redwoods districts a 20 foot contour interval is used. Although each " forty " is cruised as a unit and types are mapped as accurately as possible, tally sheets are not changed with type variations within a given " forty."

Volumes are determined by multiplying the number of trees tallied on the strip within each D.B.H. and log length class by the corresponding volume as taken from standard volume tables for the species. Volumes within each species are totaled for the strip and are converted to volumes for the forty by multiplying by the " strip multiplier." The strip multiplier is a figure based on the ratio of the total timbered acreage of the forty to the actual total timbered acreage on the strip within the forty.

Cull is estimated from observation as influenced by actual experience in timber of similar character on adjacent areas.

The total net volume of all of the forties gives the volume for the logging unit.

In the Idaho-Montana region cruise line control (tertiary control) in unsurveyed country is secured by running base lines or traverses from established points, along the tops of the ridges regardless of cardinal direction and of running the cruise lines down the slopes to the valley bottoms and up again. A variation of this is to run a base line or traverse up a valley and run the cruise lines up the slope to timber line and back again.

In the even aged stands of the Puget Sound region the procedure would be varied somewhat to meet local conditions. The cruising unit would consist of three, not two men. Due to the great density of the high and thick underbrush it has been found advisable to use a third man as rear chainman whose particular duty it is to read the Abney Hand Level and calculate the difference in the elevation. He is assisted in his Abney observation by the compassman flashing a small mirror to sight at. In this region, cruising by forties is disregarded and particular attention is concentrated on type and age class changes within the distance of each cruise line.

In the Arkansas forests with a smaller volume per acre and a less valuable and more open stand of timber a still different variation is used. The cruising unit is a one man party who runs his cruise lines by staff compass and pacing. Cruise lines originate and are tied to primary control which consists mainly of township and section lines, but in their absence may consist of compass and chain traverse of main and secondary roads. Topography is eliminated but a drainage and type map is built up. The volume of the timber is obtained from ¼ acre circular (sample) line plots spaced at distances 2½ chains apart with a cruise line interval of 10 chains thus obtaining a 10 per cent estimate. On each plot trees are tallied by D.B.H. and standard log length with volumes obtained subsequently from standard volume tables. Two miles of cruise line a day in rough country and four miles per day on level ground is regarded as a good day's work.

CASE II. A commercial cruising concern with headquarters in Seattle undertakes the appraisal of timber lands owned by a lumber company operating in the western white pine region in northern Idaho.

Exterior boundaries and all the primary control are run in with some care and accuracy. Secondary base lines may be chained or may be paced. Cruise lines, though conforming to the convention of direction up and down the slopes, are run in with hand compass and are paced for distance. Cruise lines consist of strips 1 chain wide, the width being *estimated, not measured,* except by an occasional check by pacing. The cruise line interval varies according to the intensity of the estimate so as to cover from 5 to 10 per cent of the total area. Diameters and heights are estimated, not measured, except as an occasional check. Types as designated by a rather simple schedule are mapped in. Volumes are taken from a standard volume table for the various species tallied. Cull is estimated as a flat percentage and the gross estimate is discounted by that amount. The cruising unit is a two man crew, one of whom acts as compassman, topographer and pacer, the second man estimates and tallies the timber.

CASE III. A holding company controlling some 2 million acres of pulpwood land in eastern Canada desires to place its timberland on a better productive basis. As a first step a forest survey and cruise of the whole area is made. The method adopted has this peculiarity, that all measurement of trees is reduced to a mechanical count and that the three pulp species of spruce, balsam, and hemlock are considered as one.

Primary and secondary control base lines are run in with compass and chain without horizontal correction excepting on very steep slopes where chain is broken. Cruise lines are run by hand compass and pacing at distances of 30 to 40 chains depending on the density of the timber. Cruise lines consist of strips 1 chain wide estimated, not measured, except by a very occasional check by pacing. Measurements of diameters and heights are absolutely eliminated, but a count of all the coniferous pulp trees on the strip is made. In this count, the trees are divided into two groups, those above 12 inches and those below 12 inches as measured at D.B.H. The cruising unit consists of two men, one who acts as compassman and pacer, the other who counts and tallies the timber. Types are mapped rather carefully. Once a week two cruising crews unite to run a day's calipered strip, the purpose of which is to get the average dimensioned tree below and above 12 inches. Volumes are taken from a local volume table on a straight D.B.H. basis.

CASE IV. A commercial cruising concern specializing in southern yellow pine with headquarters in Jacksonville, Fla., undertakes the examination and estimating of some 20,000 acres of forested land belonging to a lumber company operating in western Alabama. Exterior boundaries and primary and secondary control base lines are established by compass and chain. Cruise line control base lines are at intervals of 1 mile. Cruise lines $2\frac{1}{2}$ chains wide are run by hand compass and pacing at intervals of 10 chains, thus covering 25 per cent of the area. The width of the strip is estimated for the most part, with occasional check by pacing. *All* of the trees on the strip width are *counted*, but only *every fifth tree* is tallied, this mechanical selection eliminating all the factors of the personal equation in the choice. The dimensions tallied are the D.B.H. and the merchantable length as expressed in log lengths. These dimensions are for the most part estimated by the eye, with occasional check, a Biltmore stick — Merritt hypsometer being carried into the field for such purpose. Species are tallied separately. The volume for the trees actually tallied is obtained from standard volume tables for the species. This volume is multiplied by 5 to obtain the full volume on the strip, that is, on 25 per cent of the area, and is then proportionately increased to apply to the entire tract. Forest types are recognized and mapped in. The usual cruising unit is a two man crew. One man acts as compassman, pacer, and tallies the dimensions of the trees actually measured or estimated. The second man counts all of the trees on the strip and measures or estimates the dimensions of the selected trees.

Case V. Pacific Coast timber, particularly sugar pine, western yellow pine, and Douglas fir presents special problems to cruising. The large dimensions of height and diameter, the tendency of the trees to heavy butt swell, the variation in taper, and the heavy stands per acre all justify a much more intensive method of cruising than is used in other sections of the country. These areas are generally cruised in 40 acre units with surveyed section lines at mile intervals used for primary base lines. One method used is as follows: Secondary base lines between the forties are not run. Cruise lines consisting of 2½ chain strips are run at intervals of 3¾ chains thus covering 50 per cent of the area. The width of the strip is estimated by eye and distance along the strip is paced. Each tree on the strip is considered individually. The number of logs, the presence of defect affecting the volume and the possible loss in falling through breakage are also considered. After weighing all factors the cruiser tallies the total volume of the tree directly. The total volume of all trees on the strips multiplied by 2 gives the total volume on the area.

Case VI. A commercial cruising concern with offices in Montreal undertook to estimate some 2 million acres of pulpland located in the upper Ottawa valley. In this estimate all hardwoods were disregarded except insofar as they affected type classification. Exterior base lines and primary control were run with compass and chain. Driveable streams and main roads were surveyed in primary control by plane table and chain. Secondary base lines were run with compass and chain at intervals of three miles paralleled to the average direction of the main drainage. Cruise lines were run up and down the slopes at intervals of 30 chains by hand compass and pacing. These cruise lines consisted of strips theoretically 1 chain wide, thus attaining a 3⅓ per cent estimate. Actually it did not because no timber was tallied on these strips.

At the beginning of each cruise line a ¼ acre sample plot was taken, circular in shape, with its dimensions estimated. On this plot all standing timber of the pulpwood species was tallied by species and D.B.H. By reference to local volume tables carried into the field by the cruiser, the volume of these trees was determined at once and converted to terms of volume per acre. Proceeding forward on his cruise line, the cruiser, basing his judgment on the plot just tallied, made an ocular estimate of the stand per acre (pulpwood species only) for each 100 paces of distance that he traversed until he had covered ½ mile of distance when another ¼ acre sample plot was taken and the volume per acre established as before. The purpose of this and subsequent plots taken during the day was (*a*) to check his judgment or ocular estimate for the *last half mile covered*, and (*b*) to orient his judgment anew. Forest types were carefully mapped and on this particular estimate topography was taken for a topographic map by means of a standard Abney clinometer. The average volume per acre by timber types for minor drainage areas was determined and the total volume built up in this way.

The cruising unit consisted of two men parties: a compassman who paced distance and read the slope values with clinometer, and a cruiser who kept the timber record and mapped.

102. The Influence of Defect on the Estimate. — The technical forester is deficient, as a rule, in estimating cull. It is on this point that he was excelled by the old time cruiser. It is one factor of the estimate which is directly checked up by the mill scale. Experience necessary to estimate cull satisfactorily can be achieved by:

(*a*) Carefully watching the effect of cull in the sawing out of logs in the mill.

(*b*) An extended experience in identifying and estimating defect in the standing tree.

The methods for allowing for defect are three:

1. Reduction of the tally by the omission of defective trees or logs.

2. Reduction of the dimensions of the trees tallied in proportion to the amount of the defect.

3. Tallying everything as sound, and from the total volume deducting a flat percentage in proportion to the total amount of timber affected by the defect. The deduction per cent is based on:

(*a*) The observed presence and frequency of defects.

(*b*) The relative size and maturity or over maturity of the timber.

A man with a little experience in cull can do fairly well if he recognizes those factors on which defect depends, namely, site, Sheet No.

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE

Form 494-D. 6.
Revised 1920.

Section Plat

Show course by—

T........ (Logging unit) .... Compassman

R........ Type........ .... Estimator

S........ Site........ .... Date

40........ Timbered Acres on 40— / Timbered Acres on Strip— = Strip Multiplier

........M

D. B. H.	Y. P.	S. P.	W. F.	D. F.	I. C.			S.	D.
6"-11" by tens.									
12									
14									
16									
18									
20									
22									
24									
26									
28									
30									
32									
34									
36									
38									
40									
42									
44									
46									
48									
50									
52									
54									
56									
58									
60									
Vol. on Strip									
Cull %									
Net Vol. on Strip									
Vol. on 40									

FIG. 35. — Form of Tally Sheet Adopted by the U. S. Forest Service in its Timber Estimates in Western National Forests. The Trees may be Tallied by D. B. H. Alone, or by D. B. H. and Log Lengths.

size, and age. A rather extended study of the effect of these factors is of great assistance especially on the handling of uniform coniferous stands. In handling hardwoods, a much greater range of experience is required.

103. Lumber Grades in Estimating. — It is naturally a question whether a statement of the net volume of a piece of timber is sufficient. It is a recognized fact that trees do not all saw out the same quality of lumber. Lumber is classified as to quality on the basis of its grades; the highest grades of course attract the best price in the commercial markets. Hence it would be to the advantage of the timber owner if net volume of the estimate were classified according to the amount of the different standard grades that it is capable of sawing out.

Like the determination of cull the accurate estimating of lumber grades in the standing tree is based on considerable experience in local mills under the local conditions of sawing. It is also important to recognize the main factors in the standing tree on which depends the distribution of grades in the log. There are four in number:

1. Clear length.
2. Age and fullness of bole.
3. Relative dimensions in diameter of the timber.
4. Freedom from defect.

Granted this, the bigger the D.B.H., the longer the clear length, the older the tree and the freer from defect, the larger is the possible percentage of lumber of the more valuable grades.

Probably the best way* to determine the grades which a stand of timber will produce, provided reliable mill scale studies are available on the basis of log grades, is for the head cruiser to run representative quality strips. A few strips on the different sites on which the trees are tallied by diameters and the number of logs of the different grades will give satisfactory results.

There are three other methods of assigning grade values to the volume of a standard estimate:

1. Placing an approximate percentage of the total estimate in each standard grade based on observation and experience with that class of timber. This is a very poor way as it is only an ocular estimate and a personal judgment at the best.

2. Making, as the tree is tallied, a separate examination of each

* Cited from correspondence with J. W. Girard.

log of standard length and assigning a percentage of its volume to each standard grade.

3. The use of Graded Volume Tables. This has been tried out with some success in the Southern Hardwood Region and in the Puget Sound Basin on the Pacific Coast. The main objection to the tables is their excessive cost in time and money in construction and their restricted and local application to the timber for which and in which they are made.

104. Relative Speed in Estimating. — The speed of the crew is measured by the number of miles of cruise line covered by the crew per day, and is the principal factor in figuring up the expense. The actual amount of area that has to be covered by the cruise is predetermined by the desired closeness of the estimate, and is directly governed by the spacing of the strip or line plot interval. If the work is well planned and the men fairly skilled, the various parts of the organization will keep step and time with each other. The feature which most determines the actual daily speed of the crew in the field is slash in the East and brush in the West. There are also the variable factors of swamps, steep slopes and cliffs whose occurrence when met do their part to hold up the field men.

Relative Speeds per Day with a One Chain Strip

1. Very heavy timber and dense brush, 1 mile per day.
2. Western white pine, Engelmann spruce, and associated species, with good brush, 1 to 2½ miles per day.
3. Small brushy timber like the eastern white pine, lodgepole pine, eastern spruce and northern hardwoods, 3 to 4 miles per day.
4. Open yellow pine (western) no brush, 3 to 4 miles per day.
5. Open yellow pine (southern) some brush, and swamp 4 miles per day.

Maximum record in open timber, very open, 12 miles per day. To tally logs instead of trees will slow down the above 25 per cent.

105. Costs of Estimating. — There are no standardized costs in timber estimating. Costs vary from region to region, and most usually are determined for each particular job in accordance with the influence of such important factors as:

1. The numbers, skill, efficiency and speed of the cruising crew.
2. The intensity of the estimate as measured by the spacing of cruise lines, the methods of measuring and tallying the timber, the degree of accuracy desired.

3. The relative accessibility of the cruising area to lanes of an established transportation system and the utilization thereof.

4. The character, dimensions and density of the timber itself.

5. The character and kind of map and report desired and the amount of detail in field work necessary to meet this demand.

6. The relative amount of boundary survey and primary, secondary or tertiary control base lines and traverses necessary for the efficient prosecution of the work.

7. The amount of office work in mapping and computation necessary subsequent to the completion of the field work.

The expense of making a forest survey varies from a fraction of a cent to 40 or even 60 cents per acre. The latter cost should cover a very detailed type and topographic map.

Although the cost is usually figured on a per acre basis, some lumber companies prefer to have it computed by logging units and expressed as a cost per thousand feet of standing timber. This is merely a simple matter of mathematical prorating.

106. The Results of a Forest Survey. — From the work of the timber estimate and survey certain definite results in the way of a composite report are to be achieved. This report should include:

1. A map showing:

(*a*) An outline of the topography (preferably with contours).

(*b*) Roads, trails, railroads, driveable streams and all other methods of accession and transportation.

(*c*) Camps, depots, dams, lookouts, telephone lines and all other permanent improvements.

(*d*) Cuts, burns, and windfalls, classified according to the amount of merchantable timber left.

(*e*) Forest types classified according to composition and density of stand and condition (virgin or cut-over).

2. An estimate of the merchantable timber, arranged according to drainage areas or logging units, and showing in detail the amount and location of the different kinds, sizes and qualities of timber.

3. An estimate of the cost of logging each area.

4. An estimate of future timber growth (time required and volume expected).

5. A general report, including an opinion on the market value of the standing timber.

CHAPTER VIII

THE APPLICATION OF STATISTICAL METHODS

107. Definition and Scope. — The field and office operations of forest measurement deal with the collection of quantitative data which of necessity must be arranged, averaged, analyzed and interpreted before presentation in a form capable of being adequately understood. The problems so presented are similar in scope, character and adaptability of general solution as are those of the various biological and physical sciences which repeatedly deal with highly developed complexes of multiple effect due to more or less easily recognized causations of natural origin. The means to solve these problems finds its easiest expression in the application of statistical mathematics which, though most familiarly used in the fields of applied economics and sociology, finds a ready use when applied to problems dealing with the variability of any natural phenomena. According to Yule* the term "statistics" refers to " any quantitative data affected to a marked extent by a multiplicity of causes." The quantitative data may be of two classes:

1. Those referring to the presence or absence of a certain characteristic or feature among the individual units included within a certain grouping. This class of data is said to deal with *attributes*.

2. The degree or varying magnitude with which the individual units possess a certain characteristic or attribute capable of mathematical gradation, valuation, or measurement in terms of some standard. This class of data is said to deal with *variables*.

An example of the first class of data, attributes, might be cited as the absence or presence of infection with rot, as shown by the trees of a stand of virgin western yellow pine. An example of the second class of data, variables, would be the degree of rot or severity of infection, as influenced by age or size (D.B.H.) or site or crown class, etc.

* An Introduction to the Theory of Statistics, by G. Y. Yule. G. Griffin & Co., London, England. 1919.

It will have been appreciated that the work of the whole field of forestry deals with variables of living things, trees, height (or age) on diameter, diameter on age, volume on the number of trees per acre, growth in diameter, or height, or volume, on age, etc. In order to coördinate properly the diverse magnitudes of the variables recognized, it is necessary to initiate and develop a system of classification or a *class series*. The interval between each class and the next is a gradation of magnitude known as the *class interval* or the *class differential*. It is arbitrarily chosen as to value, and must be uniform in the total scale of magnitude. The number of examples of each variation of magnitude is known as the *class frequency*. It is important that there should be a method of comparing the class frequencies, and this is attainable only through uniformity of the class interval. The manner in which the recorded observations are distributed through the frequency classes is spoken of as the *frequency distribution*, and the entire body of specified observations is known as a *frequency group*.

A common example of a frequency group is a stand table, which is a tabulated statement showing the number of trees standing on the average acre of an area of forest land (see page 286). The attribute is the identification of the trees listed as belonging to, or not belonging to, a given species. The variable is the magnitude of size. Each diameter class is a frequency class, the number of trees included in each size or diameter class being the frequency. The range of the diameter classes and the numbers of items included in each being the frequency distribution.

There are three methods available for classifying frequency groups in forest measurement:

1. A measurement of every member of a specified group, as, for example, measuring *all* the trees on a sample area. A corollary of this would be the determination of the average or mean sample tree.

2. A measurement of a number of trees arbitrarily chosen as samples of a much larger group or number. An example of this is the measuring and tallying of the diameters of the trees on a series of line plots or on a series of strips in cruising. The trees tallied are accepted as a sample of all the timber standing in the forest, the greater part of which may be located on the extensive area between the cruise lines. The actual area covered by the plots or strips is but a small, though known, percentage of the

entire tract, the numerical value of this percentage being fixed by the intensity of the estimate.

3. A determination of the mathematical probability that the observed frequencies possess a minimum degree of variation, that is, fall within an arbitrarily predetermined or assumed degree of accuracy.

Whichever classification of the frequency group is recognized or applied, all may be characterized as follows:

(*a*) By a determination of its central position — the average (*the mean*).

(*b*) By a determination of the spread (*dispersion*) of the recorded observations from this center.

(*c*) By a determination of a lack of symmetry (*skewness*) in the frequency distribution about this center.

108. The Theory of Sampling. — It follows, on reflection, that the conclusions derived from the measurement of forests must be based to a large degree on but a partial examination and measurement of the whole field of data available, that is, on the practice of taking samples. When a series of observations is selected out of a mass (of living trees), it follows that there will certainly be variation of determinable character between the individual samples. But due to the influence of the laws of chance, it will be found that these variations tend to fluctuate around a certain mean (arithmetic average). The largest number of items will be found clinging close to this mean, and as the spread from the central point becomes greater, the number of items will become fewer. Increase in the number of samples chosen tends rather to augment the intensity around the mean than to extend the degree of variation.

This being the case, the application of measurement and observation to an *adequate* number of samples will reveal conclusions the stability of which permits us to apply to the whole forest generalities derived from the samples. *This is known as the application of the theory of simple sampling.*

There are three conditions, however, which must prevail before the full degree of the validity of this theory can be acceptable:

1. That there is a definite uniformity in all nature and that the samples selected from the whole mass of items available, to which the study is confined, and which is conventionally known as the "**universe,**" shall partake of this uniformity. This does not

mean that it is not necessary to have absolute information regarding the degree of or lack of uniformity within the group of items so much as it means that it is necessary to have the assurance that there is a definite limitation to the magnitude of independent variability, and that for this reason samples from the universe will exhibit fluctuation only within certain prescribed limits.

2. That the circumstances surrounding the selection of each item of the sample shall be identically the same. The most variable factor in the selection of samples is human judgment. Usually we eliminate the human equation in the selection by establishing some arbitrary procedure, such as, for example, counting all items in the universe and selecting as samples only every tenth item. An example of this is the method of cruising described on page 124 where the cruiser *counted all* the trees on the strip but *tallied only every fifth tree.*

3. That the individual items forming the sample as a whole shall be absolutely independent of one another in time, in place, and in condition.

These circumstances prevailing, all other factors of accuracy in measurement, analyzing and judgment being uniform, generalizations derived from the sample may be applied with certitude to the universe as a whole.

109. Measurement of Dispersion. — When a set of frequency classes within a frequency group are rearranged and retabulated in order of ascending or descending magnitude of the observed variable, it is known as a **Table of Dispersion,** or more briefly, a Dispersion Table. The central point of this dispersion table, as measured in terms of ascending or descending magnitude, is known as the *mean*, the *mode*, or the *median*. These are different ways of expressing the average of dispersion. They are not three ways of stating the same thing, but result from three different methods of calculating a similar value. Lack of symmetry or skewness is always present when the mean, the mode, and the median fail to coincide.

For our present purpose the " **mean** " only need be considered. Both the mode and the median belong to the field of higher mathematics in the applied science of statistics and are therefore omitted from this discussion. The mean here considered is always the weighted arithmetic average (Section 120).

The degree of dispersion, or *range*, as expressed by the mathematical differences between the magnitude of the mean and the magnitude of a given frequency class, is known as the **deviation.** Deviations derived from class magnitude values greater than those of the mean are known as *plus deviations* and those which are derived from magnitudes of less value are known as *minus deviations* (see Table VI).

To obtain some knowledge regarding the *relative fraction* of variation in the different groups, it is necessary to seek a coefficient of dispersion. This is a mathematical value which endeavors to express the extent by which the individual items of a frequency distribution differ *on the average.* It is stated in terms of the variable and indicates how typical is the average of the distribution it represents. The larger the value of this coefficient the wider the range over which it can be distributed. Small coefficients may indicate more accurate work but narrow the range over which such precision is applicable. The more common of such indexes are: (1) the average deviation and (2) the standard deviation.

110. The Average Deviation. — The average deviation is logically the most obvious measure of dispersion. It is the mean of the sum of all deviations irrespective of algebraic sign. Expressed as a formula in accordance with the values subsequently more fully discussed (Section 111):

$$\text{Average deviation} = \alpha = \frac{\Sigma x}{N}$$

where x = the actual deviations times the frequency regardless of algebraic sign.

Σ = the sum of.

Hence in the example shown in Table VI, page 137

$$\alpha = \frac{\Sigma x}{N} = \pm \frac{302}{73}$$
$$= \pm 4.13 \text{ or } \pm 4.1 \text{ square feet}$$

This means that on the basis of the assumed mean of 19 square feet per sample plot and within the range given, the average divergence per observation was $\pm$ 4.1 square feet. As such, the average deviation represents one of two things, or both working together:

(*a*) A certain inaccuracy of observation in the repeated measurement of similar items.

(*b*) The variability of the observed attribute throughout the range of observations.

As an average which expresses the deviation of each original item from the mean, it has some value and appears to be an obvious characterization of the usual lack of conformity, as would be expected. It has one great limitation, which is, that since its calculation ignores all mathematical signs it does not lend itself to any further mathematical treatment or use as can be accomplished in the use of the standard deviation.

111. The Standard Deviation. — The standard deviation is the square root of the sum of the weighted squares of all the deviations as measured from the mean, which latter is either assumed or computed. The easiest method is to assume this mean directly from the items available in a table of dispersion; and computation of its true value is reserved for a later operation. Attention is directed to the complete dispersion table, Table VI; and an endeavor will be made to describe briefly the significance of each of its subdivisions. Columns are numbered 1 to 8 from left to right.

To complete any dispersion table, the values in columns 2, 4, 6, and 8 must be totaled.

The Standard Deviation = s

$$= \sqrt{\frac{\text{The sum of weighted squares of all deviations}}{\text{Total number of observed items}}}$$

Hence
$$s = \sqrt{\frac{\Sigma d^2 f}{N}}$$

It has been found by actual experience that a more satisfactory value is computed by employing $N - 1$ rather than N as the denominator of the above equation, which then becomes

$$s = \sqrt{\frac{\Sigma d^2 f}{N-1}}$$
$$= \sqrt{\frac{2{,}448}{72}}$$
$$= \pm\ 5.831 \text{ square feet}$$

Although the standard deviation is not as easily calculated as the average deviation, it is generally more acceptable due to the fact that it is rigidly defined, is relatively stable and lends itself readily to further use and mathematical treatment.

TABLE VI

DISPERSION TABLE SHOWING BASAL AREA PER ¼ AREA SAMPLE PLOT FOR 73 PLOTS IN MIXED HARDWOOD TYPE

St. Lawrence County, New York

(Attribute) Total Basal Area in Sq. Ft.	(Frequency) Number of Occurrences of Items f	Plus Deviations		Minus Deviations		Weighted Deviations Squared	
		$+d$	$+df$	$-d$	$-df$		Deviation Squared Times Frequency d^2f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
35	1	16	16			$(16)^2 \times 1 =$	256
31	3	12	36			$(12)^2 \times 3 =$	432
27	2	8	16			$(8)^2 \times 2 =$	128
25	6	6	36			$(6)^2 \times 6 =$	216
23	10	4	40			$(4)^2 \times 10 =$	160
20	12	1	12			$(1)^2 \times 12 =$	12
19	10	Assumed Mean from Inspection					
18	9			1	9	$(1)^2 \times 9 =$	9
16	6			3	18	$(3)^2 \times 6 =$	54
14	5			5	25	$(5)^2 \times 5 =$	125
12	3			7	21	$(7)^2 \times 3 =$	147
9	2			10	20	$(10)^2 \times 2 =$	200
7	2			12	24	$(12)^2 \times 2 =$	288
5	1			14	14	$(14)^2 \times 1 =$	196
4	1			15	15	$(15)^2 \times 1 =$	225
Total	$N = 73$		156		146	$\Sigma d^2f =$	2,448

Column 1. The range of variables within the frequency group averaged by predetermined intervals into frequency classes, and these frequency classes arranged in order of descending* magnitude.

Column 2. A tabular statement of the number of individual observations of respective magnitude included within the average of each frequency class. This is known as the *frequency*. Its symbol is f. The total of all of the frequencies, that is, the aggregate of all of the individual observations or items within the frequency group, may be symbolized as N.

Column 3. Deviations of frequency classes whose magnitude is greater

* It makes no difference whether the array is in ascending or descending order as long as the gradations are constant and uniform.

than that of the origin are known as *plus deviations* and their symbol is $+ d$.

Column 4. The weighted value of each plus deviation. It is the product of the values tabulated in columns 2 and 3. Its symbol is $+ df$.

Column 5. Deviations of frequency classes whose values are less than the magnitude of the origin are known as *minus deviations*. Their symbol is $- d$.

Column 6. The weighted value of each minus deviation. It is the product of the values in columns 2 and 5. Its symbol is $- df$.

Column 7. Is generally omitted but is here used to show the step by which the values in column 8 are derived.

Column 8. The square of each deviation, which must be a plus quantity because it is a square, multiplied by its respective frequency. Its symbol is d^2f. The sum of all the values in this column gives us that value which is the sum of the squares of *all* of the deviations within the frequency group. The symbol of this total is $\Sigma^* d^2 f$.

Like the average deviation, it is an index of the average digression of each individual item from the mean. It may be used for the calculation of the standard error, the mathematical probability of success, the number of items needed to meet a specified degree of accuracy, and coefficients of skewness and of correlation.

112. Determination of the True Mean. — The mean as an expression of the average is a sum of all items of variation within the group divided by their number. This definition specifically presupposes that it must be a weighted average.

In actual practice it is even more simple inasmuch as its calculation is based, not on the items of variability in magnitude within the group, but upon the deviations of individual items from an assumed mean.

Reference is to be made to the Table of Dispersion shown on page 137.

1. By inspection select any midway value as an arbitrary mean. A little care in picking this value is to be urged. Choice should be influenced not only by its position midway in the range of deviations but also by the number of items or frequencies as they occur within this range. This value is known as the *Assumed Origin*.

2. Tabulate the deviation of all other items in the table of dispersion from the assumed origin designating plus and minus values by the proper sign.

* Greek letter Σ = Sigma (= S) symbolizes "sum of."

3. Multiply each deviation by its frequency.
4. Total all weighted plus deviations 156.
5. Total all minus deviations 146.
6. The true mean is the algebraic sum of the assumed origin and the computed difference. Expressed as a formula:

$$M = O + \frac{(+\,df) + (-\,df)}{N}*$$

where M = true mean.
O = assumed origin.
df = weighted sum of deviations.
N = total of frequencies.

$$\text{Then } M = 19 + \frac{(156) + (-\,146)}{73}$$
$$= 19 + \frac{10}{73}$$
$$= 19 + .137 = 19.137$$

113. The Maximum Error. — The maximum error may be a matter of relativity in that its value results from an arbitrary choice as to what shall be the criterion of accuracy within a given range of observations. As such, it is a statement of the maximum limits of variability in the observation or measurement of any one item as based on the mean of all observations. For the dispersion table of basal areas shown on page 137, we calculated a true mean (Section 112) of 19.137 square feet. If we arbitrarily set a standard of accuracy of 5 per cent plus or minus we get

$$\text{Maximum error} = e$$
$$= \pm\, 5 \text{ per cent of } 19.137 \text{ square feet}$$
$$= \pm\, 0.957 \text{ square feet}$$

114. The Standard Error. — The standard error is dependent upon either the desired degree of accuracy required as expressed in the determination of the maximum error, or on the standard deviation.

In terms of the desired degree of accuracy the standard error should never exceed one-third, or, at the most, one-half of the value of the maximum error. Statisticians have constructed probability

* A slightly more accurate method of computing standard deviation than that used in Section 111, is to use $\Sigma d^2f - d_0$; where Σd^2f = the sum of the squared value of all deviations and d_0 equals the difference between the assumed origin and the true mean. The result which is secured is practically the same as with the formula used, and this form is preferred.

tables, or tables of chances, which show that the chance of any error exceeding twice the standard error is 1 in 22, and three times the standard error about 1 in 370.

Hence S = standard error

= $\frac{1}{3}$ maximum error

= $\frac{1}{3}e$

= $\frac{1}{3}$ (0.957)

= $\pm$ 0.319 square feet

This may be known as the *arbitrary standard error.*

As computed from the standard deviation:

S = standard error

= standard deviation $\div$ the square root of the number of frequencies

$$S = \frac{s}{\sqrt{N}} = \frac{5.831}{\sqrt{73}} = \frac{5.831}{8.544} = \pm 0.679$$

This may be known as the *computed standard error.*

The discrepancy between these two calculations is a direct index of the absolute accuracy of the work done.

When the standard error as calculated from the maximum error according to a predetermined degree of accuracy exceeds the standard error as calculated from the standard deviation, it means that more observations have been made than will be necessary to meet the specified requirements.

When the standard error as calculated from the maximum error on a per cent accuracy basis underruns the standard error as computed from the standard deviation, it is to be taken as an index that more observations or measurements are necessary in order to meet the desired degree of accuracy. Or, in other words, these two methods of calculating standard error should coincide under optimum conditions of measurement and accuracy.

115. Number of Observations Required. — The number of observations which will be acceptable as a sample of all the items of the universe partaking in varying magnitude of the attribute observed depends most particularly upon the degree of accuracy required.

Let x = the number of observations required

Then $x = \left(\frac{s}{S}\right)^2 = \left(\frac{\text{Standard deviation}}{\text{Standard error}}\right)^2$

$= \left(\frac{5.831}{0.319}\right)^2$

$= 231.6$

or $= 232$ plots

Since 73 plots are already taken this means that in order to meet a 5 per cent accuracy standard some 159 plots should still be measured, granting that the same conditions of variability in attribute and accuracy in measurement are to be expected. Or in other words, no greater variability is to be expected for the samples still to be taken than has characterized the items measured so far.

The great advantage of this formula is that with but a relatively small number of observations completed, the computation of the standard deviation opens the way to a calculation of the precision of the work and the number of observations still to be taken in order to meet a desired degree of accuracy. In this calculation, the *arbitrary standard error* is the one commonly used.

116. Precision of Work Done. — An index regarding the precision of work done is always desirable. The following are the steps:

Calculate the computed standard error,

$$S = \frac{s}{\sqrt{N}} = \frac{5.831}{\sqrt{73}}$$
$$= \pm\ 0.679 \text{ square feet}$$

Compute the maximum error,

$$e = 3\,S$$
$$= 3 \times 0.679$$
$$= \pm\ 2.037 \text{ square feet}$$

Then, since the maximum error is to the value of the assumed mean as the percentage is to 100,

$$e : M :: \text{per cent} : 100$$

and

$$\text{per cent} = \frac{e \times 100}{M}$$
$$= \frac{2.037 \times 100}{19.137}$$
$$= \frac{203.7}{19.137}$$
$$= 10.5 \text{ per cent}$$

117. Rejection of Doubtful Measurements. — In any collection of field data there is inevitably included a number of measurements the dependability of which seems a matter of doubt. The existence of such measurements is readily revealed in graphical plotting (Chapter IX) by their abnormal location far outside the trend of an otherwise normal and regular curve. The usual practice is to disregard the more extreme and to include others of varying degrees of influence. Even then, there is a matter of doubt as to which should have been excluded and which should be included in the consideration.

The computations involved in a dispersion table, and in the calculation of standard deviation, provide a ready and easy means of solving this problem. When abnormality occurs the first resort should be to the field sheets, which should be carefully examined and rechecked for possible mistakes. On failing to find cause for exclusion by that method, recourse may then be made to the norm established by the standard deviation.

According to the law of error, there is a certain range within which the bulk of deviation falls, and beyond that range the odds that favor the probability of extreme deviation occurring are small, and become theoretically increasingly smaller. The chance of any deviation occurring outside of this range is less with a small number of observations than with a large number, and tables have been developed based on possibilities of inclusion of abnormal measurements within a given number of observations. These may be used as a guide in making a decision to reject abnormal values.

TABLE VII

STANDARDS OF REJECTION OF DOUBTFUL VALUES

Number of Measurements Taken	Reject apparent abnormalities when the deviation is n times the probable error
Up to 11	$n = 3.0$
12 to 30	$n = 3.5$
31 to 100	$n = 4.0$
Over 100	$n = 4.5$

118. Skewness. — In any frequency group which is normal, it will be found in the dispersion of the items that at points of equal deviation above and below the mean, the frequencies will be equal and the group as a whole will be termed " symmetrical." Such condition indicates a uniformity in the universe from which the samples were drawn. If such values are plotted, as in Fig. 36, there will be developed a symmetrical " bell-shaped " curve as is shown by the heavy line.

A lack of uniformity in the universe should be reflected in a lack of uniformity in the samples and a preponderance of items to

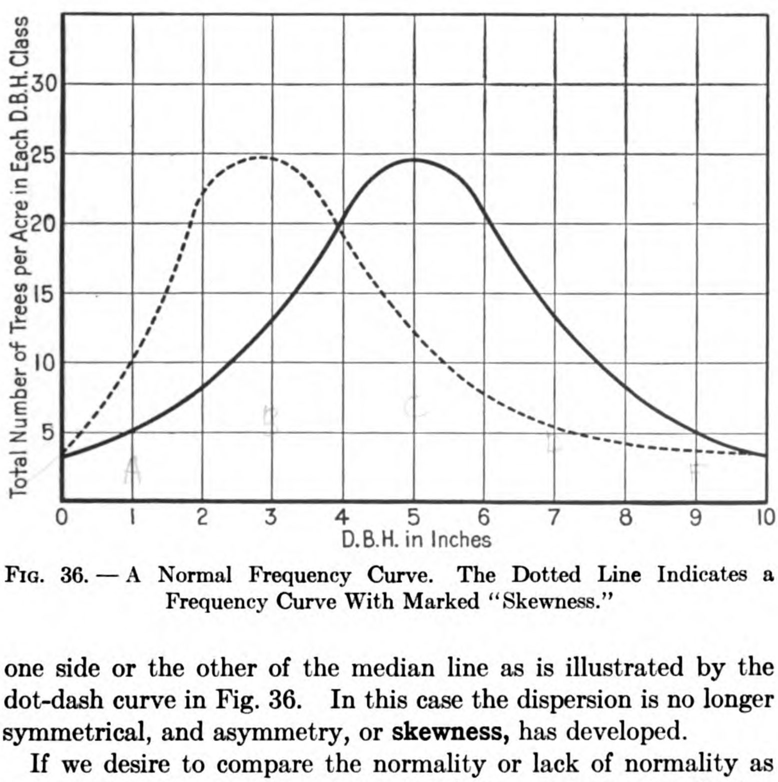


FIG. 36. — A Normal Frequency Curve. The Dotted Line Indicates a Frequency Curve With Marked "Skewness."

one side or the other of the median line as is illustrated by the dot-dash curve in Fig. 36. In this case the dispersion is no longer symmetrical, and asymmetry, or **skewness,** has developed.

If we desire to compare the normality or lack of normality as expressed by the skewness of one set of dispersion items with that of another, it is necessary to reduce their values to a common numerical quantity. Measures of skewness are reduced to coefficients for the same reason that measures of dispersion represent coefficients.

In the case of skewness, however, since the main problem is not how much the curve of normal frequency is skewed in proportion to the relative size of the items, but how much more the curve deviates on one side of the average than on the other, it seems advisable to use as a denominator in the calculation some measure either of the dispersion of the items or of their deviation.

This being the case, the coefficient of skewness may be computed from the formula:

$$j = \frac{\sqrt[3]{\frac{\Sigma d^3}{n}}}{s}$$

where j = the coefficient of skewness.
Σd^3 = the algebraic sum of deviations cubed.
n = the number of observations.
s = the standard deviation.

When this formula is applied to the x variables in Table VIII, page 147, a coefficient of skewness of $\pm$ 0.18 is obtained indicating but a minor degree of skewness in the dispersion.

Skewness in a distribution must always be either plus or minus. A value of zero would indicate perfect symmetry. The value of the algebraic sign indicates the direction of asymmetry, or toward which end the curve is skewed. It is generally regarded that a value of $\pm$ 0.1 indicates but a minor degree of skewness, whereas values of $\pm$ 0.3 or more indicate marked skewness.

The purpose of computing skewness is to determine an indication of the manner in which the items are dispersed on either side of the determined average. It serves as an index of the acceptability of the items, as a whole, as a representative sample of the universe from which they are taken.

119. The Correlation of Variables. — It often happens that in the measurement of attributes there will be developed values which seem to bear some relation to one another in that a high value in one variable seems to be associated with a high value of some other variable of determinable character. But before definite conclusions can be drawn as to the precise nature of such relationships, a correlation analysis* is necessary.

* The method described in this section holds only for those cases where regression is rectilinear. When regression is curvilinear, solution of the problem involves calculus and applied graphics.

The pairing of observations among variables is a common phenomenon. Such pairing may be direct or objective, it may be in space, or it may be in time. When we pair D.B.H. with total height, or crown class, or clear length, or any other factor by direct observation, we accomplish an objective correlation. The pairing of precipitation with current height growth in plantations is an example of correlation in space. The pairing of volume growth with age is a correlation in time.

In all cases the basic fact is that observed magnitudes in one variable are linked in some definite way with observed magnitudes of similar or opposite algebraic sign in another variable, and that between the two there seems to be a causal connection. If a high value in one variable carries the normal expectation of a high (or low) value in the other variable, it then becomes desirable to determine how great a divergence of the one from its mean is associated with a unit of similar divergence in the other, and further as to the degree of closeness with which such relationship is accomplished. This relationship when so demonstrated is known as **correlation.**

The best index of correlation is to be found in a *correlation coefficient,* which is not an expression of absolute values but is a numerical measurement of the *degree* to which correlation exists. A coefficient of + 1 indicates a perfect correlation in the two variables of the same direction. A coefficient of − 1 indicates perfect correlation but in opposite or inverse direction. A coefficient of 0 indicates no correlation whatsoever. The degree of correlation is then indicated by the approach to or regression from unity as expressed by this coefficient.

For convenience the variables observed in the same subjective material and listed in regular frequency groups are known as X and Y. We recognize such correlation in the increase of height with D.B.H., or of D.B.H. with age. If correlation is complete any change in the values of X will be accompanied by changes of similar nature, positive or negative, in Y. If there is complete independence no change will result of recognizable or associated nature.

The coefficient of correlation as an expression of the degree or relative amount of correlation between two variables may be computed from the formula:

$$r = \frac{\Sigma X \cdot Y - n \cdot \overline{x} \cdot \overline{y}}{n \cdot s_x \cdot s_y}$$

where r = the coefficient of correlation.
Σ = sum of.
X = magnitude of one variable.
Y = magnitude of associated variable.
n = number of occurences in which correlation is applied.
$\bar{x}$ = average magnitude for X frequency group.
$\bar{y}$ = average magnitude for Y frequency group.
s_x = standard deviation for X frequency group.
s_y = standard deviation for Y frequency group.

This can best be shown by referring to a definite problem. In the discussion of taper (Sections 142–144) the idea is developed that the denser the stand the higher the Form Class $\left(\text{that is, the Form Quotient as expressed by } \frac{d\frac{1}{2}}{\text{D.B.H.}}\right)$. It is also known that stand density usually affects clear length in that the denser the stand, the greater the ratio or proportion of the stem length which is clear. It is to be accepted that there has not been such clearly demarcated relationship between ratio of stem length clear and form class as to warrant any dependence of this relationship. To illustrate the probability that there is a real and positive relationship between relative clear length and form class the following study was undertaken.

More than 1400 trees of Adirondack spruce growing in the Spruce Flat Type were carefully measured. Form class (Absolute Form Quotient) was carefully determined and was denominated the *subject*, that is, the variable which was used as the standard of measurement of comparison. Within *each* form class the *relative* variable was established as the ratio of clear length in feet divided by total height in feet. The term " relative " is used to designate the variable which is compared or measured in terms of the subject. The averages for the 1473 trees are presented in Table VIII.

TABLE VIII

Coefficients of Correlation

Percentage of Clear Length and Absolute Form Quotient (Form Class)

Based on 1473 Trees of Adirondack Spruce in St. Lawrence County, N.Y.
Spruce Flat Type

X = Ratio of Clear Length in Feet to Total Height in Feet.
Y = Ratio of Absolute $d_{\frac{1}{2}}$ to D.B.H. in inches.

No. of Trees included in each Average	Ratio of Clear Length in Feet to Total Height in Feet X	Ratio of $d_{\frac{1}{2}}$ to D.B.H. in Inches = (F_q) Y	$X \times Y$	X Values Deviation from Origin 29: $d_0 = +0.013$			Y Values Deviation from Origin 70: $d_0 = -0.486$		
				$d+$	$d-$	d^2	$d+$	$d-$	d^2
22	31	70	2,170	2		4	0	0	0
26	33	75	2,475	4		16	5		25
14	27	55	1,485		2	4		15	225
28	27	78	2,108		2	4	8		64
82	29	70	2,030	0	0	0	0	0	0
47	31	75	2,325	2		4	5		25
20	26	50	1,300		3	9		20	400
36	28	71	1,988		1	1	1		1
16	32	75	2,400	3		9	5		25
17	41	79	3,239	12		144	9		81
10	28	71	2,018		1	1	1		1
16	32	70	2,240	3		9	0	0	0
17	23	60	1,380		6	36		10	100
22	31	85	2,625	2		4	15		225
33	28	70	1,960		1	1	0	0	0
26	30	75	2,250	1		1	5		25
18	24	50	1,200		5	25		20	400
11	26	55	1,430		3	9		15	225
14	23	61	1,401		6	36		9	81
8	23	50	1,150		6	36		20	400
	573	1,345	39,174	29	36	353	54	109	2,303
	Aver. 28.65	Aver. 67.25		+ 3.5 32.5	− 3.5 32.5		+ 27.5 81.5	− 27.5 81.5	

$$r = \frac{\Sigma XY - n \cdot \bar{x} \cdot \bar{y}}{n \cdot s_x \cdot s_y}$$

$$\Sigma XY = 39{,}174$$

$$\bar{x} = 28.65$$

$$\bar{y} = 67.25$$

$$s_x = \sqrt{\frac{\Sigma d^2}{N-1} - d_0} = \sqrt{\frac{353}{19} - .013} = \sqrt{18.586} = \pm 4.31$$

$$s_y = \sqrt{\frac{\Sigma d^2}{N-1} - d_0} = \sqrt{\frac{2303}{19} - (-.486)}$$

$$= \sqrt{121.113 + .486} = \sqrt{121.599} = \pm 11.02$$

Hence

$$r = \frac{\Sigma XY - n \cdot \bar{x} \cdot \bar{y}}{n \cdot s_x \cdot s_y}$$

$$= \frac{39174 - 20 \times 28.65 \times 67.25}{20 \times 4.31 \times 11.02} = \frac{640}{952} = +0.67$$

It now becomes of importance to consider the degree of precision or reliability of "r." This degree of precision can best be achieved by calculating the standard deviation of the coefficient of correlation itself. This is computed from the formula

$$s_r = \text{Standard deviation of } r$$

$$= \frac{1 - r^2}{\sqrt{N}}$$

$$= \frac{1 - (0.67)^2}{\sqrt{20}} = \frac{1 - 0.4489}{4.45} = \frac{0.5511}{4.45} = \pm 0.123901$$

The coefficient of correlation, r should then as modified by the standard deviation be read

$$r = 0.67 \pm 0.12$$

The coefficient of approximately 0.55 to 0.79 indicates a fairly high degree of correlation between these two variables. An increase in one is accompanied by a corresponding increase in the other of approximately similar degree and in similar direction.

However, the variation of from 0.55 to 0.79 is not without significance, because it designates probably that 20 pairs of averages form too meager a datum from which to draw definite conclusions other than that a trend or tendency is indicated. Probably if each pair of measurements were based on a larger number of trees more precise correlation would result.

When correlation is once established as it has been in the above, the next step is a calculation of the regression of one variable on another.

The regression of X on Y is computed from the formula

$$a_{\frac{x}{y}} = r \times \frac{s_x}{s_y} = 0.67 \times \frac{4.31}{11.02} = 0.262$$

And the regression of Y on X from

$$a_{\frac{y}{x}} = r \times \frac{s_y}{s_x} = 0.67 \times \frac{11.02}{4.31} = 1.72$$

Thus the regression of X on Y is the deviation in X resulting in a unit change in Y. An increase of Fq from 71 to 72 will be accompanied by a change of 0.262 in dead length ratio. If complete correlation is established these regression values should be approximately equal.

120. Averages. — When quantitative data of any kind are examined in their raw state, they appear as an unwieldy mass of unorganized material. The task of organizing such data constitutes an integral step in the processes of the scientific method of observation, inference, and verification. Organization seeks that form of presentation in which the significance to the purpose at hand and comparison with other data may most easily be accomplished.

The analysis of the data is an operation subsequent to the collection of measurement in the field. Rarely can conclusions be drawn directly from the field sheets. It is only when these facts have been completed, rearranged, totaled, and averaged that they are tabulated for purposes of study and comparison. Such tabulations invariably are lists of averages.

The purposes of averaging are three-fold:

1. To give a compact and complete picture of the whole group of observations or facts of which they are a sample and of which they are truly representative.

2. To serve as a basis of the comparison of different groups and to show the mathematical relationships.

3. To show these relationships on the basis of concrete facts capable of being understood by the average mind.

For our present purposes there is one form of average, namely, — the Weighted Arithmetic Average.

TABLE IX

Two Methods of Determining Average Age of Forest

Age Groups Noted in Forest	Simple Arithmetic Average		Weighted Arithmetic Average		
	Average Age		Number of Trees	Weight* in Averaging	
	Counted	Used		= Age × Number	Weighted Average Age
1–20	17	17	2	2 × 17 =	34
21–40	34	34	21	21 × 34 =	714
41–60	51	51	30	30 × 51 =	1530
61–80	67	67	50	50 × 67 =	3350
81–100	89	89	21	21 × 89 =	1869
Totals		258	128		7497

Simple Average

$$= \frac{\text{Sum of Value of Groups}}{\text{No. of Groups}}$$

$$= \frac{258}{5} = 51.6 \text{ years}$$

Weighted Average

$$= \frac{\text{Sum of (Group} \times \text{No. in each group)}}{\text{Total No. in all Groups}}$$

$$= \frac{7497}{128} = 58.5 \text{ years}$$

* The application of weighting may be made by any mathematical value common to all the groups or occurrences.

The weighted arithmetic average endeavors to give to the groups containing the greatest number of individual items a weight or influence in the direct proportion that their number or sum bears to the total of all the individual items in all groups. The norm for the weighted average will always be found in the center of gravity as indicated by the greatest number of individual occurrences. It is the result obtained when the sum of all of the measurements of all items is divided by the number of items observed. It is based on the idea of assigning such influence to each group as is equivalent to the number of times of the appearance of representatives of the group in the whole field of observation.

Tabulation is the beginning and the end of the analyzing of collected data. It seeks to present definite facts, in definite ways, for definite purposes. Its limitation is its inability to do more than state the facts. The ability to analyze, interpret and elucidate a table is rather a rare power not common to most men. For general circulation and understanding almost every table should be accompanied by written statements which clearly give the facts and definitely draw the conclusions. Even then, for the best interpretation thereof the table should be accompanied or illustrated by some form of graphic presentation.

CHAPTER IX

GRAPHS AND CURVES

121. Construction of Graphs. — When the results of observations of investigative nature have been secured in tabulated form, an important step toward an analysis and proper interpretation of the data is that of presenting these results in graphic form. The most important effect of such procedure is that it serves the very practical purpose of immediately visualizing the results, and thus opens a pathway not only to their clearer understanding but also toward the further investigation of significant relationships. To draw correct conclusions from a table of mathematical values is a task of considerable difficulty. The same figures cast in graphic form tell a simpler and much more easily understood story.

The advantages of graphic presentation are:

1. It offers a quick and ready method for the averaging of mathematical data.
2. It presents clearly to the eye the proper relationship of data, and reveals conclusions only to be achieved after a prolonged study of tabulated results.
3. Accuracy is definitely sought; errors which otherwise might be missed are at once visible to the eye.
4. It offers a ready method for quickly recognizing data which might be abnormal.
5. It is a method whereby there can be adduced general laws for the variation of quantities.
6. It offers a means for smoothing out the irregularities in such variations.
7. It offers the best method to apply a general law to values where the measurements are weak or are missing.
8. Results read from the curves of graphic projection lend themselves readily to presentation in standard tabulated form.

The disadvantages of the method are:

1. Curves so constructed may be based on inadequate premises. There may not have been enough measurements to justify a conclusion. No curve has any value in graphic presentation which is based on less than three points.

2. Difficulty is always experienced in reading from a curve decimal or fractional values, particularly so when less than unity.
3. Error in mathematics may be lost sight of, and may completely distort the trend of the curve.

122. Principles Underlying Construction of Graphs. — A brief résumé of the principles of coördinate geometry, the foundation of graphic presentation, might not be out of place. If two straight

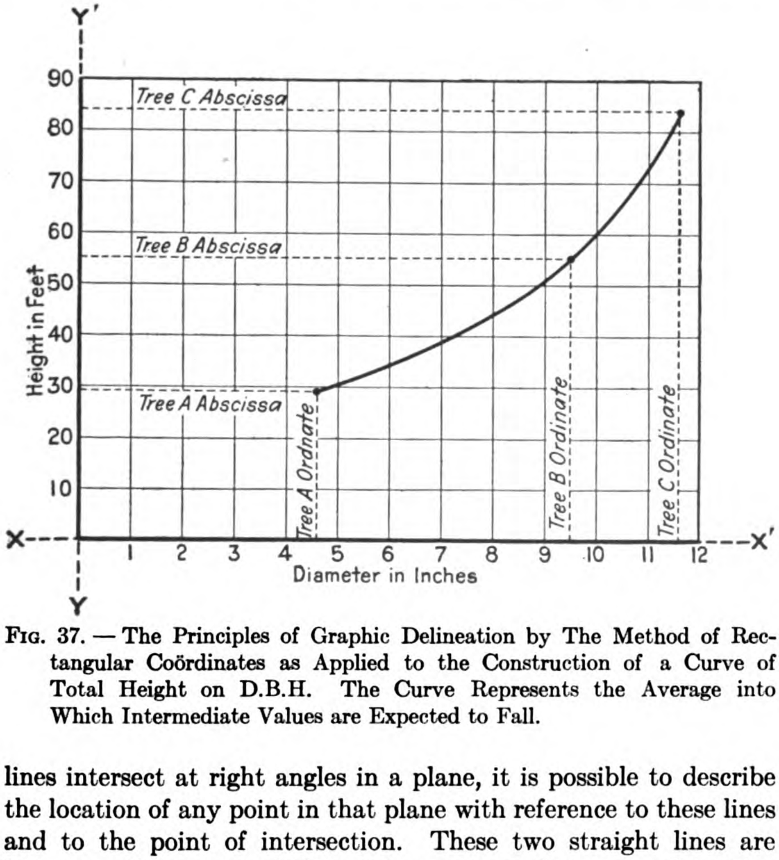


FIG. 37. — The Principles of Graphic Delineation by The Method of Rectangular Coördinates as Applied to the Construction of a Curve of Total Height on D.B.H. The Curve Represents the Average into Which Intermediate Values are Expected to Fall.

lines intersect at right angles in a plane, it is possible to describe the location of any point in that plane with reference to these lines and to the point of intersection. These two straight lines are known as the *axes*, and their intersection is known as the point of *origin*. The vertical axis is known as the *ordinate* axis, and the horizontal axis is known as the *abscissa* axis. If a certain quality of measurable magnitude possessed by a number of individual items is scaled off on the abscissa axis and if another measurable quality common to members of the group is scaled off on the ordinate axis, then the characteristics of any individual measured in terms of the scales of the ordinate and the abscissa determine its location in the plane. The point of origin is always given a value of zero on each axis.

Thus in Fig. 37, three trees are presented possessing in common the measurable qualities of height and of diameter. If " height in feet " is scaled off on the vertical axis YY', and the " diameter in inches " is scaled off on the horizontal axis XX', then the position of Tree A, and likewise of Trees B and C, can be located in the plane in reference to these characteristics as measured from the point of origin on the two axes. These characteristics (the abscissa and the ordinate of each point) are known as the coördinates, and there must be two known coördinates in order to locate a point.

123. Independent and Dependent Variables. — Every point located by graphic methods expresses a definite relation between two sets of variables as plotted on the scales of the two intersecting coördinate axes. The variable which increases or decreases by definitely determined arbitrary increments, is known as the *independent variable,* and is plotted as the abscissa. Time units form a very good illustration of an independent variable. The other variable is known as the *dependent variable* and is always plotted as the ordinate. The dependence may be real in the sense that its values are absolutely determined by the values of the independent variable, or they may be implied. In general it is the usual function of the ordinate* to indicate quantities or frequencies, and of the abscissa to locate them in relation to time, space and condition.

124. Plotting the Data. — A tabulated series of measurements lends itself readily to graphic presentation. The horizontal scale of the abscissa may be used for the measurements, and that of the ordinate for the dependencies. Care must be taken in the scale adopted in order not to overemphasize values of major weight, nor to dwarf minor ones. There is no absolute standard as to the relative values of the scales used, each series of data presenting a separate problem. In general, however, the values on the ab-

* Care should always be taken in selecting the variables to be scaled as ordinate and as abscissa. As pointed out by Bruce, Chapman and others, a considerable difference in the form of the curve may arise from an interchange of the ordinate and the abscissa.

scissa would not have a relation less than $1\frac{1}{2}$ to 1 to those of the ordinate.

Equal distances on either scale should represent facts of equal value, and the units of each scale should be adapted to a range capable of being easily included in terms of the major and minor lines on the cross-section paper used.

The purpose of a graph is an endeavor to represent the general relation of facts as they actually exist. This relation will be shown, not by a series of straight lines connecting the points plotted, but by a smooth curve intended to *fit* the average of the samples observed and measured.

It may be of assistance to connect the points plotted by light dash-dot lines for the purpose of noting their trend, and as a guide to the smoothed curve, which is first sketched in free-hand and later drafted in permanent position by the use of standard curved edge or spline. In drawing in the curve, attention must be paid to the points having the greatest weight, as based on the greatest number of measurements. A point based on the averaging of 39 trees is 13 times as important as one based on the averaging of the measurements of 3 trees. But in graphic plotting since the values of the points must vary, as do the square roots of their weights, the curve representing mean values would fall approximately at one-sixth of the distance from the 39 trees, and five-sixths of the distance from the 3 trees.

It must ever be kept in mind that a curve is drawn to express an average. From this curve there can be read and made a regular tabular statement of the average values within each group.

125. Cautions to be Observed in Constructing Graphs.* —

1. The arrangement of a graph, as ordinarily constructed, should proceed from left to right, and from bottom to top (Fig. 38).
2. All lettering and figures should be so located as to be easily read when the abscissa and ordinate are in their respective horizontal and vertical positions (Figs. 38, 39, 40, 42 *et al*).
3. Figures for the scale of the ordinate should be placed at the left of the diagram, and at the bottom for the scale for the abscissa (Fig. 38, etc.).
4. The lettering of the title and legend should be located preferably in the right upper quarter of the graph sheet, but only when

* Adapted from the Haskell Report to the American Society of Mechanical Engineers, 1919.

such placing is convenient to the graph at hand and neither interferes nor offers any confusion to its clear interpretation. The name or the initials of the man who constructs the graph should be entered in the lower right-hand corner (Fig. 39).

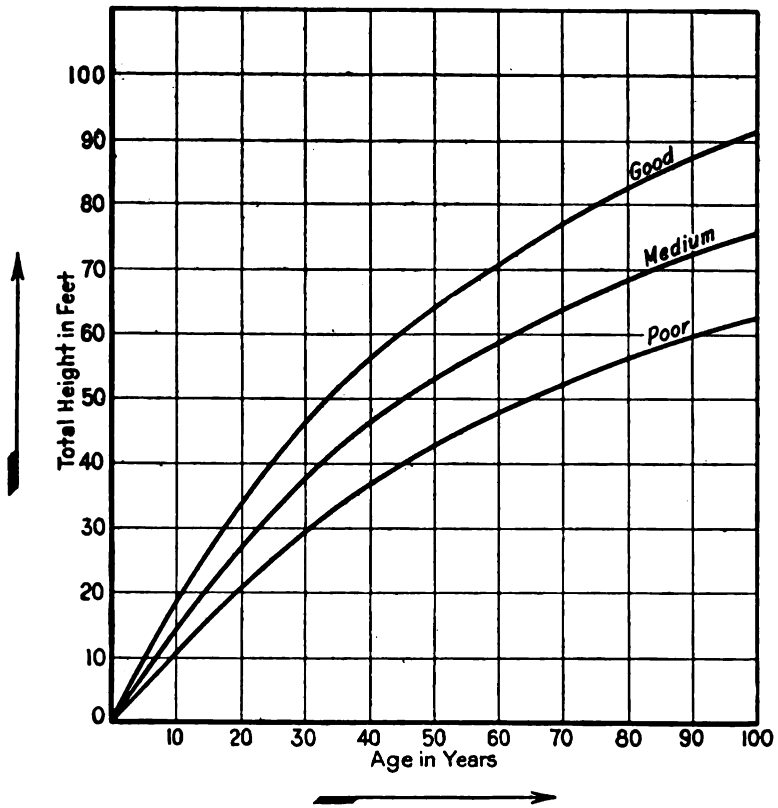


Fig. 38.

5. Particular care should be taken to designate clearly the specified characteristics represented by the values scaled off on the ordinate and the abscissa, such as: " Height in Feet," " Number of Trees per Acre," " Age in Years," etc. (Figs. 38, 39, 40, 41 *et al*).
6. The title of each graph should be neatly lettered on the sheet and should be made as clear and complete as is compatible with the desired degree of conciseness that is dictated by the necessity for definite information and the limitations of the space available (Fig. 39).

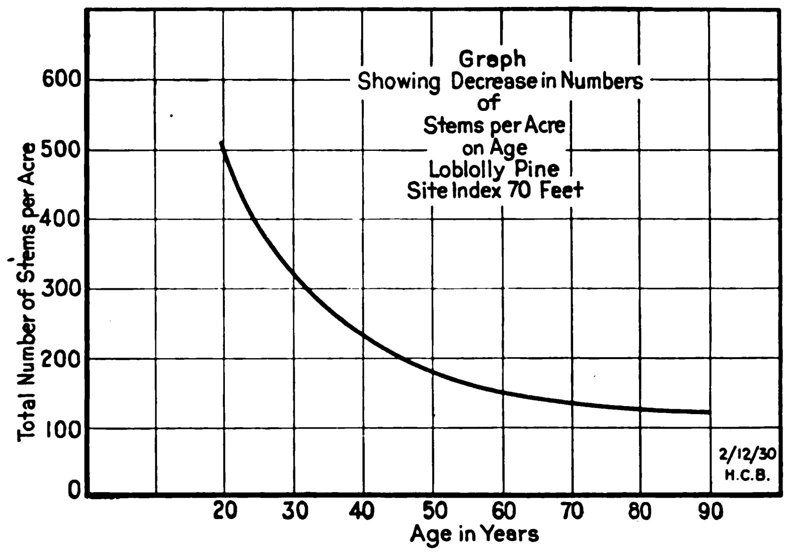

Graph
Showing Decrease in Numbers
of
Stems per Acre
on Age
Loblolly Pine
Site Index 70 Feet
Total Number of Stems per Acre
600
500
400
300
200
100
0
20
30
40
50
60
70
80
90
Age in Years
2/12/30
H.C.B.

Fig. 39.

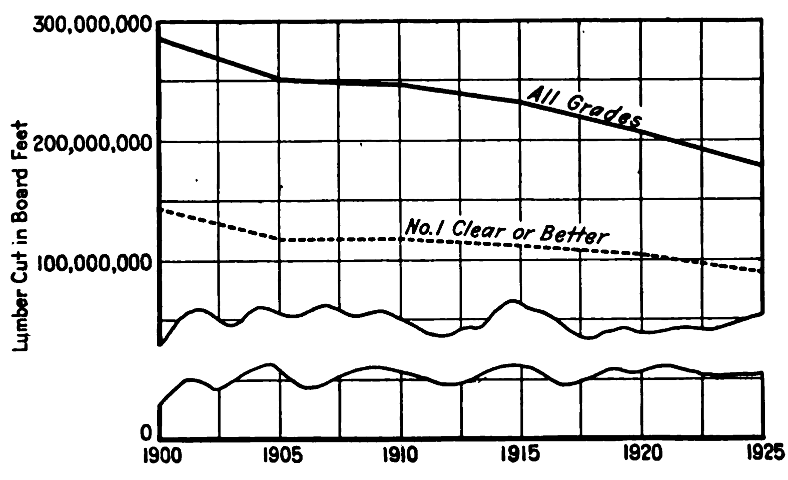

300,000,000
200,000,000
100,000,000
0
Lumber Cut in Board Feet
All Grades
No. 1 Clear or Better
1900
1905
1910
1915
1920
1925
Year

Fig. 40.

7. The zero lines of both the vertical and horizontal scales represented by the ordinate and abscissa axes should be ruled in heavily in order to facilitate their easy identification from whatever other coördinate lines may appear on the sheet (Figs. 38, 39, 41 *et al*).
8. In selecting a scale for either the ordinates or the abscissas choose one which will serve the double function of including all of the items listed and at the same time fitting the paper.

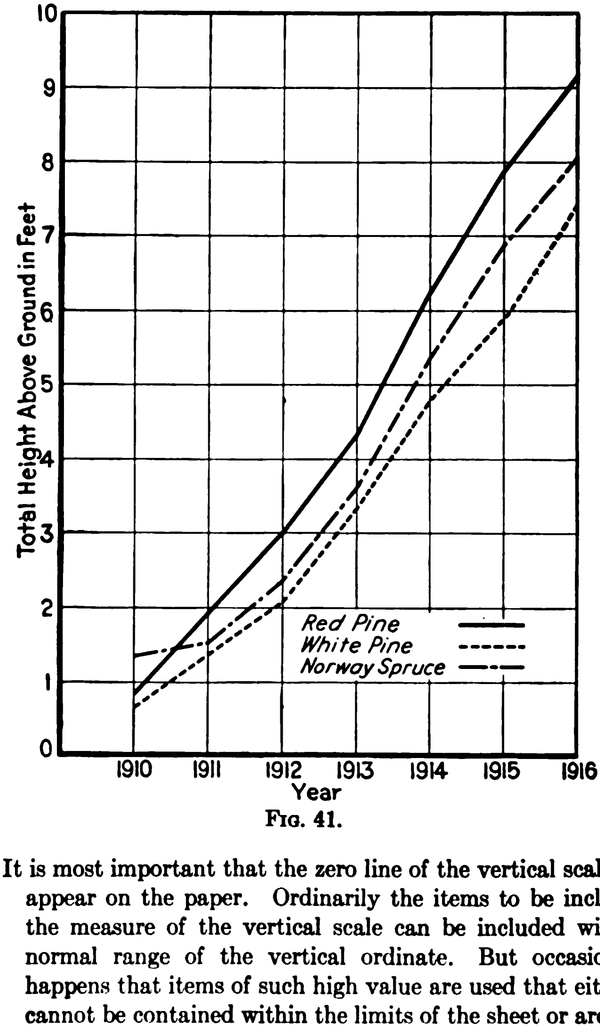


FIG. 41.

9. It is most important that the zero line of the vertical scale should appear on the paper. Ordinarily the items to be included by the measure of the vertical scale can be included within the normal range of the vertical ordinate. But occasionally it happens that items of such high value are used that either they cannot be contained within the limits of the sheet or are at such distance from the zero line as to make their scaling and coördi-

nation a difficult matter. Under such circumstances the actual distance between the zero line and the curve of the graph may be reduced by omitting several intermediate non-used values and indicating in their place a horizontal break in the diagram (Fig. 40 and Fig. 84).

10. On the whole, no more coördinate lines should be employed than are necessary to guide the eye in reading the values of the diagram.

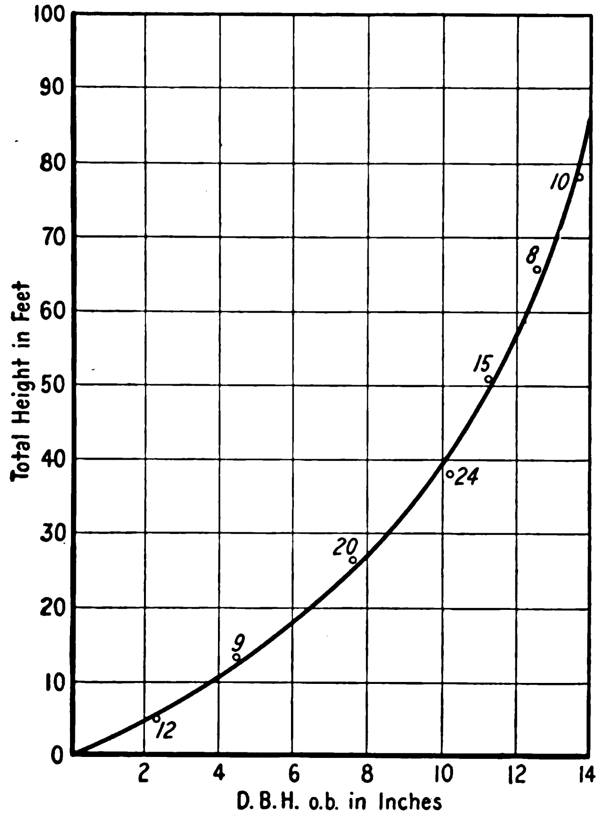


FIG. 42.

11. The line representing the graph curve should be drawn in firmly and distinctly by one single line. Smudging, weak, faint and double lines are to be avoided (Figs. 39, 42 *et al*).

12. Where values affecting two or more different principles are included on the same sheet, such as height values for different species, by means of two or more curves, they may be indicated

by a difference in the form of the lines of the curves; solid lines, dash-dot, dash, dash-dot, etc. (Figs. 41, 47, 48 *et al*).

13. Do not try to include too much on a single graph sheet, especially where two or more principles are affected. There is great danger of including so much as to lead to confusion. The

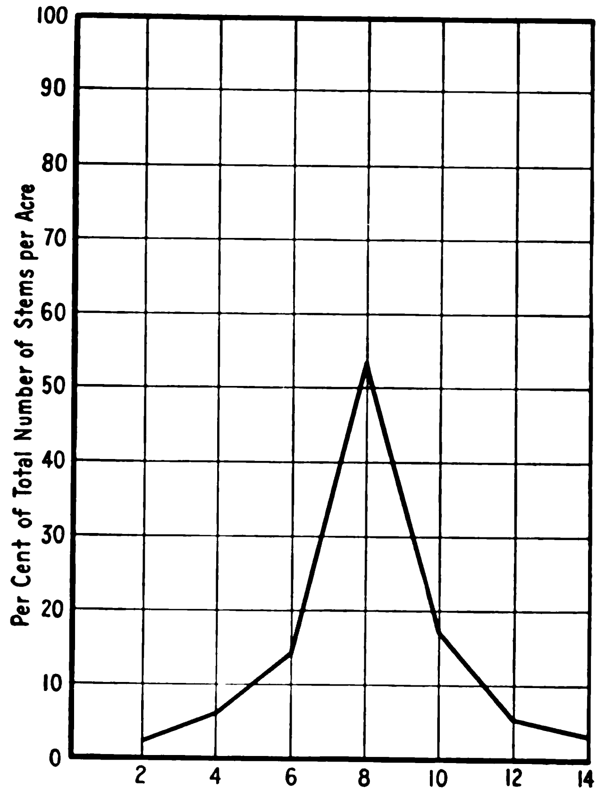


FIG. 43.

fundamental purpose of a graph is to present a picture. Just as soon as the *main* purpose of the picture is drawn the inclusion of too much detail weakens it. It is better, on the whole, to place the data on several sheets, than to try to include too much on one.

14. The size of the graph is a compromise between accuracy and convenience. Large graphs offer a greater facility in exact reading, but are more difficult to handle in their actual con-

struction, and also in filing. Small graphs are difficult to read with any exactitude but are easy to handle and have the added advantage of presenting a more concise picture, such as can be taken in at a single glance.

15. The degree of accuracy to be used in constructing and reading graphs depends upon their purpose. In general, in construction, that degree of accuracy should be sought which will make the graph illustrate sufficiently the principle it is desired to bring out. Needless refinement and absolute precision in every detail are not necessary. In reading a graph, due regard should always be given to the basis of construction and to the relative closeness of the scale used. It is foolish, for example,

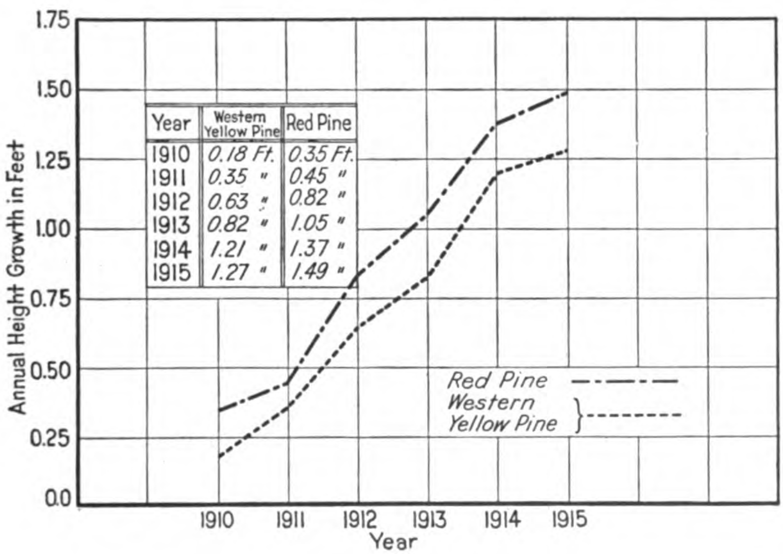


FIG. 44.

to read to thousandths, data based on ordinary ciphers measured and graphed on a scale whose smallest subdivision is a tenth of an inch.

16. In curves representing a series of observations, the points representing the separate values should be indicated on the diagram either by dots, small circles, or small crosses (Fig. 42).

17. In order to guide the construction of the curve so that it may best fall in the plane of the average, it may be advisable to indicate at each of the points so represented the number of observations on which the location is based. This may be done by insert-

ing at each of the points the value in small figures, provided that in so doing, the diagram does not become cluttered up and confused and its utility destroyed (Fig. 42).

18. When one of the scales employed in a diagram indicates per cent values, it is best to emphasize the 100 per cent line (Figs. 43 and 87).
19. When either scale of the diagram refers to a time interval, as indicated by dates, be careful to scale them off in their exact value (Figs. 40, 41, 46, 48 *et al*).
20. In selecting the scale to be used in a graph, be sure to use one adapted to the units of division on hand. Do not, for example, endeavor to work with a scale divided in tens or units of tens, by assigning to each unit a value, say, of 66 feet. Where the paper is divided on the decimal plan, use 1, 5, or 10, or multiples thereof as the unit.
21. It is seldom desirable to include in a graph the numerical data on which the curve is based.
22. Where some statement of such numerical data is desired, it is generally best to include within the form of the graph, or closely adjacent thereto, a neat tabulation of such amount or length as is pertinent to the figure (Figs. 44 and 66).
23. It is well to remember that every graph occupies a medium position between two sets of averages or tabulations, namely, the mathematical averaging from which it is constructed, and the tabulation representing a "smoothed average" which is derived directly from it.
24. It is thus important that every graph sheet shall be carefully lettered and indexed in relation to its origin and development. In an extensive piece of work, sheets have a habit of getting mislaid or lost or out of sequence, and every care and check must be used to keep them in their proper places.

126. Logarithmic Graphs. — The advantage of graph construction by rectangular coördinates is that it immediately presents a picture of the increments or decrements of absolute change of the primary variable in terms of arbitrary units of measurement. Its limitation seems to be that it does not reveal *rates* of change — information which may be desirable especially when time units form the basis of the study.

In his study of "Some Public and Economic Aspects of the Lumber Industry,"* W.B. Greeley, then Assistant Forester of the United States, brought out the fact that the differences existing

* Report 114, U. S. Dept. Agr., Washington, D. C., Jan., 1924.

between the retail prices of southern yellow pine lumber and southern yellow pine stumpage were as given in the accompanying table (Table X).

TABLE X

RETAIL LUMBER AND STUMPAGE PRICES, SOUTHERN YELLOW PINE 1901–1915

Year	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914	1915
Average retail price for all grades	11.20	12.00	12.50	11.80	14.00	16.30	16.00	12.60	13.80	13.70	14.50	15.90	16.30	15.80	13.20
Average stumpage price	1.15	1.25	1.60	2.00	2.90	3.75	3.77	3.79	3.82	4.50	5.20	5.20	5.15	5.00	4.90

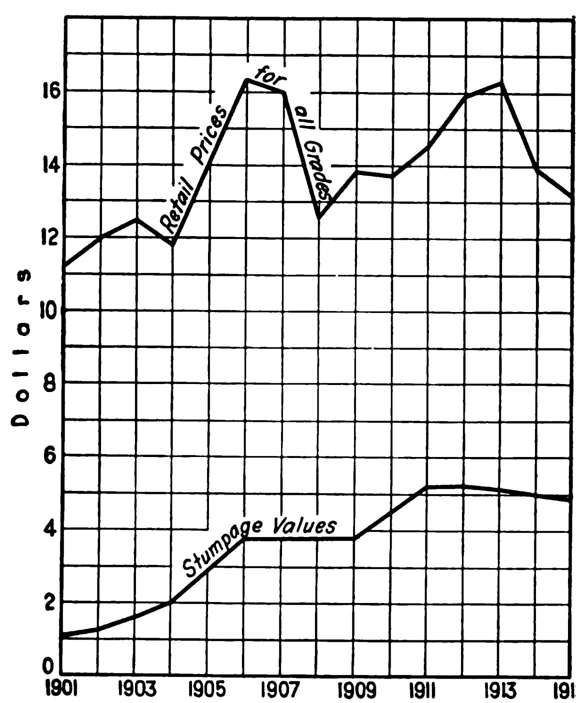


FIG. 45. — Rectangular Coördinate Chart Showing Retail Lumber and Stumpage Prices of Southern Yellow Pine 1901–1915.

If these figures are plotted in a simple graph, that is, one constructed by means of regular rectangular coördinates, on the basis of actual changes in price, it would appear as Fig. 45. From this graph it would seem that the changes in the retail prices of lumber have not only been more violent than but seem to bear little relation to the changes in stumpage prices. Increases in retail prices should be reflected by a similar trend in stumpage. This is not definitely visualized in this chart.

What is desired is one which will picture the *degree* or *scale of change* rather than the actual change. It can best be accomplished by a logarithmic chart in which equal *proportional* changes will be

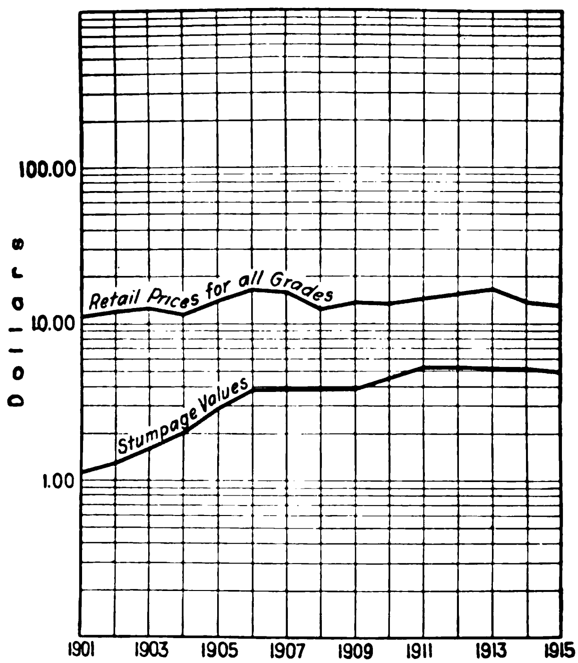


FIG. 46. — Semi-logarithmic Chart Showing Retail Lumber and Stumpage Prices of Southern Yellow Pine 1901–1915.

represented by equal changes in the ordinate. This brings out the fact that the changes have not been so violent as would seem and that movement in one has been followed by similar movement in the other. See Fig. 46.

A logarithmic chart is one in which one or both of the ordinates is laid off, not to the natural scale of the values, *but to the scale of the logarithms* of those numbers. When both ordinates are so

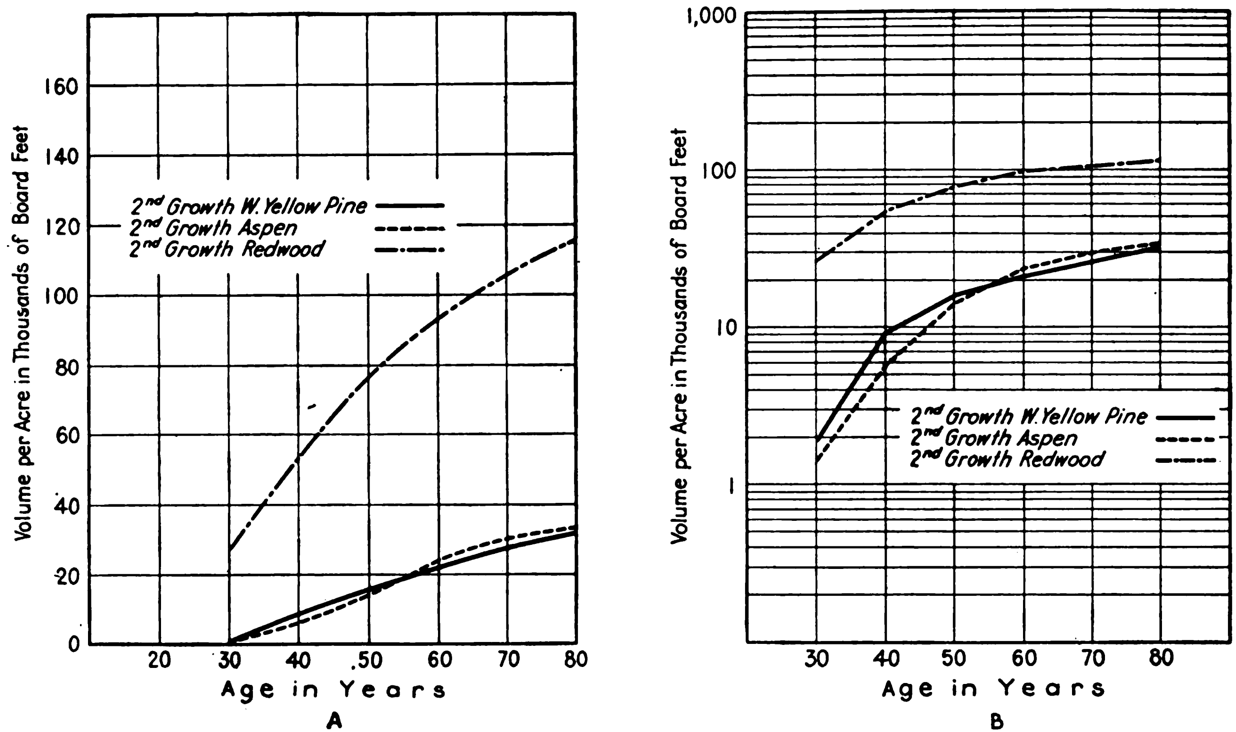


Fig. 47. — Rectangular Coördinate and Semi-logarithmic Charts Showing the Per Acre Yields in Board Feet at Different Ages of Second Growth Redwood in California, Second Growth Aspen in Lake States, and Second Growth Western Yellow Pine in Idaho.

scaled, the resulting chart is known as a double logarithmic chart. When one ordinate is scaled by logarithms and the other is in terms of the natural scale, it is known as a semi-logarithmic chart. Semi-logarithmic charts are by far the more common, and on these charts it is the usual practice to use the logarithmic values on the vertical ordinate.

Logarithms drawn to scale as representative of numerical values are often used to advantage by mathematicians and engineers. The ordinary Mannheim slide rule which is commonly used in calculations is a good example of this. The graduations on a logarithmic chart are constructed in the same manner. Although graduations are *labeled* as numbers, they are *scaled* by logarithmic values. Each deck or cycle of the graduations ends with a power of ten. The chart may be graduated in terms of one cycle, two cycles or more. The number of cycles used is of minor importance provided that the proper scale is maintained within each succeeding cycle.

The most useful application of logarithmic charts in the representation of a time series lies in the fact that the significance of a given change depends upon the magnitude of the base from which the change is measured. An increase of 1000 board feet per year or per decade means 100 per cent when plotted on a base of 1000, and is no less significant than an increase of 10,000 when plotted on a base of 10,000. This is brought out in Fig. 47, which shows the yields of 80 foot site index timber for three second growth species showing an entirely different significance when plotted on the logarithmic scale. On the natural chart the absolute changes in the redwood for the last three decades are from 5 to 10 times greater than for the aspen, but on the logarithmic chart they are shown to be of but approximately equal importance.

A distinct advantage of the logarithmic chart is found when the range of comparison of a group of variables covers an extremely wide scope. This is perhaps best illustrated by Fig. 48, which shows the lumber production of the entire United States and the relative production of the main four groups of lumber-producing states. If the five series are to be presented on a single rectangular chart, scaled arithmetically, a scale must be used which will permit the entry of the largest single item of 40,018,282,000 board feet. Were it further desirable to display the lumber production of one or two or more individual states, such a scale would reduce the

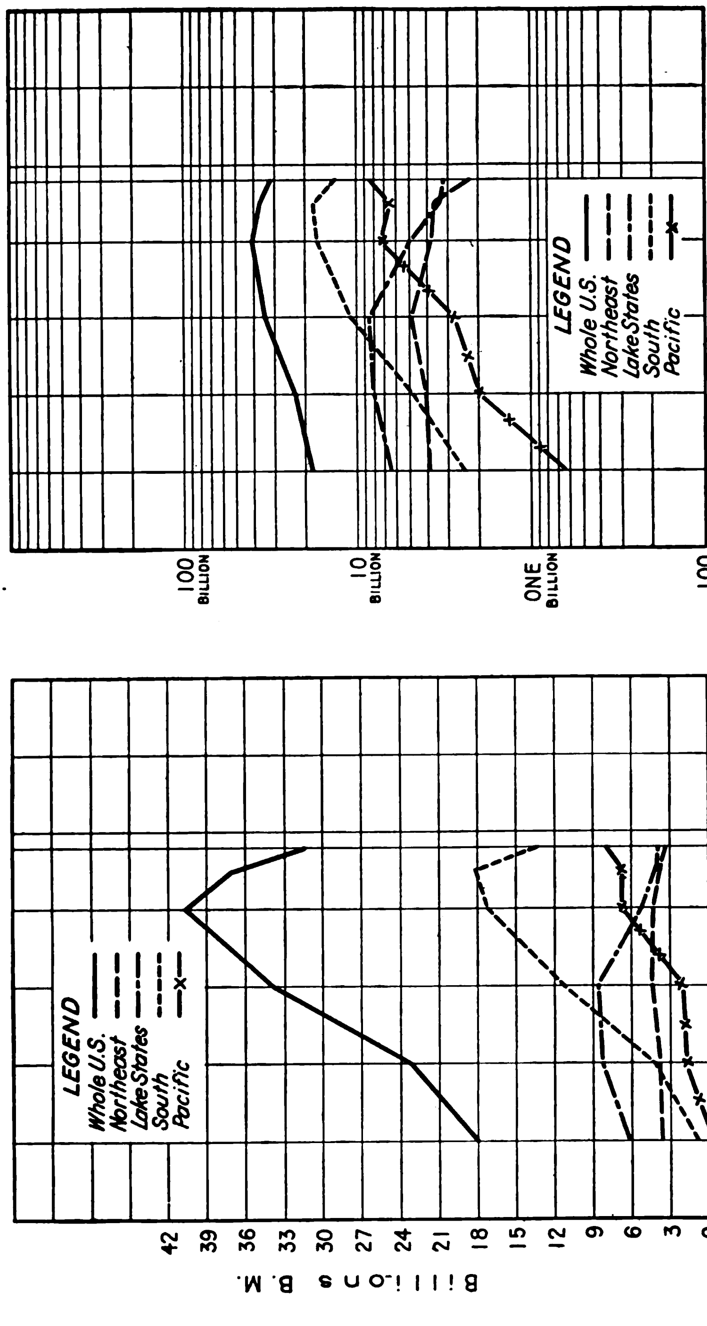


FIG. 48. — Rectangular Coördinate and Semi-logarithmic Charts Showing Figures of Lumber Production in the United States During the Period 1880–1918.

relative importance of the smaller magnitudes to a point where they might be completely lost.

For the purpose of comparing groups or series which differ markedly in respect to magnitude, the rectangular coördinate chart with the natural scale is weak. In contrast, the logarithmic

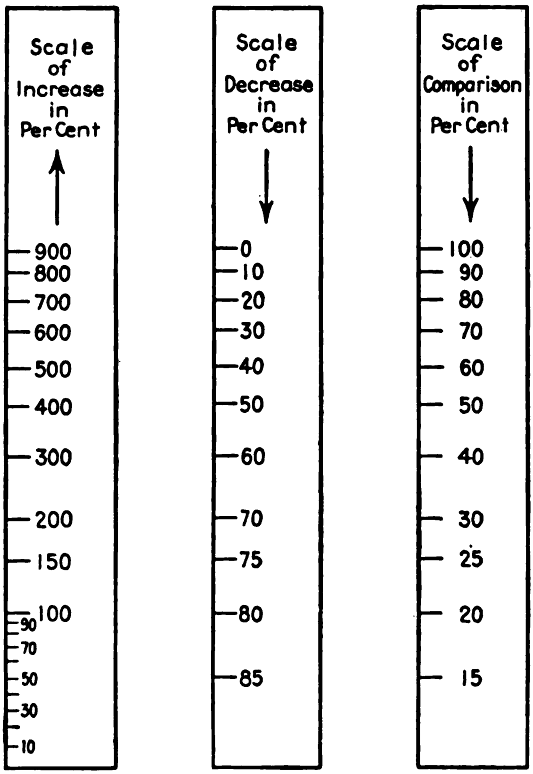


FIG. 49. — Scales of Increase, Decrease and Comparison Percentage Values for Use With Logarithmic and Semi-logarithmic Charts. (To be Used in Directions Indicated by the Arrows.)

scale permits accurate or rapid comparison. Reference is made to Fig. 49. These scales, constructed on the basis of a single logarithmic cycle, are for use with semi-logarithmic graphs in determining the increase, decrease, and comparison percentages of different series. They are to be used in the directions indicated by the arrows. The vertical distance between any two points in

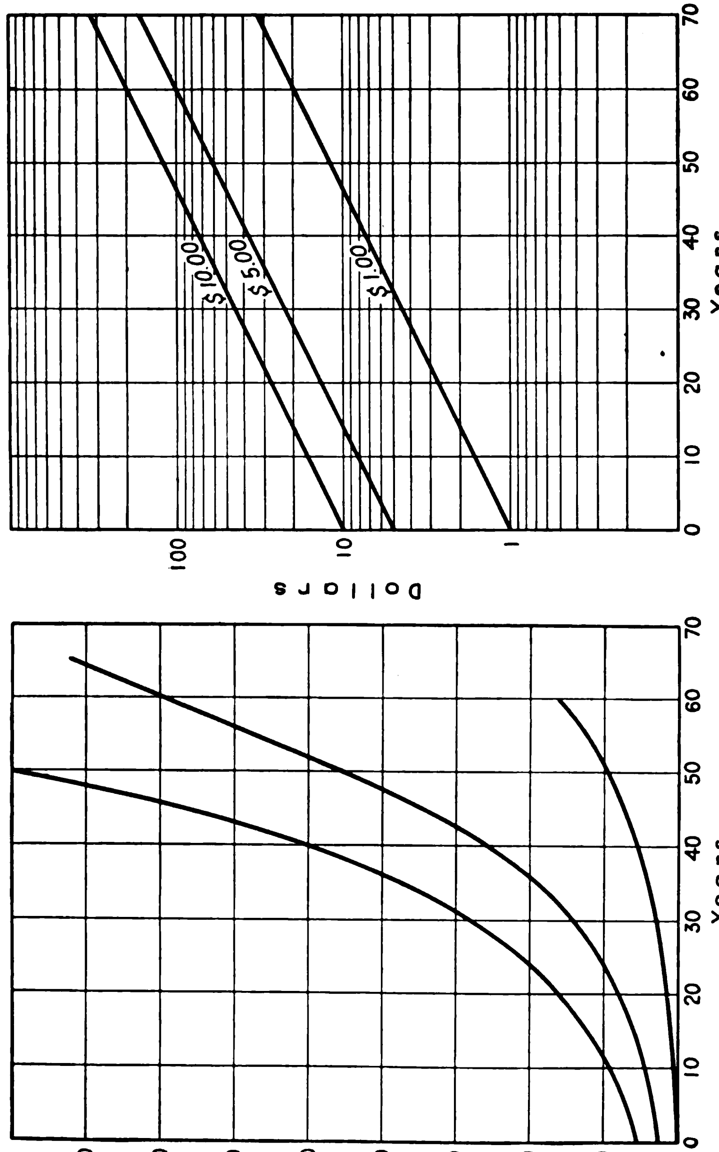


FIG. 50. — Graph Showing That Quantities which Increase at a Definite Rate and which are Drawn as a Curve on the Natural Scale of the Rectangular Coördinate Chart are Graphed as a Straight Line on the Semi-logarithmic Chart. These Curves Represent Compound Interest Calculations at 6 Per Cent.

a given series measured upward on the " Scale of Increase " or measured downward on the " Scale of Decrease " gives this value as a percentage of the base from which it was derived. The " Scale of Comparison " is to show percentage relations of one series to another, as derived from a common base, as for example the different amounts of lumber cut in various sections of the United States for any one year as illustrated in Fig. 48.

Logarithmic charts can also be used to advantage where a quantity is increasing at a definite rate as, for example, a sum of money at compound interest, as in Fig. 50. When the rate of increase is constant, the logarithmic chart shows a straight line. In any curve or series of curves equal changes of increase will be shown by equal slopes and a comparison of rates of change can always be made by comparing the slopes of the plotted lines. A further advantage of logarithmic charts of semi-logarithmic character is that they permit the plotting of absolute magnitudes and the comparison of relative changes at one and the same time. Percentage changes can be read directly and percentage relations definitely established. Care must be taken, however, in using logarithmic charts to keep in mind the conditions under which the scale was established. Such a chart shows facts relative to conditions of the base and nothing else.

Logarithmic charts are sometimes called ratio charts.

CHAPTER X

ALINEMENT CHARTS AND ANAMORPHIC CURVES

127. Introduction. — It is only on rare occasions that the final product desired from forest measurements is computed on the spot directly from the field data. Scaling is an admitted exception to this statement as are also some systems of timber estimating. The usual practice is to collect a large quantity of the pertinent measurements and by subsequent computation in the office determine the required information. The office work then becomes a more or less routine process in which computing machines, adding machines, slide rules, and other mechanical contrivances are brought in to assist the mental efforts of the computers. Alinement charts and anamorphosed curves are two more such helps and as such they are presented.

128. The Theory of Alinement Chart Construction. — An alinement chart is one which graphically endeavors to visualize the relations of the three contributory items in a given equation in such a way that the projection of a straight line connecting any two of the variables must of necessity intersect the third. It is accomplished by projecting these values to scale on three parallel axes which are so spaced and arranged that corresponding magnitudes are in linear relation with one another. The axes on which the independent variables are scaled are known as the *primary* or initial axes. The axis carrying the dependent variable is known as the *final axis.*

Although the explanation to be offered regarding the basis of construction of alinement charts for the greater part involves but three separate variables, in actual application they are by no means confined to three, and more may be accommodated if necessary. Many computations involve several variables. This is provided for by establishing separate systems for related variables in which the final axis of one set of variables becomes a primary axis for the succeeding set, and so on.

In the actual construction of an alinement chart three factors should be kept in mind:

1. A clear delineation of the significance of the ultimate product.

2. A selection of the proper scales for the graduation of the axes. The units must be large enough to enable readings within a reasonable degree of accuracy but must not be so large that the range of values desired will expand the chart beyond convenient size.

3. A proper spacing and grouping of the axes which carry the values of the several variables. Acute intersections and oblique readings are always to be avoided. The aim should be to secure a compact chart which can be easily and accurately read and which is of moderate size.

Alinement charts may be used for problems of addition, subtraction, multiplication, and division, or for any combination of two or more of these processes as may be desired. The more complicated the problem the more complex the chart. The main purpose of the subsequent discussion is to outline the application of the alinement chart method to certain of the simpler problems that the forester is called to solve.*

129. Addition. — Algebraically the process of addition may be expressed by the formula $a = b + c$. If then, as in Fig. 51, the independent variable b is scaled off in measurable units on an initial vertical axis B, and the other independent variable c is scaled off on an initial axis C, a straight line joining b to c must intersect the final axis A at a point a the numerical value of which on the scale of graduation on its own axis is equal to $b + c$. Of necessity, the graduations on the a axis will not be of similar magnitude as those on the b and c axis, but will bear a proportional relationship in accordance with the relative distance or space between the initial and the final axes.

In Fig. 51, the graduations of the two primary axes B and C are of equal magnitude. The A or final axis is placed equidistant between them. The straight line MNP connects b units (12) on the B axis with c units (18) on the C axis. Then by application

* For a more extended discussion of this subject the student is referred to the following:

1. Alinement Charts in Forest Mensuration, by Donald Bruce, Jour. of For., Vol. XVII, No. 7, Nov. 1919. p. 773.

2. A Modification of Bruce's Method of Preparing Timber Yield Tables, by L. H. Reineke, Jour. of Agr. Research, Vol. XXXV, No. 9, Nov. 1927. p. 843.

of the laws of similar triangles, it can be demonstrated that $NO = \frac{1}{2} PS$, and $OR = \frac{1}{2} MQ$, and that $NO + OR = \frac{1}{2} (PS + MQ)$, and hence if the A axis is graduated with a scale whose magnitude

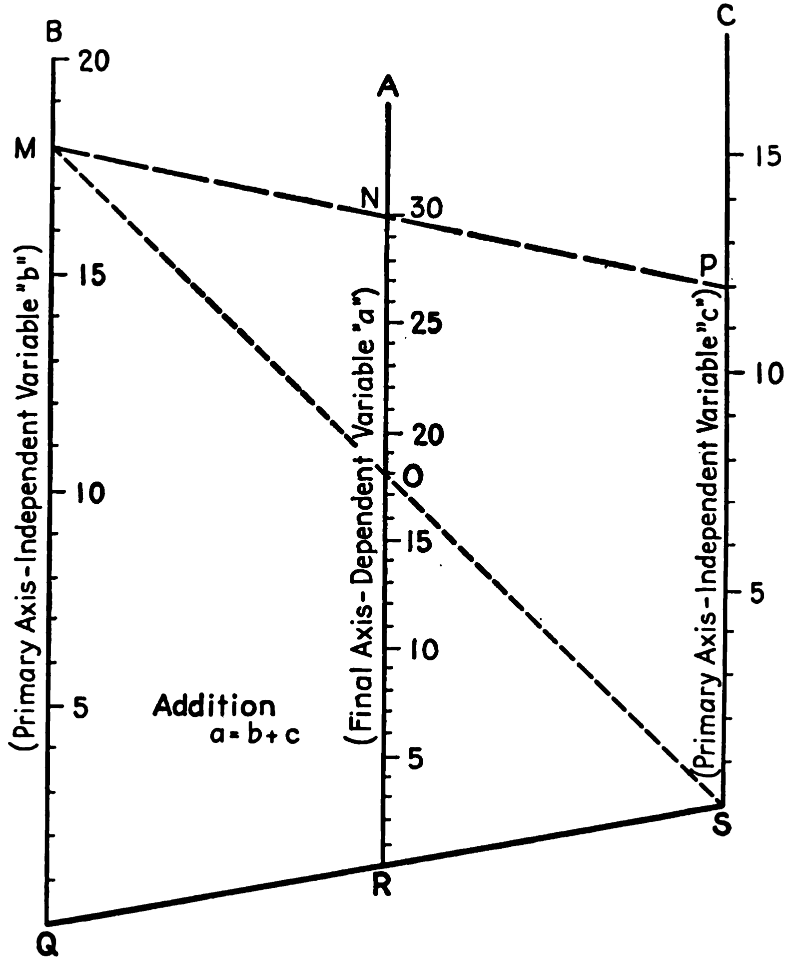


FIG. 51. — Alinement Chart for Addition.

is equal to half of that used on the B and C axes, the intersection of the line connecting b and c will fall at a or at 30 divisions on that scale of magnitude.

If the spacing of the final axis is placed at $\frac{1}{3}$ of the distance between the two primary axes then the graduation on the final will be in similar ratio, that is, ⅓ magnitude; if ¼ the distance, then ¼ of the magnitude; and so on.

However, in practice the graduations of the final axis are seldom calculated in this manner, but are calibrated by intersections from the primary axes. In the illustration shown in Fig. 51, we see that 12 + 18 = 30, and this intersection is marked on the final axis.

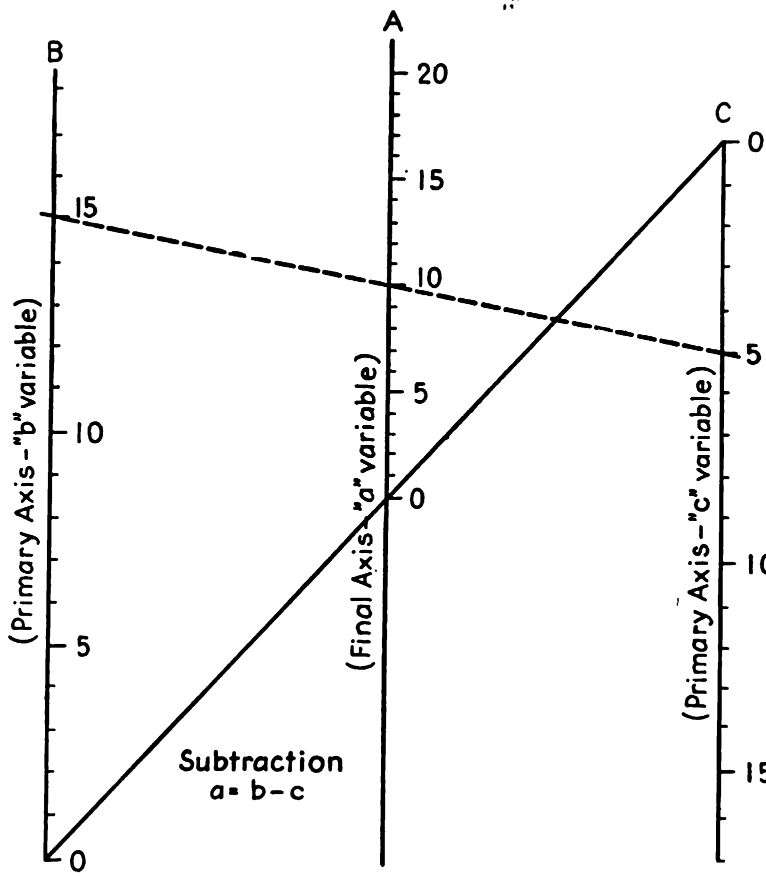


FIG 52. — Alinement Chart for Subtraction.

If then the straightedge is connected with 12 on the *B* axis and 17 on the *C* axis, its sum or intersection on the *A* axis can be calibrated as 29 and the unit of graduation for the axis is determined. Further cross check will calibrate other values if desired until the axis is entirely graduated.

130. Subtraction. — The process of subtraction may be considered as addition in a reverse direction in which $b = a - c$. Hence, by modifying the designations of primary and final axes in accordance with the changed character of the dependent and in-

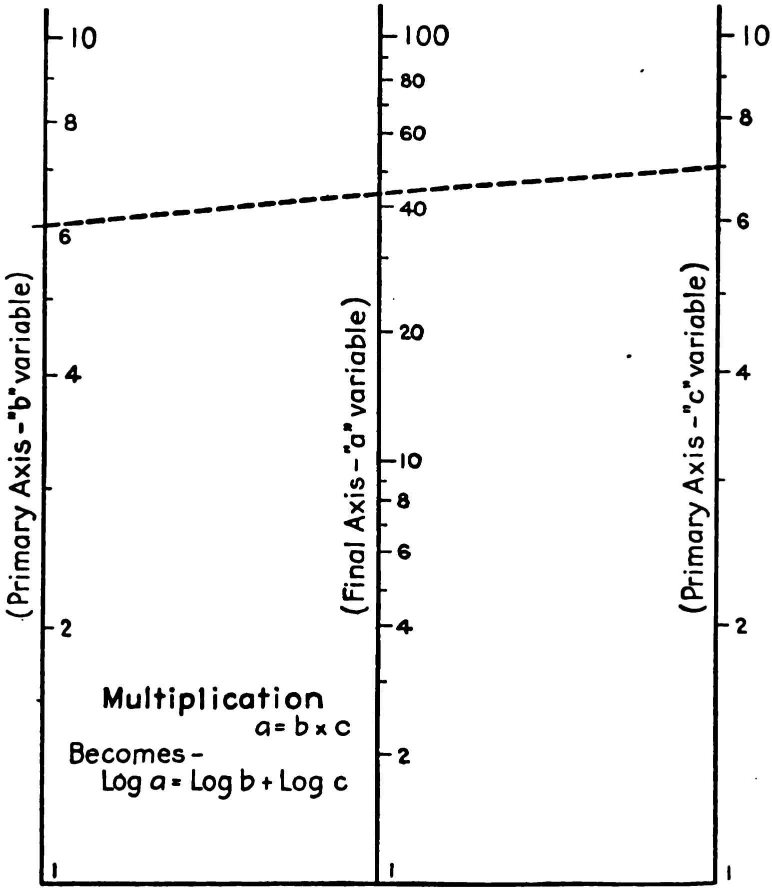


FIG. 53. — Alinement Chart for Multiplication with Logarithms.

dependent variables and also reversing the *direction* of application, an addition chart similar to that of Fig. 51 may be used.

However, it may be more desirable to graduate one of the primary axes — the C axis, in a reverse direction, and the problem becomes $a = b - c$. But $a = b - c$ in reality is equivalent to $a = b + (-c)$. Referring to Fig. 52, we see this illustrated.

The straight line connecting the zero of the *B* axis with the zero of the *C* axis must at its intersection with the *A* axis designate the

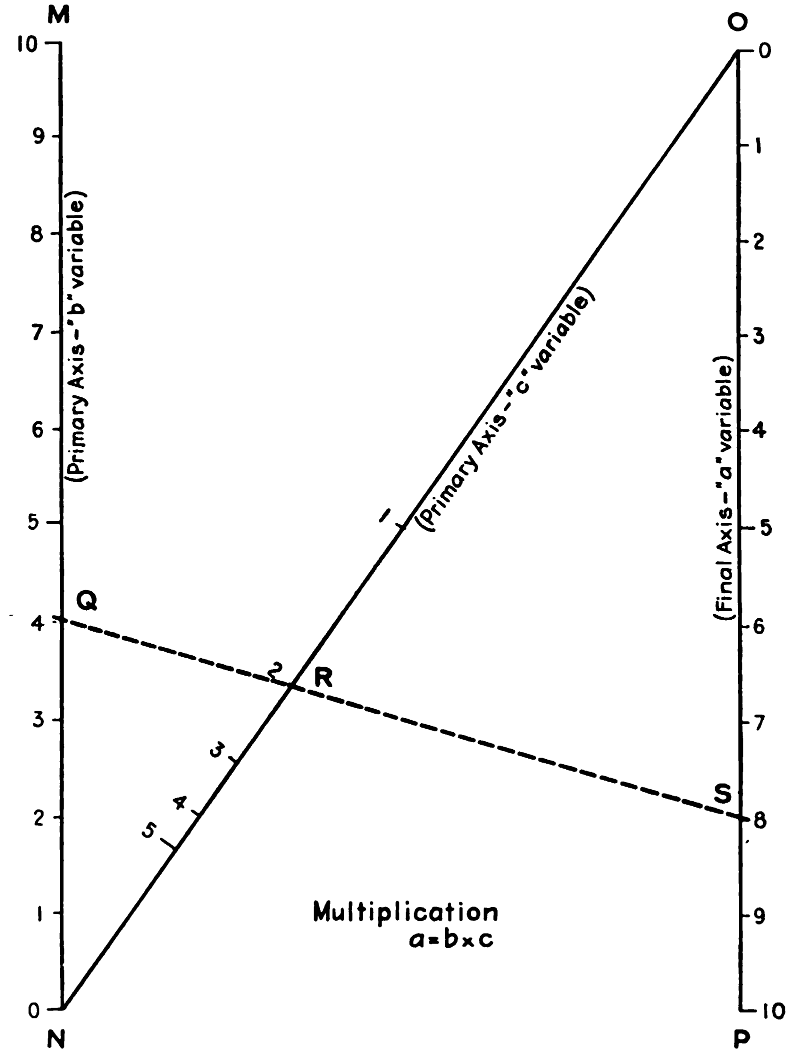


FIG. 54. — Alinement Chart, "Z" Form, for Multiplication.

zero of its graduations. Applying the laws of triangles to the figure, it can then be shown that when the *A* axis is mid-way between the *B* and *C* axes, the magnitude of its graduations upward from its determined zero will be equal to half of the magnitude of the graduations on the other two axes.

131. Multiplication. — The process of multiplication is equivalent to $a = b \times c$. It is a common practice for mathematicians to use logarithms in multiplication, in which case the equation becomes $\log a = \log b + \log c$. If then the B and C axes are scaled off in accordance with the value of the logarithms of numbers rather than the absolute values of the numbers themselves, the process follows exactly that for addition, and the intersection of the alinement with the A axis will derive the logarithmic value of the product, as in Fig. 53. When the final axis is placed mid-way between the other two, its graduations, as before demonstrated, will be of a magnitude equal to half of that on the others. With logarithmic graduations this means that equal distance on the final axis will be graduated in two cycles of logarithmic values as compared with one cycle on the other two axes.

It may not, however, be either convenient or advisable to use the logarithmic method of multiplication. If direct multiplication is desired, it can be done by means of a Z chart as in Fig. 54. In this case the two parallel axes MN and OP represent the first primary and the final axes respectively. Their graduations are equal in magnitude, but with this difference that those on the primary axis are graduated upward from its origin, whereas those on the final axis are graduated downward, N and O being the respective points of origin. The second primary axis is the inclined line NRO connecting the two points of origin as scaled. The alinement QRS represents the multiplication, the product of which is read on the final or A axis.

The easiest and quickest method of graduating the second primary axis (the Z axis) is by intersections of related values. However, its graduations may be calculated from the relations of the sides of similar triangles.

In the triangles SOR and QRN

$$OS : QN : : OR : RN$$

$$\frac{OS}{QN} = \frac{OR}{RN}$$

But $\quad RN = ON - OR$

Hence $\quad \frac{OS}{QN} = \frac{OR}{ON - OR} = OS = QN\left(\frac{OR}{ON - OR}\right)$

In the equation $a = b \times c$

$$OS = a$$

$$QN = b$$

and $$c = \frac{OR}{ON - OR}$$

In any given multiplication chart ON becomes a constant of determinable scale to which arbitrarily a given value such as 10 (or 100 etc.) units may be applied.

Hence $$c = \frac{OR}{10 - OR}$$

and $$OR = \frac{10\,c}{1 - c}$$

Hence on the scale of 10 units for the distance ON

when $x = 1$, OR the scaled distance $= 5$
2, OR the scaled distance $= 6.7$
3, OR the scaled distance $= 7.5$ and so forth.

132. Division. — The alinement chart process of division may be deduced from that of multiplication. Where logarithms are used, the operation $a = \frac{b}{c}$ becomes $\log a = \log b - \log c$ and the process is directly akin to that of subtraction in which the scales of the axis are graduated in terms of logarithmic instead of absolute values, as in Fig. 55A. The projection of the divisor on the central axis, in this case a primary axis, will require a scale one half the magnitude used on the other two axes.

If the division is desired in terms of the absolute scale, an alinement chart will be constructed as in Fig. 55B. The left-hand axis is the first primary axis. The right-hand scale which is graduated in the *opposite* direction is the final axis.

The bar of the Z connecting the zeros of these two scales is the second primary axis. Graduations on this axis may be laid down by intersections or by computations based on the relations of the sides of similar triangles as has been described in the preceding section.

Still another method of using an alinement chart for division is to substitute for division the multiplication of the dividend by the reciprocal of the divisor, in which case $a = \frac{b}{c}$ becomes $a = b \times \left(\frac{1}{c}\right)$ and a direct multiplication chart may be used.

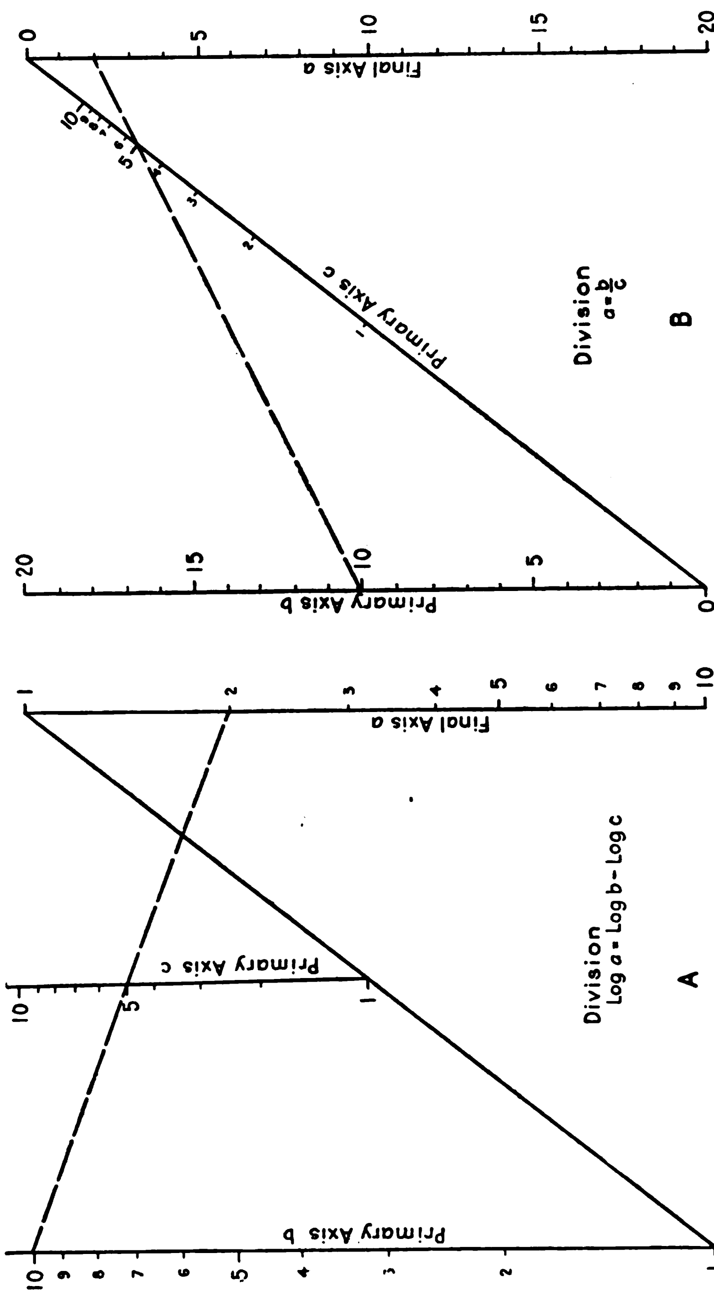


FIG. 55. — Alinement Chart for Division.

133. Constants, Coefficients, and Exponents. — When these appear in the equation the usual practice is to relate them, so far as it is possible, to one of the variables, with a consequent modi-

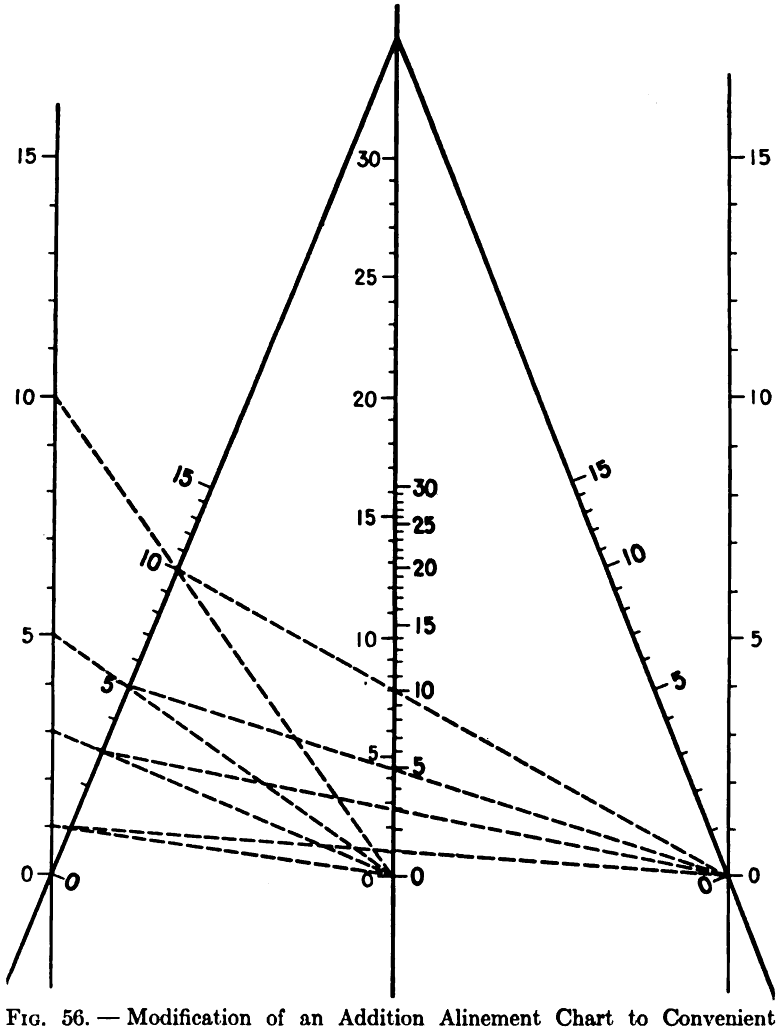


FIG. 56. — Modification of an Addition Alinement Chart to Convenient Dimensions.

fication of the graduation of the axis carrying that particular variable. Thus $a = \frac{1}{2} b^2 + c + 1$ becomes $a = (\frac{1}{2} b^2 + 1) + c$, and it will be solved by an addition chart with one axis carrying graduations modified to correspond with values of ($\frac{1}{2}$ $b^2 + 1$). When a variable exponent such as $a = b^c$ is introduced it may be resolved into $\log a = c (\log b)$ and as such may be charted as a multiplication.

Modification of the scales for purposes other than the mathematical delimitations of the several factors is also a common practice. Modification may be made for purposes of conciseness in the chart itself. For example, as in Fig. 56, an addition chart, the three termini of whose axes theoretically meet in infinity, is made of determinable dimensions by an arbitrary focus of the three axes within measurable limits. This is made possible by the pivoting of the two outside axes on points of similar mathematical value. The subsequent modification and graduation of the scales of these two axes is indicated by the dotted lines. In a similar way the modification of the scales of graduations of a multiplication chart of the Z type may be accomplished by pivoting the bars of the Z at their point of intersection with the diagonal. In any such modification of scales what is sought is the most efficient delineation of values in the significant portions of the chart. No attempt should be made, however, to effect any vital changes of scaled magnitudes by this plan since any graphic inaccuracies are correspondingly magnified.

134. The Application of Alinement Charts. — The foregoing is a brief statement of the fundamentals involved in the construction and graduation of alinement charts as applied to the four primary mathematical processes. The uses of these methods in solving the problems of the forester are many. Particularly useful are they in those operations where a mass of field data is resolved by computations involving the repeated application of rather simple formulas. Such examples may be cited as Huber's formula for computing the volumes of logs (Section 87); Smalian's formula for computing the volumes of logs (Section 87); Schiffel's formula for computing the volumes of standing trees (Section 141); the form factor method of computing volumes for volume tables (Section 153); presentation of volume tables (Section 157); computation of per acre yields (Section 185). There are many others. That the alinement chart method of solution is applicable is justified by the fact that the accuracy of the field measurement is not of such high degree as to place the moderate precision of graphic solution beyond the limits of consistency or acceptability.

Only one of the previously mentioned computations to which alinement charts are applicable will be briefly discussed. This one is chosen because it illustrates a solution of addition and multiplication from one chart.

135. Solution of Smalian's Formula by Alinement Chart. — This formula expresses the volume of a log as $V = \frac{B + b}{2} \times L.$ By further resolution this becomes

$$V = \left(\frac{\pi D^2}{4 \times 144} + \frac{\pi d^2}{4 \times 144}\right)\frac{L}{2}$$

$$= (D^2 + d^2)\frac{\pi L}{1152}$$

In this form the formula is subject to solution by a combined addition and multiplication chart. Three parallel and equidistant axes are constructed. The two outer of these three are graduated in terms of the squares of any convenient unit. That is, the 2 is placed at 4 units from the origin, the 3 at 9 units, the 4 at 16 units and so on. It is advisable to place the graduations on the same side of the two axes as is signified by the Roman numeral I at the top of the column. The reason for this will be explained later. The middle one of these three axes need not be graduated. It is the final axis of the first part of the computation, but it is also to serve as a primary axis for the second computation. Intersections based on the addition of the diameter values fix a point necessary for the multiplication.

At some convenient distance from the first three axes a fourth parallel axis is erected and graduated from the *top downward* in terms of the natural scale of some convenient unit as in I on that scale as shown in Fig. 57. This is the final axis of the multiplication and of the entire computation. It will express volume in cubic feet. A diagonal, forming the second primary axis for multiplication in a Z chart, connects the zero end of the volume scale with the lower or zero end of the ungraduated final scale of the addition chart. The diagonal is graduated from the upper end downward in terms of $\frac{\pi L}{1152}$. In actual practice the graduations can be more easily calibrated by intersections between known values of the first primary and final axes of the multiplication chart.

For small sized logs, with which otherwise annoyingly acute intersections will be experienced, it is advisable to recalibrate the right-hand sides (indicated by the Roman numeral II) of the two

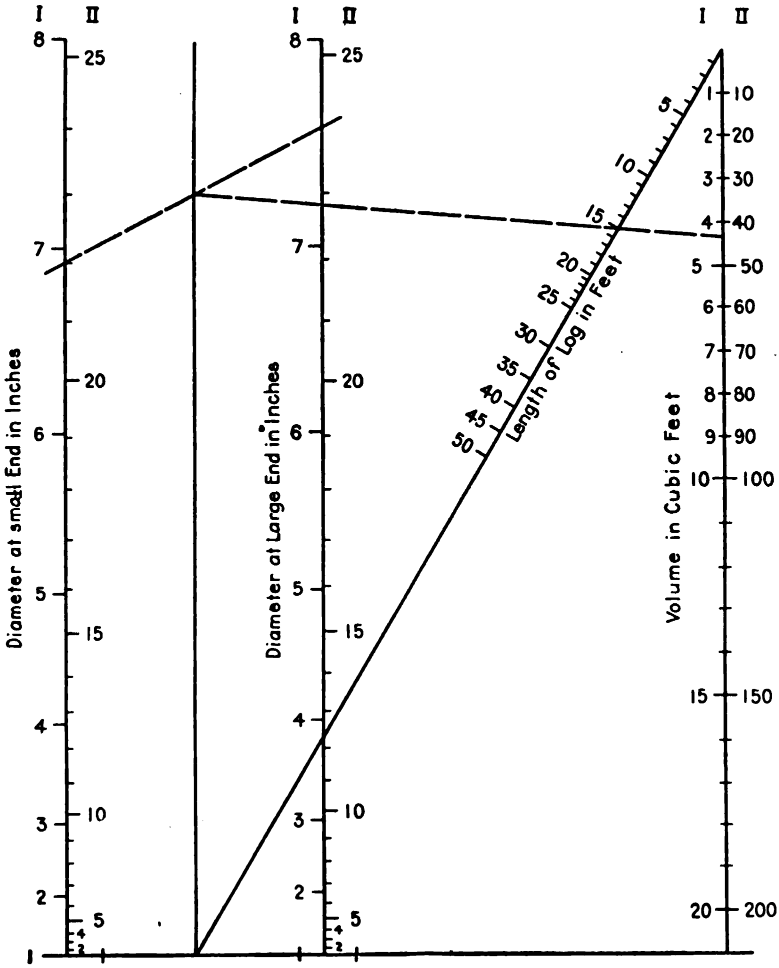


FIG. 57. — Alinement Chart for Computation of Volume by Smalian's Formula for Cubing Logs

diameter scales and of the volume scale with a unit 10 times as great as before.

136. Advantages of Alinement Charts. — The great advantages of alinement charts lie in the simplicity of their construction and the ease with which they can be used. As contrasted with methods involving the use of basal area as well as other tables, with or without slide rule, alinement chart methods will cut by more than half the time allotted to office computation. An alinement chart can be constructed rapidly, easily, and with a degree of accuracy quite consistent with the problem at hand, granted that the principles involved are clearly understood and fairly delineated, and granting a desirable degree of manual skill and accuracy in actual chart draftmanship. The superiority of the method over any system of graphs or charts based on rectangular coördinates is also obvious. They are simpler in construction and easier to read. They can be constructed more quickly and cheaply and lend themselves better to an accurate interpolation of variables. Their limitations are those inherent in all forms of graphic delineation in that beyond a specifically defined range their degree of accuracy becomes relative rather than absolute. Furthermore, error in chart construction will result in cumulative errors in computation.

137. The Theory of Anamorphosis. — The word anamorphosis comes from two Greek words which mean " to form anew." Its relation to graphics and particularly to forest graphs lies in the possibility of *expressing a curve by a straight line.* A curve (Section 122) shows the relations of two sets of variables as scaled on two rectangular coördinates. Equal distances on each scale represent corresponding variations of magnitude. The scales of magnitude for each ordinate are equispaced and constant.

When a straight line is substituted for a curve, the immediate effect is to shift the gradation of one or other of the ordinates so that no longer are equal changes in magnitude represented by equal distances on the scale. Such distances are arbitrary rather than absolute. The coördinate which is usually changed in graduation is the abscissa.

Suppose, as in Fig. 58, we have a curve XYZ, showing relations of height in feet attained by trees 7 to 12 inches D.B.H. Draw a straight line $XY'Z$ across this curve in any convenient position that approximates or coincides with its general trend. In order to make this straight line the equivalent of the curve all that is necessary is to revise the graduations on the abscissa axis in such a way that stated magnitudes as measured on the arbitrary scale for the straight line are equal, at similar points on the height scale of the vertical ordinate, to absolute values for the curve.

In Fig. 58 the values at 7 inches and at 12 inches are the same for both the straight line and the curve and will remain unchanged. At 8, 9, 10 and 11 inches certain shifts are made as follows. Beginning at the base of the 8 inch ordinate of the curve proceed upward as indicated by the arrow to its intersection. From this point a line is projected horizontally to intersect the straight line. Two points, one on the curve and one on the straight line, have been established which are the same number of vertical units above the base. Hence, if from the latter point (on the straight line) a perpendicular, as indicated by the arrow, is dropped to the base, it will mark a calibration of the scale which can be

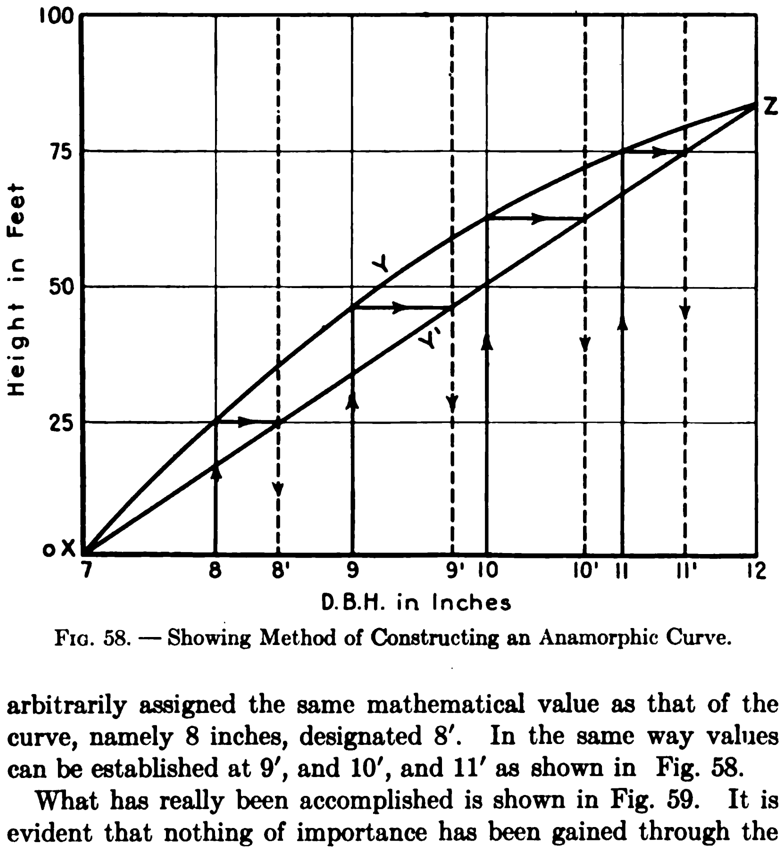


FIG. 58. — Showing Method of Constructing an Anamorphic Curve.

arbitrarily assigned the same mathematical value as that of the curve, namely 8 inches, designated 8′. In the same way values can be established at 9′, and 10′, and 11′ as shown in Fig. 58.

What has really been accomplished is shown in Fig. 59. It is evident that nothing of importance has been gained through the anamorphosis of a single curve. It is true that a straight line with an *apparent* regular rate of increase has been substituted for a curve. But this is out-balanced by the complexity introduced in the irregular system of the new graduations on the abscissa axis. If there was a series of two or more *independent* curves radiating from the origin, their anamorphosis would have produced as many sub-graduations of the horizontal axis as there were curves, with resulting confusion.

However, it is with this last type of curve projection that anamorphosis finds its most efficient use. A series of *related* curves,

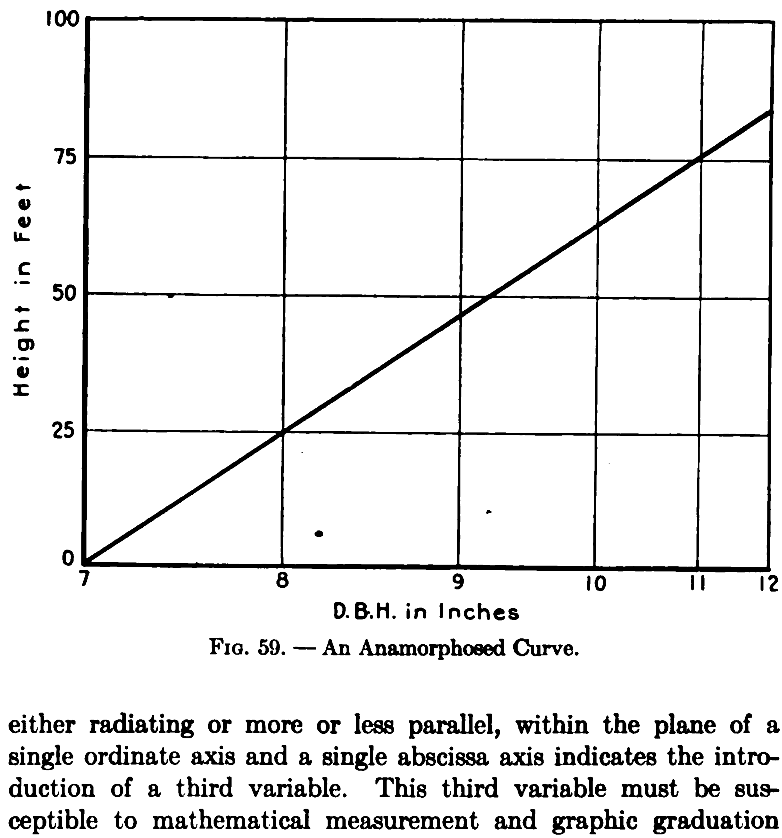


FIG. 59. — An Anamorphosed Curve.

either radiating or more or less parallel, within the plane of a single ordinate axis and a single abscissa axis indicates the introduction of a third variable. This third variable must be susceptible to mathematical measurement and graphic graduation as were the other two variables. Since this third variable is a common characteristic of all items included, its delineation is usually accomplished in graphics through the equispacing of a series of curves of similar trend. Such is known as a set of *harmonized curves.* We have such a set when we plot volume or form factor on diameter and height, or volume on height and diameter, or volume on age and on site class values, etc.

If one of the curves in a harmonized series is anamorphosed, the resulting regraduation of the abscissa axis often serves as an automatic means of transforming all the other curves of the series to straight lines. Thus, since the complexity associated with the spread and trend of a series of curves is replaced by the minor complexity of a regraduation of the abscissa axis, there are evidenced obvious advantages of construction, adaptability and use.

The simultaneous anamorphosis of a series based on three variables is best accomplished by the use of a *graduating curve* drawn, in obverse relation, downward from the common point of origin of ordinate and abscissa, both of which are scaled, though in opposite directions, on the vertical axes of rectangular coördinates. The graduating curve serves as the basis of magnitude determination in terms of all three variables. This can best be illustrated by a concrete example.

An examination of the relations of total height on age as influenced by site quality for jack pine in the Lake States reveals the data given in Table XI.

TABLE XI

AVERAGE HEIGHT OF DOMINANT TREES

Jack Pine in the Lake States

Age in Years	Height of Tree in Feet		
	Good Sites	Medium Sites	Poor Sites
10	17.5	13.2	9.5
20	29.5	23.2	17.2
30	46.0	37.0	27.8
40	56.5	47.0	36.0
50	64.0	53.0	43.5
60	70.2	58.1	47.5
70	76.5	63.5	52.2
80	82.1	67.9	56.0
90	87.1	72.1	57.8
100	91.2	75.1	61.9

A sheet of standard rectangular cross-section paper is prepared for plotting. Any convenient point approximately midway on the left-hand vertical or ordinate axis is selected as the origin or zero. Total height in feet is scaled on the vertical or ordinate axis above the zero in conventional form. But the abscissa axis, instead of being scaled as a horizontal axis, *is scaled downward from the point of origin* in terms of equal magnitude. The whole process is simply that of considering the abscissa axis scaled first as a horizontal axis and then pivoted downward on its origin through a horizontal arc of 90 degrees to the right.

The next step is the construction of the graduating curve. The horizontal axis at the point of origin is first temporarily graduated in conventional form but *in terms of the dependent variable.* The intersection of coördinates representing mean values as projected vertically downward from the temporary axis with corresponding horizontals will determine a series of points through which the graduating curve is drawn. (See Fig. 60.) As soon as the graduating curve is constructed the temporary axis is erased.

To anamorphose the series one proceeds as follows. Start at any convenient point, such as 50 years, on the downward axis representing the abscissa and proceed horizontally to intersect the graduating curve. Thence proceed vertically to intersect corresponding values for the three site variables as scaled on the height scale of the ordinate. In the figure, the heavy line with the arrows indicates this process. Other age values will develop other points. Straight lines, radiating from the point of origin, are then drawn through the averages of these intersections to represent heights on age classified by similarity of site quality. As shown by the points, the straight lines do not always fit the data. In any series of curves the several sets of data are of varying reliability. In such a series the strongest should always be the basis of the anamorphosis, and divergence from the data of weaker curves may not be taken too seriously. The essential thing is that the shapes of the curves and the variables they represent are identical, that is, they show harmonization. If desired, the anamorphosed curves may be redrawn in conventional form as in Fig. 38.

138. The Application of Anamorphosis. — The greater number of curves in forestry are based on measured data. When more than two variables have been measured some process of harmonization often on very meagre data is at once indicated. Harmonization by inspection or approximation is rarely successful. Us-

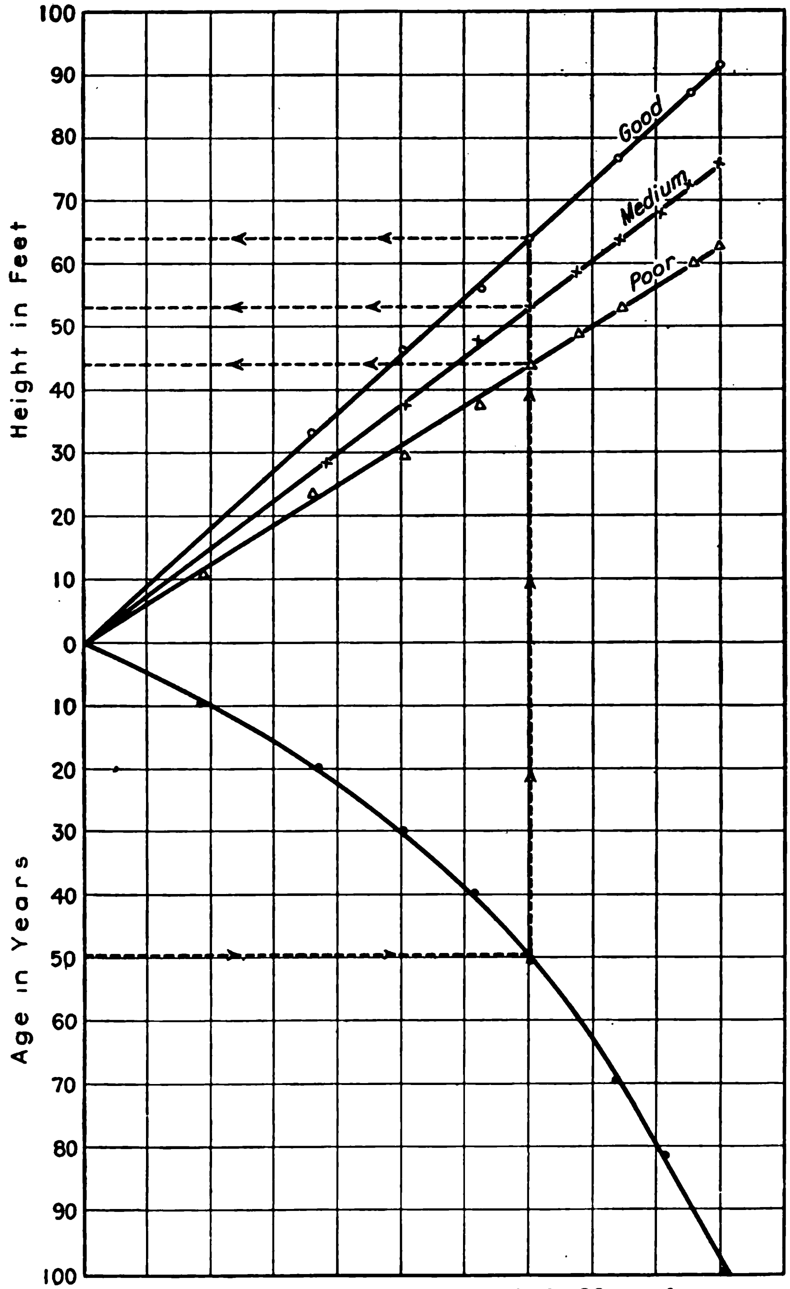


FIG. 60. — Anamorphosis of a Curve Series by Means of a Graduating Curve.

ually the central curves being based on the greater number of measurements show some reliability, but weakness is apt to develop toward their extremities, and lack of harmony will be observed in the outer members of the series. The function of anamorphosis is to eliminate human judgment and reduce approximations to a minimum, and to place the whole process of harmonization on a mechanical basis.

Anamorphosis should be regarded as a means to an end and it must not be considered as applicable to every condition. There are a great many cases where anamorphosis will not work. Curves which first rise and then fall, one or more times, are not susceptible to this treatment. Constant vigilance must be maintained to recognize unfavorable conditions in application. The chief danger sign is not so much a failure of the lines to fall in a linear zone as a definite regularity in nonconformity. Sometimes anamorphosis will present a condition contrary to fact. When yields per acre data by site qualities and age are anamorphosed, the harmonization may be so complete as to indicate the culmination of mean annual growth and current annual growth as occurring on all sites at the same age. This is contrary to experience, and as such appears to indicate a limitation of the method.

Anamorphosis involves a considerable degree of care and labor. Moreover, the picture of straight lines is seldom as clear a visualization of the data as that presented by curves. On the other hand, with due recognition of its limits of application, it is probably the best available means of perfecting curve harmonization with a minimum misinterpretation of basic data. A certain amount of interpolation and extrapolation is also made more easily available.

CHAPTER XI

FORM AND TAPER

139. The Variable of Tree Form. — Theoretically, all trees in their main stem approximate the form of a cone-like solid. Practically there are wide variations in this form due to variations in the *rates* of diminution in diameter from the base to the tip of the tree. This progressive diminution in diameter is known as *taper*. Within the same forest or stand, taper varies with species, with size (D.B.H.), with height, and with age, particularly in the portions of the tree where the stem penetrates the crown.

One of the main problems of the forester is that of the general application of the results of his tree studies and investigations. Any set of figures, no matter how carefully constructed, such as a volume table or a growth study for a given species, which is based on the principle of averaging measurements of trees whose dimensions fall within certain limits of height and diameter, is limited in its general and specific application by the factors just cited. Variations in taper are fundamental reasons for variations in volume, and hence it is of utmost importance that this factor be given recognition in all studies which relate to volume in trees and in stands of trees.

140. Methods of Studying Tree Form. — There are three different methods for studying tree form.

1. By comparison of Standard Form Ratios.

   This is the oldest of all form study methods and endeavors to express variation by means of:

   (*a*) Form Factors.
   (*b*) Form Quotients.

2. By a Classification of Form on the Basis of Form Ratios.

   This method, which has been developed to its highest point by forest investigators in Sweden, endeavors to classify all trees as members of arbitrarily established " form classes," that is, trees whose surface lines follow the same form or hyperbolic curves, as determined on the basis of:

(a) Form Quotient Ratios.
(b) Form Point Ratios.

3. By compilation of Taper Tables.
This is a method which seeks to show the actual form of trees on the basis of a tabulation of the average diameter dimensions at arbitrarily selected points on the tree stem.

141. Form Factors. — The first studies seeking a solution of the problem of form variations were made in an effort to coördinate form and volume by Form Factors, that is, by establishing a series of mathematical ratios between the volumes of trees and the volumes of some standard geometric solids such as cylinders or cones which had the same dimensions of diameter and height. The main object of such investigation was to seek a standardized law of form in trees, and from this to derive a correction factor, applicable to trees in the computation of their volume. In this, the study failed, due mainly to the fact that the basic diameter (used for both tree and solid) was subject to the variations of butt swell. The accepting of diameter at breast height did not solve the difficulty since it introduced into the already over-complex problem a new variable, not due to form, but to varying proportion or ratio between D.B.H. height and total height.

A Form Factor is a statement of mathematical relationship or proportion; a ratio between the volume of a tree and the volume of a conventional geometric solid, such as a cylinder, a cone, or the frustum of a cone, a paraboloid or a neiloid which has the same dimensions of height and diameter.

In formula form this is written as follows:

$$\text{Form factor } (Ff.) = \frac{\text{Volume of tree}}{\text{Volume of Geometric solid}}$$

hence Volume of tree = Volume of Geometric solid × Form factor.

The form factor thus becomes a reducing figure by which the volume of a geometric solid of the same dimensions as the tree is multiplied to derive the volume of the tree.

When the geometric solid to which the volume of the tree is compared is a cylinder, this reducing figure is necessarily a fraction or decimal, since the stem of the tree is not cylindrical but approximately conical in shape, and has a smaller volume. When the compared geometric body is either a cone or a frustum of a cone, this reducing figure is near to unity. Hence there are three types of form factors whose classification is based on the form of the geometric solid to which the volume of the tree is compared:

(*a*) Cylindrical form factors.

(*b*) Conical form factors.

(*c*) Frustum form factors.

The first two are best adapted to cubic volume and are mainly so used. The last, that of frustum form factors, is mainly adaptable to board foot volume and is to be used in the computation of volume of the merchantable portion of the stem of the tree. The applicability of this type of form factor is easily understood when it is remembered that the used portion of the stem of any tree is in reality the frustum of a conoid.

Form factors are always classified by species, each separate species having a different set of form factors. They are further classified on the basis of diameters and heights, either merchantable height or total height. In any case a diameter measurement is necessary, and such measurement, though measured *outside the bark* must be reduced to terms of *inside the bark* at that point, in order to obtain the correct basal area dimension

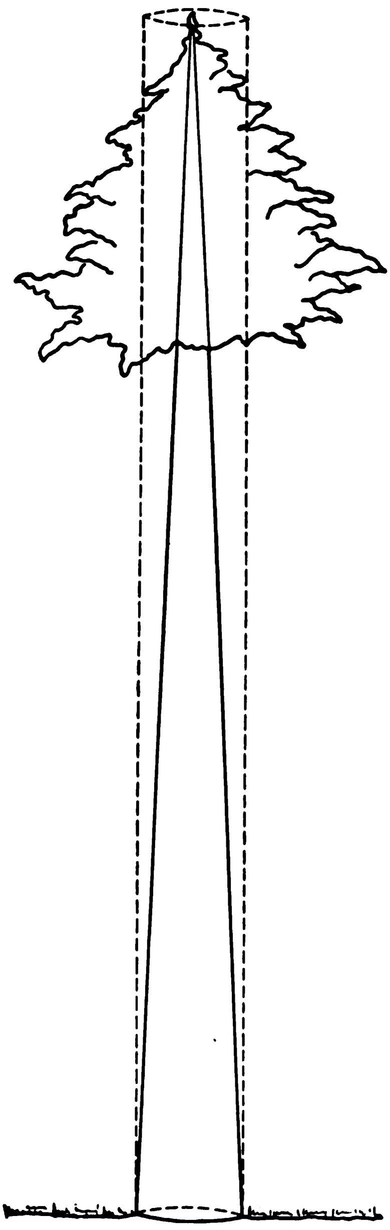


FIG. 61. — A Form Factor (*Ff.*) is a Ratio of Tree Form which Expresses the Relation Between the Volume Contained Within the Tree and the Volume of a Geometric Solid (Such as a Cylinder) of the Same Dimensions of Basal Diameter and Height.

requisite for the calculation of volume of both the tree and the solid.

According to the location of the basal diameter measurement on the standing tree there is developed another classification of form factors:

(a) *Breast Height Form Factors.* — When the diameter measurement accepted for the basal area determination of both tree and solid is the D.B.H. It is sometimes assumed that the diameter breast high outside of the bark is the same as the diameter inside the bark of the stump. This assumption is scientifically inaccurate. If D.B.H. is used it must be reduced to inside bark dimensions and the portion of the tree below and outside the D.B.H. dimension is disregarded.

(b) *Absolute Form Factors.* — When the diameter used is that at the base of the *used* tree stem.

(c) *Normal Form Factors.* — When the diameter dimension used for the basal area determination of both the tree and the solid is taken at a fixed and relative position above the ground. The height of this diameter measurement should bear a definite and predetermined ratio to the total height of the tree, as for example, $\frac{1}{20}$ of the total height.

It is also important that trees must be of the same form height and that the amount of stem length included in the computation of volume must be stated or implied. According to the amount of stem height used, there is developed still one more classification of form factors:

(a) *Merchantable Form Factors.* — When the height used in the volume computation is equivalent to the merchantable length.

(b) *Stem Form Factors.* — When the full length of the total height is used.

(c) *Tree Form Factors.* — When the total height is used for the computation of the volume of the solid and when the entire volume of the tree including stump, logs, top and branches is determined. This is seldom or never used in American forestry practice but is not uncommon in Europe.

Construction of form factor tables can best be done by following the tree fellers on a logging job and measuring the trees on the ground. In this way more accurate measurements can be taken, and more easily, especially those required for diameters in the upper portions of the stem. Results of trees of similar D.B.H.

TABLE XII

BREAST HEIGHT CYLINDRICAL STEM FORM FACTORS
ADIRONDACK SPRUCE

Township 2, McCombs Great Tract No. 2, St. Lawrence County, New York
(Based on 751 trees)

D.B.H. Class	Total Height Class of Tree in Feet					Average *Ff.* in each D.B.H. Class
	30	40	50	60	70	
4	0.567	0.544	0.528			0.549
5	.559	.556	.502			.547
6	.530	.511	.497			.504
7	.526	.506	.492	0.446		.498
8		.502	.498	.487		.496
9		.512	.492	.487		.495
10		.509	.494	.465	0.462	.485
Average form factor for each height class	0.540	0.517	0.495	0.474	0.462	0.506

and height are to be averaged together. These subsequently are curved by diameter classes or by diameter class over heights for the purpose of evening off irregularities.

142. The Form Quotient. — According to the definition* of the Society of American Foresters, a form quotient is the quotient of the breast high diameter of a tree into the diameter measured at any height above the D.B.H., usually at ¼, ½, or ¾ of the height of the tree. The form quotient which is most commonly used as a means of expressing the relative form of trees is that at ½ height. As such it is classified in two ways:

(*a*) *Normal Form Quotient.* — Where the upper diameter measurement is taken at a point on the tree stem at ½ its total height.

* Jour. of Forestry, Vol. XV, No. 1, Jan. 1917, p. 81.

(*b*) ***Absolute Form Quotient.*** — Where the point of upper diameter measurement is taken exactly half way between the tip of the tree and the D.B.H. point.

The Form Quotient, then, is a ratio which expresses the mathematical relation between two diameter dimensions in the same

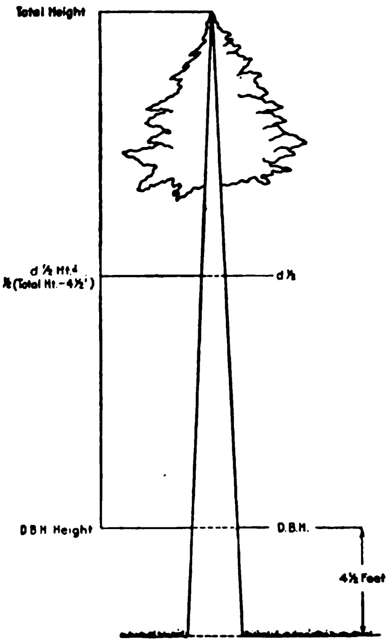


Fig. 62. — A Form Quotient (*Fq.*) is a Ratio of Tree Form which Expresses the Relation or Ratio Between the D.B.H. and Some Upper Diameter Such as the Diameter at One Half of the Total Height.

tree. It is thus quite distinct from a form factor, which expresses the volume relation of a tree and a geometric solid of similar length and basal area dimensions. As a mathematical expression:

$$Fq = \frac{d}{\text{D.B.H.}}$$

or more commonly

$$Fq = \frac{d\frac{1}{2}}{\text{D.B.H.}}$$

For several years after its development, the idea of the form quotient as an expression of tree form was regarded as a mathematical novelty. In 1899 the Austrian forester, Schiffel, made

TABLE XIII

ABSOLUTE FORM QUOTIENTS FOR ADIRONDACK SPRUCE

Township 2, McCombs Great Tract No. 2, St. Lawrence County, New York
(Based on 751 trees)

D.B.H. Class	Total Height Class of Tree in Feet					Averages for all D.B.H. Class
	30	40	50	60	70	
4	78.2	76.9	74.7			77.0
5	76.3	72.5	71.3			72.9
6	74.6	73.0	71.2			72.9
7	74.2	72.6	71.5	65.3		72.2
8		73.7	71.7	70.4		72.0
9		73.0	71.2	70.9		71.4
10		73.0	71.0	69.0	67.0	70.4
Averages for all height classes	75.6	73.0	71.1	69.3	67.0	72.3

practical use of it in his studies of tree form. He deduced the fact that variation in the form of trees indicated by this ratio could be used in the computation of volume. In an involved and complicated calculation he derived the formula for cubing the entire contents of standing trees in one computation as:

$$V = (0.16\,B + 0.66\,b)\,H$$

where B^* = basal area in square feet corresponding to the D.B.H. measurement.

b^* = basal area in square feet corresponding to the $d\frac{1}{2}$ measurement.

H = height of the tree in feet.

V = volume in cubic feet.

A subsequent study by Tor Jonson, a Swedish forester and mathematician, corrected a slight error inherent in Schiffel's

* A set of tables giving these values will be found in the Appendix. Table XXV, page 297.

formula, namely, the relative variation in the height of the $d\frac{1}{2}$ point. That is, the height of D.B.H. measurement was a constant whereas the $d\frac{1}{2}$ was variable in position and was measured relatively much closer to the D.B.H. point in short trees than it was in tall trees. Jonson eliminated this as a variable by insisting that the $d\frac{1}{2}$ must be taken at a point not half way between the ground and the top but *exactly half way between the point of D.B.H. measurement and the top.* When this adaptation was applied to Schiffel's conclusions, there was finally secured an adequate and consistent expression of the tree volume based on diameter, height and form.

143. Form Class. — The researches of Tor Jonson in his study of form were carried to further conclusions. With height and diameter thus taken care of, the recognition of the Absolute Form Quotient as a standard of form permitted the distinguishing or classification of the third variable of volume, namely, *taper.* This is known to be dependent upon environment, in that the main factor of environment which affects tree form is density or crowding.

FORM CLASS AS INFLUENCED BY STAND CROWDING

Character of Stand	Form Class (Form Quotient)
Poor density	0.575–0.625
Fairly good density	0.625–0.675
Good density	0.675–0.725
Overcrowded	0.725–0.750

From this, two very obvious conclusions can be drawn, first, the denser the stand the larger the diameter (that is, less taper) in the upper sections of the tree; and second, the fuller the bole in the upper sections the less the development of the crown in the proportion that its length bears to the total length of the tree stem. It may be pointed out that European investigators have long established the conclusion that the real taper of the bole of a tree does not begin until the stem penetrates the crown.

Form Class was to be expressed through the medium of and is the equivalent of the Absolute Form Quotient. All volume computations may be classified by form class as well as by diameter and height.

144. Form Point. — It was further pointed out by Jonson, Maas, and other Swedish investigators that the development of the form* of a tree as exemplified in the longitudinal section of the stem which graphically represents its dimensions, depends absolutely upon the mechanical stresses to which the tree is exposed, and not upon its characteristics of height and of diameter, not upon its species, age, site, nor any other factor, except as these factors affect the shape and development of the crown. The stresses to which the tree is subjected are two: first, in the minor amount, that of gravity from the dead weight of the stem and limbs, comparable to the stresses of compression in a vertical column; and second, that of the force of wind, or rather the resistence to wind force, which operates and is focused or concentrated in point of maximum pressure. The stresses resulting from wind pressure are such as to compel the tree to construct its stem in such a way that with the smallest possible amount of material being used, there shall be developed at all points on its longitudinal axis the same relative resistance to fracture or shear. As the concentrated force of the wind strikes a point lower or higher on the tree (dependent upon the relative position and shape of the crown), there is developed quicker or smaller taper respectively.

When it was recognized that the form of the stem of a tree was directly resultant to the mechanical laws governing stress, it was possible to calculate the normal relation between crown form and stem form. The main determinative of the form of the stem is the position of the point of greatest resistance to wind bending. Since it is the crown of the tree and not the stem which offers this resistance, the point where the wind offers its concentrated pressure should be centered approximately at the center of gravity of the crown. This is known as the "*form point*," when its height above the ground is expressed as a percentage of the total height of the tree, and its value will determine the Form Class of tree.

* The formula used by Jonson was Höejer's equation for tree form: $\frac{d}{D} = C \log \frac{c+h}{c}$, where D = D.B.H.; d any diameter above D.B.H. and C and c are constants; and where h is equal to $\frac{\text{Total height} - \text{height of "}d\text{"}}{\text{Total height} - \text{breast height}}$

In practice, the method consists of measuring* or estimating in the field a sufficient number of trees to get a good indication of the average " form point " of the stand. Reference is then made to a set of tables which shows the corresponding form class for given form points, that is, absolute form quotient.

There are two limitations to this form point method of estimating the form class of standing trees. The first is due to the

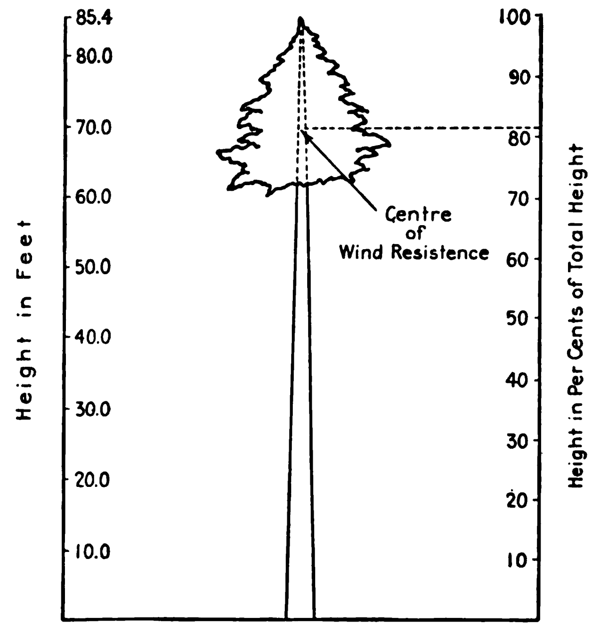


FIG. 63. — Form Point (*Fp.*) is an Expression in which the Center of Wind Resistance in the Crown is Stated in Terms of a Percentage of Total Height.

natural distortion of the position of the true form point. Based on the idea that it is the apparent center of wind pressure, it presupposes the fact that the apparent and the actual center of the wind pressure are the same. However, due to surface friction, the wind near the ground does not exert so great a pressure as in the upper part of the tree. In consequence of this fact, there re-

* A simple instrument for form point determination can be secured by graduating the scale of a Chrysten Hypsometer on a percentage basis rather than in feet.

sults an actual raising of the true center of wind resistance above the apparent center. Furthermore, the actual point of main resistance to wind within the crown varies with its density and the degree of exposure, both of which will affect the true, rather than the apparent, center of wind resistance. The second limitation is the personal equation in identifying and estimating the position of the form point itself. Inasmuch as it becomes a matter of personal choice, two persons may differ considerably in its selection and identification, thus presuming different values for the same ratio.

Appreciating these difficulties, an interesting form point determination has recently been worked out by Gevorkiantz and Hosley.* They decided that the growing space available had a profound influence on the form of trees. Growing space not only determines the relative dominance of the tree, but also offers opportunity for lateral development. Hence the crown as a solid should be considered in at least two dimensions. They were able to show a strong relation between the form quotient, the dead length and the crown width. Dead length was defined as the length of the main stem above the ground free of living branches, and was expressed as a percentage of total height. Crown width was expressed in terms of an index based on the standard measured widths of crowns of trees with a breast height diameter of 10 inches (o.b.). Reference to a set of tabulated indexes determined the proper one for a tree of given D.B.H. and crown width. The normal equation for the value of the form quotient was then determined as:

$$Fq = 0.4868\ L + 1.5697\ I + 18.2591$$

where Fq = the absolute form quotient, that is, the form class.
L = dead length as a percentage of total height.
I = crown index as taken from the table.

Thus, they point out that a tree which has a total height of 60 feet, a dead length of 42 feet, a crown width of 10 feet and is 8 inches in diameter breast high, shows:

$$L = \frac{42}{60} = 70 \text{ (per cent).}$$

$$I = 12 \text{ (as taken from table).}$$

$$Fq = (0.486 \times 70) + (1.57 \times 12) + 18.259 = 0.71.$$

* Form and Development of White Pine Stands in Relation to Growing Space, by S. R. Gevorkiantz and N. W. Hosley, Harvard Forest Bull. 13, Harvard Forest, Petersham, Mass., 1929.

A form class table was then constructed on the basis of dead length and crown index, by which when the dead length has been measured* and the crown index has been determined, crown class may be rapidly established. According to the appropriate form class of the tree, reference is then made to a set of form class volume tables for the establishment of the volume of the tree or the stand. The limitation of the method seems to lie in the difficulty of accurate determination of the crown index and the establishment of a proper and accurate set of crown index values as a basis for form class. The method has worked with apparent success with the one species, eastern white pine, and appears to have merited a more extended trial with other species and in other regions.

145. Purpose of Taper Tables. — One of the methods of showing the relationships and differences in the forms of trees is by means of a *taper table*. This is a tabulated form which records actual diameter values in inches at definitely established points of measurement more or less equispaced along the main stem between the butt and the tip. Since it immediately supplies all the required data of diameter and length according to size classes it may be used as a basis of volume computation in terms of any desired unit of measurement. In addition, such a table, not being limited by local customs or standards of stump height, usable top diameter, or by any other factor, enables the computation of volume to meet any standard of utilization and for any form of product. The definite endeavor of a taper table is to demonstrate the average form of trees as a step toward the accurate compilation of their volume. In this it actually falls short of its objective since the volumes obtained are not the volumes of the average trees but are the volumes of trees of average dimensions.

146. Form Class Taper Tables. — According to Tor Jonson, taper within a given form class is in constant ratio progression from the bottom to the top of the tree. Hence, by collecting together all trees which show the same absolute form quotients (that is, form class) and determining their respective diameters at regular intervals, it becomes possible to derive taper tables in which all the variables of diameter, height, and difference in form have been given due and proper consideration. When trees of varying height are

* A Chrysten hysometer graduated in percentages again proves to be a handy instrument for such purpose.

thrown together that variable is coördinated by expressing the heights of the different points of measurement, not in terms of the actual height in feet above the ground as they occur, but at equal fractional distance on the bole, expressed in terms of percentages of total height. Similarly, diameter values at the different points of measurement, that is, the taper measurements, will not be expressed in inches but as percentages of D.B.H. When the D.B.H. point is accepted as the base of the tree, the stump and diameter below D.B.H. being disregarded, height above ground values are then determined, not as a percentage of total height above the ground, but as a percentage of the total height *above D.B.H.* This places the determination of the percentile tapers on exactly the same basis as that of the absolute form quotient and derives their consideration as Subordinate Form Quotients.

The simplicity of the conception of the idea becomes apparent when it is appreciated that, by reducing both diameters and heights to their respective percentage bases, taper becomes the absolute criterion of form, and taper tables designed to show form are placed on a true and proper basis.

147. Construction of Form Class Taper Tables — Field. — The measurements necessary to compiling a form class taper table may be taken by following any logging operation. Inasmuch as all measurements for taper must be taken *inside the bark*, the trees are measured at the points where such opportunity is offered, namely, at the ends of the logs.

On pulpwood jobs, where the bark is removed in the logging operation, the log end measurements may be totally disregarded. D.B.H. is determined at the true D.B.H. point. The portion of the stem above D.B.H. is divided into ten equal sections, and tapers, or average diameter measurements in inches, are determined and recorded in inches at these points. This process automatically scales the tree at the conventional 10, 20, 30, 40, and 50, etc., percentile points for taper measurements. The D.B.H. point represents the zero percentile point and the top of the tree the 100 percentile point of the height percentage determination. The D.B.H.o.b. in inches and the total height of the tree in feet, which latter will include the portion of the tree below D.B.H., will be obtained and recorded for purposes of classifying the data.

On logging jobs which do not require the removal of the bark before shipment from the logging area, it will be necessary to deduct bark* thicknesses at every point of measurement. This, especially with thick-barked trees, may slow up the work too materially to commend its general use.

In taking field measurements for taper, extra care must be used not only to get average diameters at each point measured but to make sure that such measurements are not influenced by local swelling due to branch egress, fork, burl, knot, rot or eccentric form. The objective of the work, namely, the regular measurement of the successive diminution in diameter, must be kept constantly in mind, and all factors not incidental to its regular progress should be disregarded.

148. Construction of Form Class Taper Tables — Office. — All field sheets are sorted and collected by D.B.H. and total height classes. Within each height class all measurements *taken at the same percentile height above the ground are averaged.*

Curve A. — FORM BASED ON DIAMETER.

The results determined by averaging within each height class are then cast into a rough chart or graph of the taper for the class. The independent variable or the abscissa will be the diameter in inches, and the dependent variable or the ordinate will be height in feet. It is a composite or average curve which shows the plotted average of form within the class per diameter. See Fig. 64. A separate curve must be constructed for each separate height class. Stump diameters are not plotted in this graph, the curve simply being brought down on its main trend from the lowest points plotted to the base line by eye. Stump diameters, and, in large trees, even breast height diameters, are so subject to the eccentricities of butt swell that their introduction at this time may cause both confusion and error.

On the vertical ordinate of the graph as constructed, measure up $4\frac{1}{2}$ feet above the base line representing the ground, and scale off the value of D.B.H. inside the bark.

Divide the distance between the D.B.H. height level and the top of the tree exactly in half and scale off the value of $d\frac{1}{2}$.

Divide the $d\frac{1}{2}$ as scaled by the D.B.H. to determine the approximate Absolute Form Quotient for this height class.

Thus we have determined its relative *Form Class.*

This is the fundamental purpose of this Curve A.

* The Swedish bark-measuring instrument (Section 25) will probably be the most satisfactory means of obtaining this measurement with consistent accuracy.

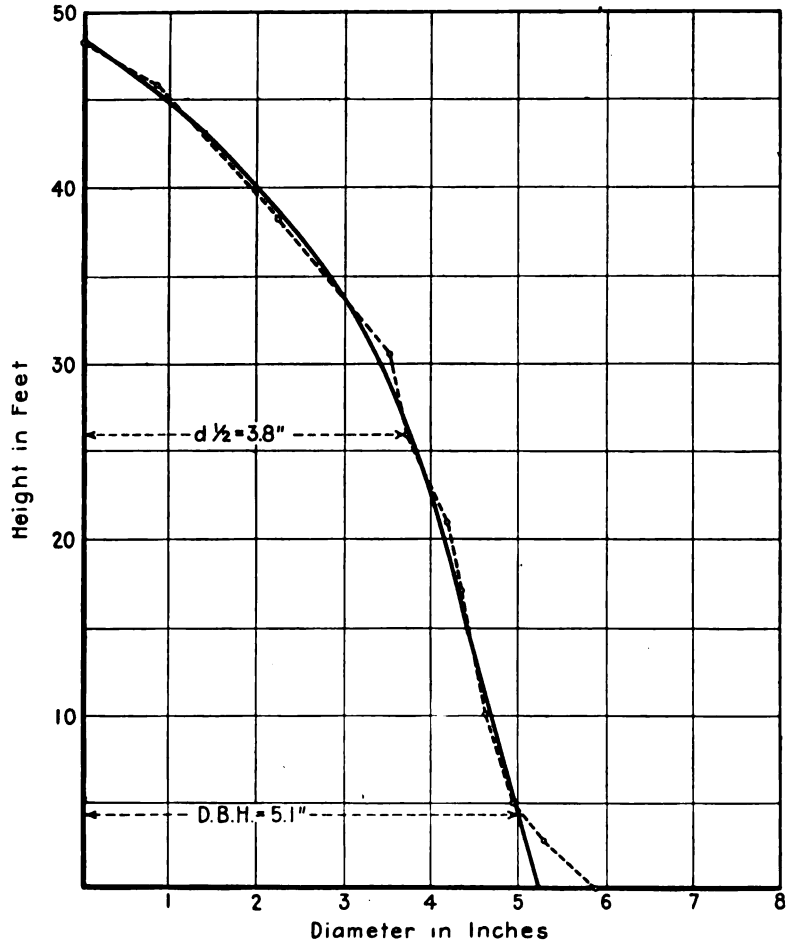


FIG. 64. — Form Class Taper Curve A. Preliminary Taper Curve for the 5 Inch 50 Foot Height Class, Red Spruce (New York). The Dotted Taper Curve Represents Mathematical Averages of the Measurements of 57 Trees. The Absolute $d\frac{1}{2}$ and the D.B.H. are Both Indicated. Form Class ($Fq.$) = 0.745.

Curve B. — Approximate Form Quotients on Size Class.

As was seen in Table XIII, Form Quotient values in our wild unmanaged forests are not absolutely constant for the stand according to its density as claimed by Jonson, but vary from D.B.H. class to D.B.H. class and from height class to height class. A series of harmonized curves which will even off the irregular progression from size class to size class is now desirable. See Fig. 65. Values are taken from the original graph sheets as plotted and computed. From this, curve values of true absolute form quotient or form class are read off and are entered as such on the respective sheets in place of the approximate values.

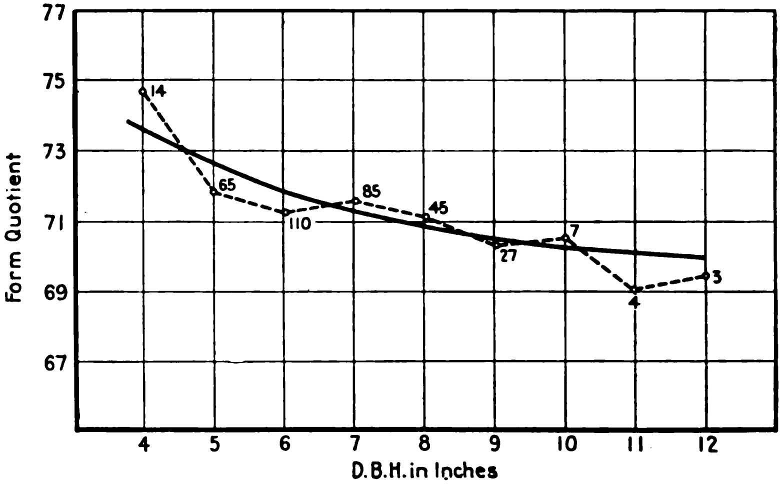


FIG. 65. — Form Class Taper Curve B. Method of Smoothing Out Irregularities in Form Class Values ($Fq.$) From D.B.H. Class to D.B.H. Class in 50 Foot Trees of Red Spruce (New York).

Curve C. — APPROXIMATE TAPER.

With true absolute form quotients thus determined, recourse is taken to the original rough taper charts — Curve A. Select 20 to 30 of these sheets, seeking those ranged throughout the size classes which display the average of a goodly number of trees.

Taking each of these sheets in succession, divide the ordinate, the distance between D.B.H. height and the top of the tree, into 10 equal parts.

At each of these points on the ordinate erect a perpendicular and prolong it until it intersects the curve representing the taper of the tree. These are the percentile height levels.

Scale the distance in inches from the ordinate to the taper curve.

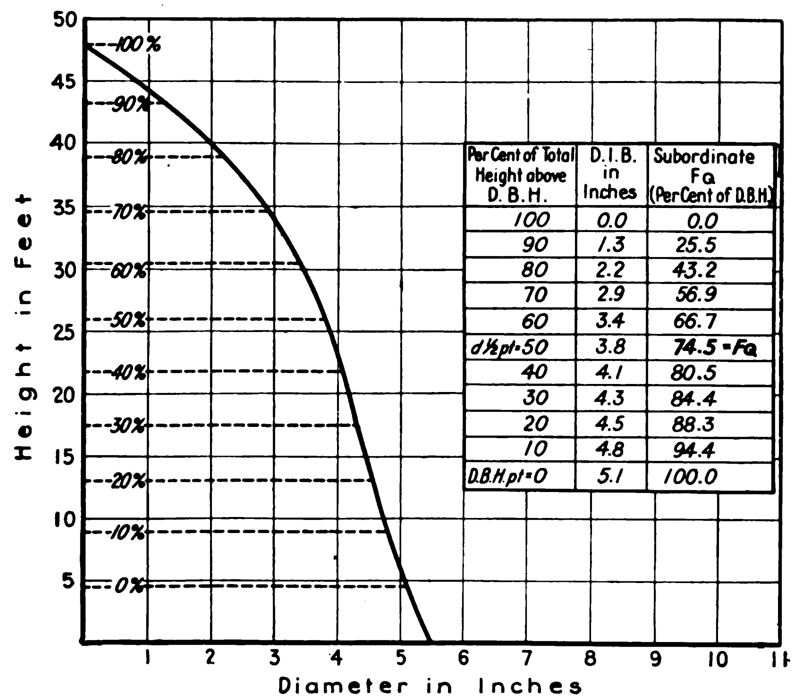
Reduce these distances in inches to percentage values of D.B.H. by dividing each one successively by the scaled D.B.H. This can be done readily and quickly by use of a slide rule.

These are the percentile tapers or subordinate form quotients.

It will be noted in Fig. 66 that the value 74.5 at 50 per cent of the height distance is the absolute form quotient.

Curve D. — Percentile Tapers on Absolute Form Quotient.

There will naturally be some irregularity between corresponding values on the different sheets, and in order to smooth out such irregularities, these values are cast into a set of harmonized curves. The independent



Per Cent of Total Height above D. B. H.	D. I. B. in Inches	Subordinate Fa (Per Cent of D.B.H.)
100	0.0	0.0
90	1.3	25.5
80	2.2	43.2
70	2.9	56.9
60	3.4	66.7
d½pt·50	3.8	74.5 = Fa
40	4.1	80.5
30	4.3	84.4
20	4.5	88.3
10	4.8	94.4
D.B.H.pt = 0	5.1	100.0

Fig. 66. — Form Class Taper Curve C. Approximate Taper at Percentile Heights Expressed as Subordinate Form Quotients, that is, Percentages of D.B.H., for Trees of the 5 Inch 50 Foot Height Class, Red Spruce (New York).

variable plotted on the horizontal scale of the abscissa will be the Absolute Form Quotient, while the vertical ordinate will be scaled as Subordinate Form Quotients. See Fig. 67.

All values are to be plotted on one single graph sheet. The curve at 50 per cent of the height distance should approximately approach a straight line or flat curve. Below that height these curves will mostly fall in straight lines, but above they will show rather easy, smooth curves.

Percentile tapers are now available for any established form class and for any diameter and height class within that form class.

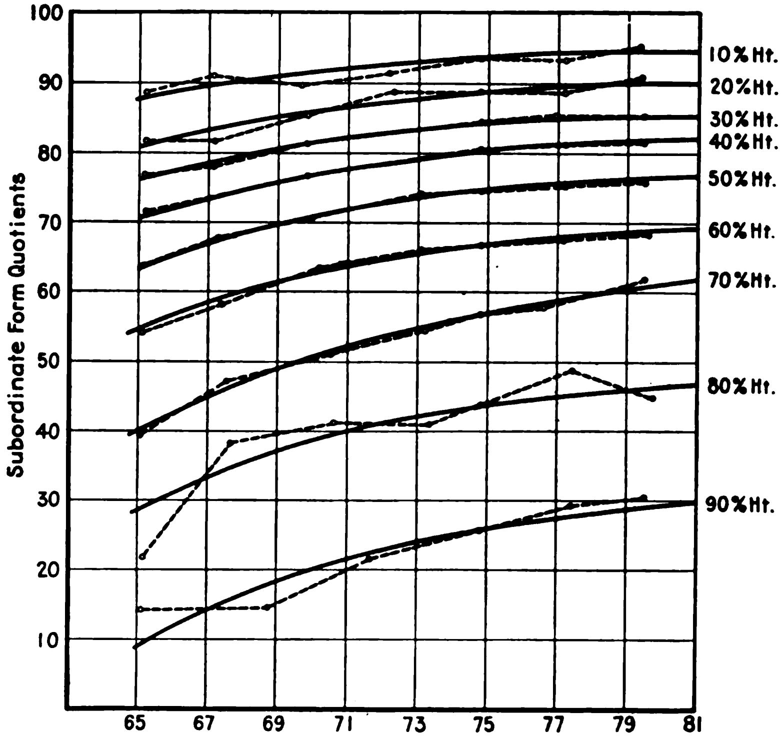


FIG. 67. — Form Class Taper Curve D. Method of Smoothing Out Irregularities in the Subordinate Form Quotient Values for Red Spruce (New York). The Chart Shows Variation in Form Class (Absolute Form Quotients) from 65 to 70.

Curve E. — D.B.H. INSIDE BARK ON D.B.H. OUTSIDE BARK.

Before we can cast these results into a table, it is necessary to harmonize tapers based on measurements inside the bark with diameter breast high measured outside the bark. With D.B.H.o.b. as the abscissa and D.B.H.i.b. as the ordinate this can readily be done by plotting the scaled D.B.H.i.b. measurements from the A set of curves against the averaged D.B.H.o.b. measurements taken from the field sheets.

The object of this curve is to obtain a true value purposely ignored in the Curve A, namely, the D.B.H.i.b.

Curve F. — FORM CLASS TAPER CURVES.

Now, with correct D.B.H. inside the bark, correct subordinate form quotient values, and correct absolute form quotients, or form class, it is a simple matter to draw a set of exact taper curves within each form class.

This graph is of precisely the same form as the first sets of curves. "Height Above Ground" in feet is the abscissa and "Diameter in Inches" scaled off at the percentile heights is the ordinate. See Fig. 69.

FIG. 68. — Form Class Taper Curve E. Relation of D.B.H. inside bark to D.B.H. outside bark, Red Spruce (New York).

The advantages of this method are:

1. The work progresses toward a definite end by direct methods.
2. Errors directly due to the process of harmonization are avoided.
3. A permanent record of correct tree form is established.
4. The method is mathematically accurate and dependable.
5. The construction of volume calculations and volume tables according to any specified standard of utilization is easy, and any reconstruction to meet changing standards is simplified.

149. Form Factors from Percentile Tapers. — If a set of percentile tapers are available, there is offered a very simple method of computing form factor values. Let D be the D.B.H. and let it be expressed as a percentage value of D.B.H., namely, 100 per cent. Let the subordinate form quotients at the percentile heights be known as d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, and d_9.

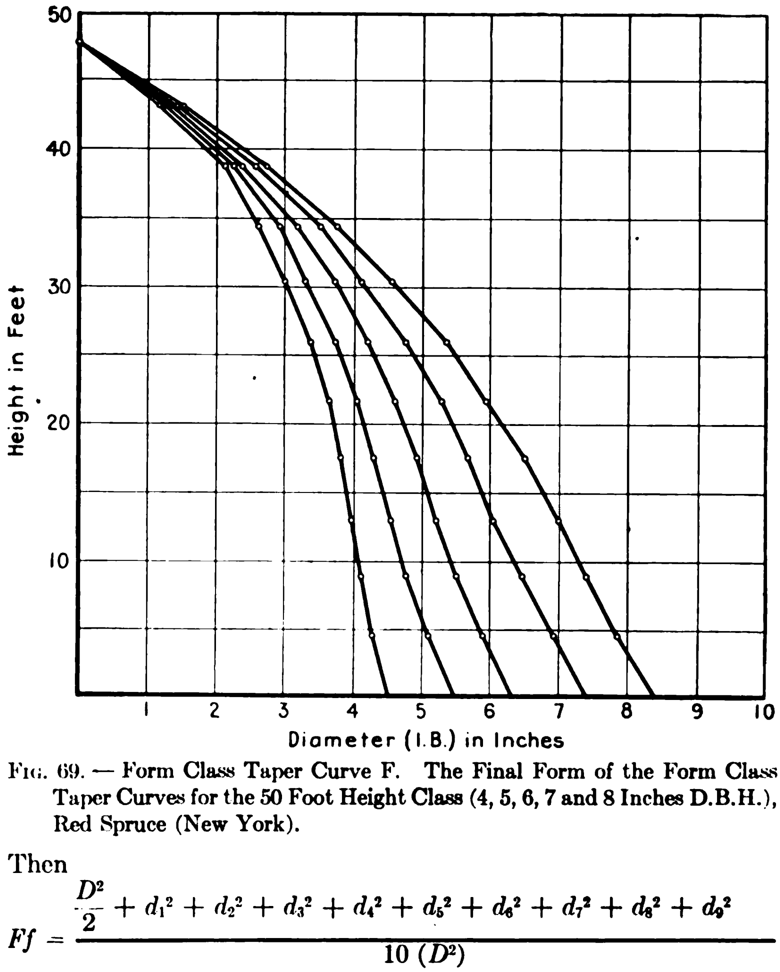


FIG. 69. — Form Class Taper Curve F. The Final Form of the Form Class Taper Curves for the 50 Foot Height Class (4, 5, 6, 7 and 8 Inches D.B.H.), Red Spruce (New York).

Then

$$Ff = \frac{\frac{D^2}{2} + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2 + d_8^2 + d_9^2}{10\,(D^2)}$$

$$= \frac{\frac{10{,}000}{2} + {}^*(94.4)^2 + (88.3)^2 + (84.4)^2 + (80.5)^2 + (74.5)^2 + (66.7)^2 + (56.9)^2 + (43.2)^2 + (25.5)^2}{10\,(10{,}000)}$$

$$= \frac{51{,}065}{100{,}000} = 0.511$$

This form factor, which is known as Rinicker's Form Factor, applies only to the portion of the tree *above D.B.H.*, and error will be incurred if applied to the entire tree.

* Values from Fig. 66, page 207.

CHAPTER XII

VOLUME TABLES

150. The Purpose of Volume Tables. — A *volume table* is a tabulated statement which shows for a given *species* the average contents of trees of different sizes. The tabulation may be in any unit desired, but the units most commonly used are the board foot, cubic foot and cords. With proper converting factors it may be possible to transfer the values from one unit to another as desired.

Volume tables are used in timber estimating chiefly to determine the volume of the timber standing on a given area.

Volume tables for the most part are capable of application solely to the species for which they are made, and to that species only within narrow local limits. A volume table made for red spruce at Newcombe, N. Y., for example, might be applicable all over the Adirondacks, but not in Maine nor in West Virginia. It could not be used for Engelmann spruce in Colorado, nor for Sitka spruce in British Columbia. In Europe and in India several attempts have been made to construct general or universal volume tables applicable wherever a given species is grown. But so far, in this country, no great success has attended such efforts.

151. Standard Volume Tables. — Standard* volume tables are so called because by custom and by use they have become *standards* in forestry practice all over this country. They are based on the measurement and determination of the volumes of trees under actual conditions of logging practice. The volumes recorded are the average volumes of a number of separate measurements. The number of trees required as an adequate basis for a

* Several editions of standard volume tables for the more important species in the United States are available. Special mention may be made of:

(*a*) Technical Bulletin 39, University of Minnesota, St. Paul, Minn., 1928.

(*b*) Bulletin 14, New York State College of Forestry, Syracuse, N. Y., 1923.

(*c*) Volume Tables for Important Timber Trees of the United States, U. S. Dept. of Agric., Parts I and II; Washington, D. C., 1925.

good volume table depends upon the accuracy and intensity of the methods used in its construction. It requires the collection of data for from 100 to 2500 trees depending upon the species of tree and the kind of table.

Standard volume tables may be classified in several different ways, according to the method in which the trees have been classified in the field, namely:

1. By diameters* alone.
2. By diameters and total heights.
3. By diameters and merchantable heights.
4. By diameters and log lengths.
5. By diameters and tree classes.

Volume Tables Based on D.B.H. — In this form of table the trees are classed only by diameter breast height outside of the bark. All that the table seeks to show is the average volumes of trees of different diameters regardless of the variation of heights. The volumes of tall trees are averaged with those of short trees so long as all have the same D.B.H. value, and this average volume is tabulated as the volume value for the class. Hence, volume tables constructed on such a basis are capable of accuracy in the long run only when applied to the aggregate of a large number of trees. Even then they are of but local application.

Volume Tables by D.B.H. and Total Heights. — In two trees of equal diameter but different total height, the taller tree will have the larger volume. This is true in the case of cubic volume and is also true in board feet volume. Taller trees have greater merchantable length, and having fuller taper, produce larger logs with more content. In a volume table where the tabulation is made on the basis of diameters classified by different heights, it is therefore possible to express the volume of the contents more accurately. A table of this kind is much more applicable to general conditions.

Volume Tables Based on D.B.H. and Merchantable Length. — The volume which is of most concern to a buyer or seller of standing timber is the merchantable or usable volume. The factor of length determining the used volume of the tree is, of course, the merchantable length. By adopting this unit greater accuracy can be attained in the construction of a volume table. It has several

* The diameter mentioned here is always D.B.H.o.b.

drawbacks, however, in that its height classification must be a constant multiple of the two foot increase in which logs are cut, so as to meet any combination of log lengths that might be devised. This requires a great many more columns in the tabulation, as many as 30 to 50. The construction of such a table is very laborious, and its application, notwithstanding its accuracy, is unwieldy. In order to warrant its use in the field, the heights of the trees to which it is to be applied must be constantly and accurately measured with hypsometers. One other drawback is that the acceptable merchantable length varies from region to region and from job to job.

Volume Tables on D.B.H. and Standard Log Lengths. — These are similar to diameter-merchantable length tables, but are more convenient for the practical cruiser. The standard log length usually accepted for these tables is 16 feet. A skillful cruiser needs no measurement of height as he can with reasonable accuracy train his eye to judge the merchantable portion of the stem of a tree in log lengths.* Trees can easily be tallied and the computations are not difficult. It is a much better type of table than the preceding one. Its drawback is that logs are not cut in standard 16 foot length all over the country. In New York State, Vermont and Quebec 13 feet is a common length. Pulpwood† is commonly cut in 4 foot lengths. On the Pacific Coast 30 to 62 foot log lengths are not uncommon. All of these vary far enough from the 16 foot standard to make its application unhandy.

Volume Tables Based on D.B.H. and Tree Class. — In many cases, for some kinds of work, particularly in estimating pulpwood or fuel wood, where the actual cubic contents are of more consequence than the form of the species, volume tables are prepared for trees of different diameters and crown classes:

Class I. — Trees growing in the open with large spreading crowns occupying 50 to 75 per cent of the full stem length.

Class II. — Trees in uncrowded stands, more or less irregularly forked with many branched stems, the crown occupying 50 to 75 per cent of full stem length.

Class III. — Trees in crowded stands, with crowns occupying

* It is sometimes customary to recognize half logs (8 foot lengths). These may be interpolated if desired.

† In the Lake States, pulpwood is often cut in 8 foot lengths.

less than 20 per cent of the full stem length, and with very irregular, deformed, crooked stems. Trees badly suppressed as a result of insufficient light and growing space are examples in this class.

152. Construction of Volume Table — Field. — Volume tables are constructed from the careful measurement of the dimension and accurate computation of the volume of actual trees. Hence, in selecting trees for measurement, only those trees should be taken that are found to be normal and representative of the average trees in the stand. This means that decadent, malformed or broken trees will be disregarded. Under ordinary conditions, a tree in a forest will usually produce a stem which is rarely absolutely straight, practically never of truly geometric form; yet, nevertheless, the average tree of specified and given dimensions for a given species will approach closely to a certain definite mean which may be accepted as the norm for the species. It is the volume of this *average tree* that is sought from the measurement, computation of volume and averaging of the volume of many actual trees which approach more or less closely to the conformity of its lines.

The field work, with but minor exceptions which will be pointed out in turn, is essentially the same in all forms of construction.

Data may be collected by following any logging job and taking the necessary measurements on the felled trees. Trees which show the condition of forked top, abnormal crook, broken top, and heavy lean, and trees with turpentined or fire-scarred butts which distort the D.B.H. values, should not be measured as a basis for volume table construction.

Diameter Breast High is measured to the nearest tenth of an inch at 4½ feet above the average ground level. Outside bark measurements, the average of two measurements at right angles, are taken, but bark thickness to the nearest $\frac{5}{100}$ inch should be measured* and the corresponding *inside* bark measurement determined.

Diameters should be measured at definite intervals *above the stump*, which in virgin timber should not exceed 1.5 feet in height, and in second growth, 1.0 feet. The length of these intervals depends upon the relative height of the tree as follows:

* The Swedish Bark Measurer will prove to be a most convenient instrument for this purpose.

(*a*) At 8.15 feet intervals where the maximum height of the timber does not exceed 5 standard (16 foot) logs.

(*b*) At intervals of 16.3 feet for taller timber.

In addition, intermediate diameter measurement should be taken on the butt log at 1 foot above the stump, 2 feet above the stump, 6 feet above the stump, 8.15 feet above the stump, and 12 feet above the stump. This is for the purpose of getting accurate data on the stump taper and the effect of butt swell. All of these diameter measurements will be average diameters taken outside the bark, and inside bark values must be determined from accurate measurements of bark thickness to the nearest $\frac{5}{100}$ inch at the various points of measurement.

Dimensions of length or height should be measured with a tape or measuring stick to the nearest half foot. The tape is to be preferred. If possible all height measurements should be taken before the tree is sectioned into logs, but this is not always feasible. When length measurements are taken after the logs have been cut, care must be used to add together the lengths of all the several sections in getting height values. Total height is measured from the average ground level to the tip of the tree. Merchantable length is measured from the top of the stump to an arbitrarily fixed top diameter limit which will be determined according to local standards of utilization, and the log rule used. If the International log rule is used, this may be as small as 5 to 6 inches. With the Scribner rule or the Scribner Decimal C rule, it may vary from 6 to 8 or even 10 inches.

If age is desired, it may be obtained from stump ring counts on from $\frac{1}{10}$ to $\frac{1}{4}$ of the trees measured, such count being uniformly scattered through the diameter range. This, however, belongs more to the purview of yield and growth studies than to volume table construction.

All of these data may be entered in special forms or sheets, one for each tree measured. Glass grouping may be prepared for in advance and all trees whose dimensions fall within specified limits may be entered on the same sheet. A common class grouping would be by D.B.H. and total height classes. The U. S. Forest Service standard form sheet No. 334 (see Fig. 77, page 247) could be used, although this special form is perhaps more adapted to growth study than to volume compilation. Reineke's special form sheet (U. S. Forest Service standard form 558*a*) could be used.

This will, however, be discussed later (Section 154). A form similar to the one illustrated in Fig. 70 would be quite adaptable to the present purpose.

Tree No.
Locality
Timber Type
Species

TREE VOLUME SHEET

Date
Form Class
Recorded by

D.B.H. Class Height Class

	Tree Data							Diameter Measurements in Inches													
No.	D.B.H. Inches	Absolute d ½ Inches	Form Class	Total Ht. Feet	Stump Ht. Feet	Merch Length Feet	Length of Top Feet	1 Ft.		2 Ft.		4.5 Ft.		6 Ft.		8.15 Ft.		17.3 Ft.		25.45 Ft.	
								o.b.	i.b.	o.b.	i.b.	o.b.	i.b.	o.b.	i.b.	o.b.	i.b.	o.b.	i.b.	o.b.	i.b.
Total																					
Average																					

	At Measured Heights above Ground														
	34.00 Ft.		42.15 Ft.		50.30 Ft.		58.45 Ft.		and etcetera		Cubic Feet		Board Feet		
	o.b.	i.b.	o.b.	i.b.	o.b.	i.b.	o.b.	i.b.			Merch	Total	Int'n'l	Scribner	
Total															
Average															

FIG. 70. — Tally Form Adapted to Recording Essential Field Data in Volume Table Construction.

153. Construction of Volume Tables — Office Computation. — All tree records are grouped by D.B.H. and height classes. For a diameter range up to 36 inches, 1 inch diameter classes should be used. For a diameter range which exceeds 36 inches, 2 inch diameter classes will be found more convenient. With trees less than 100 feet in height, the height class should be grouped on a 5 foot interval, and for trees 100 feet and more on a 10 foot interval. Within each diameter height class the average dimensions at each significant point of measurement should be carefully determined. Further procedure then varies depending upon whether the unit in which the table is to be compiled is in cubic feet or board feet.

Volume Tables in Cubic Feet. — Within each D.B.H. and height class the average volume in cubic feet is established. Whether this is done by averaging the computed volumes of each tree, or by averaging the measured dimensions of the several trees and then computing the volume of the average tree makes no essential difference. In cubing, the stump is considered as a cylinder with a basal area equal to that at stump height. Logs are cubed by Smalian's formula, in short logs (8.15 feet) for small sized timber, and in long logs (16.3 feet) for taller trees. It is more desirable, however, to compute the butt log always in two short lengths (8.15 feet) no matter what the relative height of the trees. The top should be computed as a paraboloid. These computations may be performed by basal area tables, basal area slide rule or alinement charts (Chapter X) and should be checked for accuracy. In cubing, three significant figures should be retained. The total volume of the tree is obtained by totaling the computed volumes of the several sections.

In addition, within each D.B.H. and height class, the average D.B.H. should be determined to the nearest $\frac{1}{10}$ inch, the average total height to the nearest $\frac{1}{2}$ foot, the volume of the cylinder corresponding to these dimensions, and the average cylindrical stem form factor.

These form factors are computed by dividing the tree volumes obtained above for each D.B.H. and height class by the corresponding cylinder volumes.

The form factors should be curved over height and diameter.

The final volume table is prepared by multiplying the curved form factor by the corresponding cylindrical volume for each D.B.H. and height class, and entering this volume value into the proper place in the tabulated form of the table.

Volume Tables in Board Feet. — Trees are grouped in height and diameter classes as before; 1 inch classes with trees whose diameter ranges does not exceed 36 inches, 2 inch classes for larger timber; $\frac{1}{2}$ log (8 feet) classes for trees not exceeding 5 standard logs of merchantable length, standard log classes (16 feet) for taller timber. Within each size group, the average D.B.H., the average used length (in terms of logs and half logs) and the average volume are found. Volume is computed by scaling each log as if it were sound in conformity to standard scaling practice. Scaling will be based on the lengths and top d.i.b. actually cut. The log rule used will be that in general use in the region or on the timber sale. The final volume table will be in terms of that log rule.

Within each size group, the merchantable portion of a tree is then considered as a frustum of a cone. Its basal diameter is that on the stump at the predetermined stump height (1.0 or 1.5 feet). Its top diameter is that of the predetermined merchantable top d.i.b. (5–10 inches). Its length is the measured distance on the tree stem between these two points. These values may or may not coincide with similar dimensions actually cut in the logging operation according to the closeness with which actual standards of utilization agree with the predetermined ideal. The volume of this frustum in board feet should then be determined using the same log rule as before. The length of the frustum is sectioned into standard log lengths, and board foot values are interpolated to fractional top d.i.b. and length values. The top log is extended to coincide with the merchantable top d.i.b.

Within each size group the frustum form factor is found. It is obtained by dividing the total merchantable volume for the tree by the scaled volume of the corresponding frustum. These form factors are then curved over D.B.H. for the purpose of eliminating irregularity from class to class. The final table is obtained by multiplying the frustum volume for each group by the curved frustum form factor for the D.B.H. class.

Every table constructed from field data should be checked for accuracy by comparing the actual volumes of basic trees with corresponding values taken from the table. For important tables a satisfactory limit of error is ½ of 1 per cent, plus or minus. With less important tables 1 per cent error plus or minus will be acceptable. The Average Deviation of individual tree volumes should also be computed. In this computation values as read from the volume table should be interpolated to the nearest tenth of an inch for diameter and the nearest foot for height.

If subordinate tables are desired, they should be derived from the basic table. Such subordinate tables may involve a closer utilization — for example, to a 3 inch top; or a statement of volume in terms of some other unit such as cords; or a classification of the table on some other basis, such as total height, rather than by log lengths. When two or more tables have been independently derived from the same set of data, the values should be checked against each other by computing the cubic foot-board foot ratio, or the per cent difference in volume when two different log rules have been used.

The method of constructing volume tables by means of form factors is mathematically sound and derives surprisingly accurate results. Since the method is dependent on the measurement of but a relatively small number of trees (100–200 will derive a good table as compared with other methods dependent upon 300–2000 trees), it offers further advantages of cheapness and speed.

A statement of certain supplementary information should always be published with every volume table. This should include:

1. Species, common and scientific name.
2. Region and locality in which the measurements were taken.
3. Author and date.
4. Unit of volume — cubic or board feet; the log rule used and the closeness of utilization.
5. Portion of the tree measured and closeness of the measurement. Some statement should be made regarding the instruments used and their standards of accuracy.
6. Basic data, number of trees measured within each D.B.H. and height class. In presenting the final table, it is often customary to enclose the basic data within heavy black lines.
7. Methods of computation.
8. Average deviation.
9. Aggregate difference.
10. Age of the trees measured.
11. A statement regarding site quality especially if Field tables exist. If not, something should be said regarding the physiographic location of the material from which the data is drawn.

154. U. S. Forest Service Volume Record Form 558*a*. — The U. S. Forest Service has recently adopted a field form for volume table work which has been devised by L. H. Reineke. See Fig. 71. The fundamental idea seems to have been the elimination of certain difficulties heretofore experienced in keeping a proper record, by actually drawing to scale, as the measurements are taken, a diagram representing the longitudinal section of the tree. Additional speed and accuracy are attained in the subsequent office work in computing volumes, not by mathematical formulas, but by determining the *area* of the tree diagram by means of planimeter, and of transforming figures of area to figures of volume by means of a conversion factor, the value of which is dependent upon the scales chosen for the X and the Y axis of the graph.

The form consists of a specially designed graph paper with a tabular schedule of square inch-cubic foot conversion factors in the lower left-hand corner. Four different scales are available on each of the two axes, that combination being chosen which is best adaptable to the size of the timber at hand. The combination

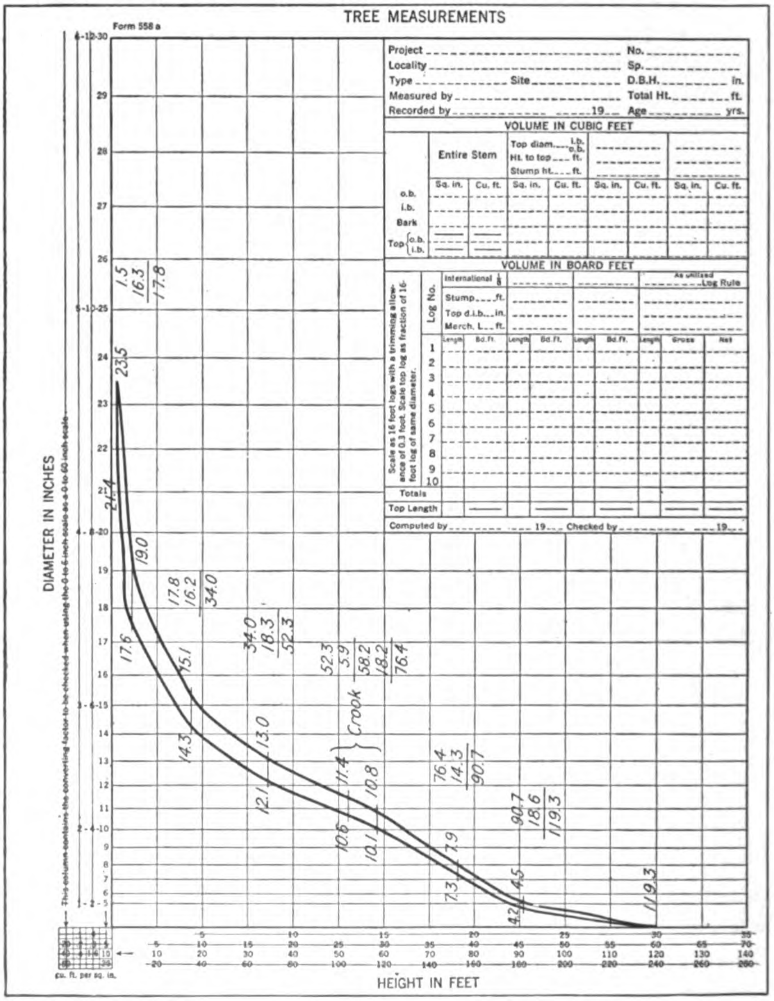


FIG. 71. — U. S. Forest Service Form No. 558*a* (Reineke's Field Form) for Recording Field Data for Volume Computation.

of scales chosen should be indicated either by checking the corresponding cubic foot-square inch converting factor on line with the diameter and height scales used, or by *lining out* all the scales *not* *used*, as in Fig. 71. These lines should be carried across the conversion factor schedule.

The average inside bark and outside bark diameter measurements are taken at each point of section as previously described and are plotted to scale directly on the field sheet at their corresponding heights above the ground.

For the purpose of absolute record and guidance, the inside bark and the outside bark diameter measurements, and the dimension of height should be printed in at each point of section. A reference to the figure will illustrate this point. Beneath the height record the length of the next log or section is entered and added, thus obtaining the height of the next section. Curves drawn exactly and smoothly through the several points indicate the form of the tree, inside and outside of the bark. In constructing this taper diagram, the various points should never be connected by a series of straight lines. The straight line form of tapers, besides being inexact, incurs error both in form and volume.

The ends of the logs actually cut may be shown by straight lines connecting the i.b. and o.b. points. Breaks, rot, defect or cull, the base of the crown, forks or crooks may be indicated by brackets and brief descriptive lettering. If the forester believes a more efficient utilization or better scale can be obtained by a sectioning other than actually used, he may designate that sectioning by means of dotted lines with appropriate lettering. Care must be taken not to overcrowd the graph with explanatory material, thus obscuring the purpose for which it was originally intended.

To determine the cubic foot volume, the method is as follows: Consider the stump as a cylinder by drawing a horizontal line from the curve at 1.0 feet (or 1.5 feet) above the ground to the vertical ordinate. Measure the area between the X and Y axis and the taper curve by means of a polar planimeter (Fig. 72). In following around the diagram proceed in *clockwise* direction. Without resetting the planimeter make a second circuit to check the first. The acceptable discrepancy between the two readings should not exceed 1 per cent of the measured area. Multiply this area in square inches by the appropriate cubic foot converting factor corresponding to the ordinate scales used as previously checked.

In the upper right-hand corner of the graph paper a double set of columns is provided for entering and recording volume values, one for cubic feet and one for board feet. For cubic foot values those columns permit the entry of the entire stem volume and also merchantable volume in one, two or three standards of utilization as influenced by top d.i.b., length of top and height of stump.

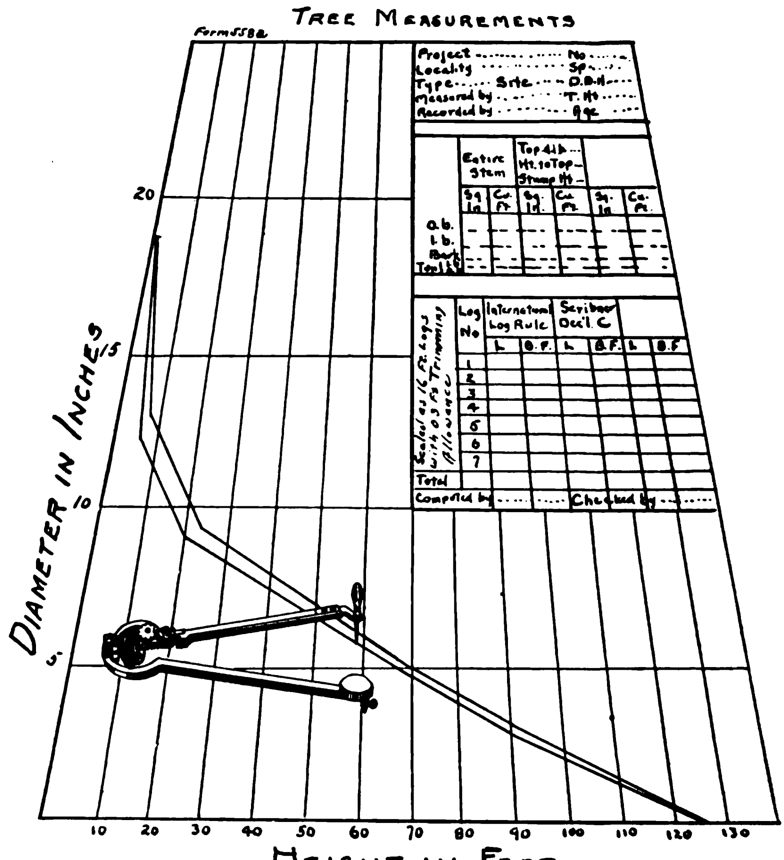


Fig. 72. — Showing Method of Computing Volume from Reineke's Field Form by Means of Polar Planimeter.

Board foot volumes are determined by means of specially prepared transparencies which are superimposed over the graph paper and tree diagram so that the left-hand edge of the transparencies coincides with the ordinate indicating stump height, the two base lines coinciding. The board foot volume for each separate log can be read from a vertical scale at the intersection of the taper curve with vertical ordinates indicating standard log lengths. The minimum top d.i.b. recognized is 5 to 6 inches. Top logs shorter than 16 feet are scaled as fractional values of full-lengthed logs of similar top diameter dimensions. Columns are provided for entering and recording the board foot scale according to different log rules and different standards of utilization.

When board foot-cubic foot ratios, frustum form factors, cylindrical form factors, bark volume percentages, or any other ratios or percentage relations are desired, these should be obtained from *totals* of volumes of diameter height classes rather than from the values for individual trees. Computations from group totals tend to eliminate errors incurred in the processes of averaging ratios and percentages. In addition, the work necessary to such operations is greatly reduced.

There is no doubt that the use of the Reineke form enables a considerable saving in the time and effort incidental to volume computation of trees from field measurements. Chances of error especially in reading and recording measurements are greatly reduced and a complete visualization of the form of the tree according to its outside and inside bark dimensions is immediately presented. The scale of the chart, however, is rather small for large sized trees and unless care is taken error and confusion may be incurred. The great advantage is the saving in office work. Determination of volume by planimeter can be done with greater accuracy in from a fourth to a fifth of the time taken to *compute* the volume of the same tree by mathematical means.

155. Form Class Volume Tables. — As we have already seen, one of the factors which most profoundly influences the volumes of individual trees within a given diameter-height group is taper or form (Chapter XI). More particularly, it is the taper in the upper sections of the tree. If, in addition to a classification of trees and volumes on the basis of diameter and height, there is a further classification on the basis of taper as expressed by form class (absolute form quotient) a higher degree of accuracy is possible in the determination of individual volumes.

Form class volume tables are constructed directly from form class taper tables (Sections 146–148). Form class taper curves are plotted on ordinary cross-section paper, a separate curve being required for every diameter class and height class within a given form class. Supplementary curves should be drawn showing: (*a*) the relations of D.B.H.o.b. to D.B.H.i.b. and (*b*) the relation of used top d.i.b. to D.B.H.o.b.

Within each D.B.H. class, height class and form class, the following are in turn determined:

(*a*) Diameter breast high outside and inside of the bark.

(*b*) Height of the tree above the ground and above breast height.

(*c*) Length of the section below breast height, in feet and as a percentage of the height above breast height.

(*d*) Merchantable top diameter inside the bark expressed as a percentage of breast height.

(*e*) The heights of successive logs and half logs expressed as percentages of the total height distance above breast height. These values can be read directly from the curve if desired.

(*f*) From the taper curves the percentile tapers of each log and half log, that is, the diameters in inches inside the bark at the top of each log and half log read as percentages of D.B.H. (inside bark).

(*g*) A conversion of these percentage diameters to inches by multiplying D.B.H.i.b. by corresponding percentage diameter.

(*h*) Volumes for each log or half log as below.

Unlike the standard volume table which can be tabulated within a single table on a single sheet, a form class volume table requires a number of tables and sheets, there being as many volume tabulations as there are form classes. Within each form class they are tabulated by D.B.H. and height as before. Before form class volume tables can be used the form class of the forest must be actually ascertained. This may be done by one of the " form point " methods described in Section 144, or by felling and measuring the absolute form quotients of a number of test trees.

Form class volume tables have been developed to their widest usefulness by Swedish foresters, whose enthusiastic use extends to them almost the powers of universal volume tables. The general consensus of present opinion seems to be that these claims for universality are not borne out in application to American trees, and that if form class volume tables are to be used, a separate set must be constructed for every species. This fact combined with the arduous labor of preparing them does not seem to commend their early adoption over the more easily constructed standard table.

TABLE XIV

RED SPRUCE IN NEW YORK

Form Class 70 — [2]D.B.H. Class 24 — [1]Ht. Class 110

Half logs (8.15 feet) Expressed as percentages of Total Height Above Breast Height		Top d.i.b. of the Several Logs		Volume	
No.	Percentage of Total Height	Percentage[3] of D.B.H.	Inches	Board Feet (Scribner Decimal C)	Cubic Feet (Smalian)
1st ½ log[5]	10.1	90.1			
2nd ½ log	17.6	83.7	[5]18.8	240	38.12
3rd ½ log	25.2	79.1			
4th ½ log	32.8	75.2	16.9	180	27.84
5th ½ log	40.2	73.1			
6th ½ log	47.8	70.1	15.7	160	23.20
7th ½ log	55.5	57.5			
8th ½ log	63.0	48.0	10.8	70	15.44
9th ½ log	70.0	41.4			
10th ½ log	84.0	31.1	7.0	30	.81
Totals				680 b.f.	[4]105.41

[1] Total Height = 112.1 feet.
[2] D.B.H.(o.b.) 24.3 inches.
[3] D.B.H.(i.b.) 22.5 inches.
[4] If total cubic volume is desired cube the stump as a cylinder and the top as a paraboloid.
[5] Stump d.i.b. = 109.6 per cent of D.B.H. = 24.9 inches.

156. Local Volume Tables. — Strictly speaking, a local volume table is one made from one specified locality to conform with local customs of logging and milling. Herein, however, it is to be understood to be a volume table *derived* from a standard volume table which will conform and apply to the species in a definite locality. As usually constructed it takes the form of a table classified by D.B.H. only, and becomes a very useful table on a cruising job, since it eliminates the height differential and permits tallying the trees by species and D.B.H. directly.

It requires, in the first place, that there must be a volume table existent for the desired species, preferably classified by both diameter and height classes. Total heights are better than merchantable height values or log lengths values inasmuch as they are easier to identify and measure. It is fundamentally important *that the height class values of the original table* be known, so that the maker of the local volume table may take height measurements to the same relative point on the tree stems and relate these measurements to the same unit of volume.

Field Work. — Diameters breast high outside the bark are measured and recorded to the nearest tenth of an inch, and the corresponding heights are measured and recorded to the nearest 5 feet. Endeavor should be made to cover the entire range of the diameter classes in such a way as to get approximately the same number of measurements for each class. It would seem that ten to twelve trees per class ought to be sufficient. Some trouble ordinarily will be experienced in doing this due to the laws of distribution and of chance. The ordinary run of diameter values crowd close to a norm represented by the dimensions of the average tree. Difficulty will be experienced in getting a sufficient number of trees for the smallest and largest dimensions.

Office Work. — 1. Within each D.B.H. class determine the mathematical average D.B.H. value to the nearest tenth of an inch, and the average height value to the nearest foot. This height value average should be carried out to decimals of a foot.

2. Procure a standard volume table of the species. Be careful to note that its height classification is the same as has been measured in the height measurement taken in the field.

3. Interpolate from the standard volume table the volume of the tree which will meet the specifications of each of the averages, for D.B.H. in inches and for the corresponding averages of height in feet. This requires interpolation both up and down and across. In practice it seems best to interpolate for height first and diameter later.

This process of interpolation, in spite of its simplicity, usually puzzles the student, and an example may be well in point.

Suppose in the 27 inch diameter class that the mathematical calculation shows that the average tree of this class is 27.3 inches, D.B.H. and 114.6 feet high. From a standard volume table in hand we note the volumes for the full inch class both *above and below* our average D.B.H. value, and for the height class both *above and below* our averaged height values. That is, we would note that a 27 inch 110 foot tree has a volume of 895 board feet, and a 27 inch 120 foot tree has a volume of 937 board feet. But we are not interested in 110 foot trees, nor in 120 foot trees, except as they will help derive the volume of a tree 114.6 feet high. In the 27 inch class, the increase in volume from the 110 foot class to 120 foot class is 4.2 board feet for every foot increase in height. For 4.6 feet of height the increase in volume will be 19.13 board feet, rounded off to 19. Therefore a 27 inch 114.6 foot tree will have a volume of 895 + 19 or 914 board feet. Similarly, a 28 inch 114.6 foot tree will have a volume of 946 + 20 or 966 board feet. The conclusion is that in trees of this height, for every increase of one tenth of an inch in diameter there is an increase in volume of 5.2 board feet. Hence 27.3 inch 114.6 foot trees will have a volume of 914 + 16 or 930 board feet.

4. When all the interpolations for all D.B.H. classes have been completed, cast the results into a graph of Volume on D.B.H. and even off the irregularities by a smooth curve.

5. Read from the curve the volumes for each full inch and tabulate the results. This table is the local volume table. Being in terms of volumes classified by diameters alone, it is an extremely easy table to use, and commends itself to greater local use than did the original standard table from which it was derived.

157. Alinement Chart Volume Tables. — The alinement chart technique offers not so much a method of *constructing* volume tables as it does a manner of *presenting* volume tables already constructed, so as to promote greatest facility in manipulation in the computation of tree and stand volumes. As such it would seem that their best expression is found when the element of taper has been eliminated as a variable, as in the construction of form class volume tables. This reduces the problem to the consideration of two independent variables, the D.B.H. and height of trees, and the dependent variable of their volume in terms of some unit of measure, board feet, cubic feet, cords, etc. If the two former are scaled off on the initial or primary axis of a conventional alinement chart, the proper scaling of the dependent variable on a final axis set in correct relation to the other two will aline corresponding volumes.

In appearance such a chart takes the form of a conventional alinement addition chart (Fig. 51, page 173) with three vertical axes. Actually it is a chart of arbitrary values since it seeks to visualize volume in terms of two factors arbitrarily adopted as standards of classification rather than a computation in the terms of this volume. Such is quite in harmony with the fundamentals of alinement chart construction.

In constructing an alinement chart volume table the two outside axes are accepted as primary or initial axes, and the third placed in

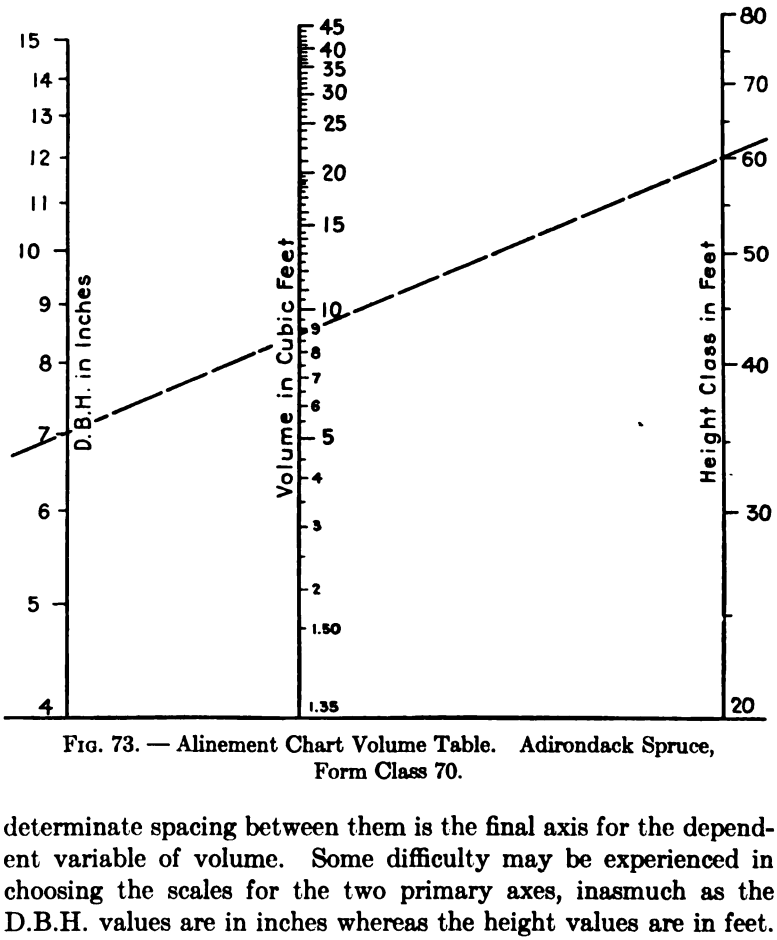


FIG. 73. — Alinement Chart Volume Table. Adirondack Spruce, Form Class 70.

determinate spacing between them is the final axis for the dependent variable of volume. Some difficulty may be experienced in choosing the scales for the two primary axes, inasmuch as the D.B.H. values are in inches whereas the height values are in feet. This probably can be accomplished most easily by the employment of moduli, the arbitrary values of which are indicated by the actual space limits desired in the final chart. A modulus may be defined as a multiplier by which the values of one system of logarithms are transformed into those of another set. The problem is to seek a scale of graduation that will encompass a given D.B.H. range, or height range, within a fixed distance on the axis, and to graduate this axis in units of magnitude of modified terms. Thus the primary axis (the left-hand vertical axis) carrying the D.B.H. scale in inches may be graduated from the formula

$$S_d = M(\log D)$$

and the second primary axis at the right carrying the height scale in feet may be graduated from

$S_h = M(\log H)$

where S_d = modified values of the D.B.H. scale.

S_h = modified values of the Height scale.

M = scale modulus as fixed arbitrarily by the desired limits of the chart.

D* = D.B.H. value in inches.

H = height value in feet.

The position of the third and final axis in relation to the other two may be fixed by intersections. This is accomplished by seeking a series of volume values which are of equal magnitude, say 15.0 cubic feet (or board feet) but which are derived from different D.B.H. and different heights. Alinement of two, three or more of these corresponding independent variables will derive a common volume point, or at least a series of points falling within a small radius. Similar points derived higher and lower on the volume scale will indicate the position of this final axis. The test of the accuracy of its location is that it must be truly parallel with the other two axes.

The graduations of the volume scale may be located by a more extended use of the principle of intersections. As pointed out by Reineke,† with the determination of a few of the graduations by intersection, he distances of these graduations over any fixed point may be curved over the value of the graduation and the intermediate distances may then be transferred from the curve to the axis.

158. Graded Volume Tables. — There is another form of volume table which might be mentioned. This is the graded volume

* It will probably be advisable to confine the scale graduations to the significant range within which volumes are computed. Thus the smallest D.B.H., the smallest height and the smallest volume will all coincide with the zero or base line of the chart.

† L. H. Reineke, *op. cit.* No. 58.

table. It is one which endeavors to show quality yields or grades as well as quantity yields and volume. Logs are measured and followed through the mill, and the yield in board feet in each grade is carefully noted and totaled. This type of table is difficult to make. In use, they are restricted to the locality in which they are made, and to the given mill conditions prevailing, as influenced by the type of machinery and the standards of utilization. Vary these conditions in any detail and the volume table becomes useless. However as timber becomes more valuable and logging and milling conditions become more standardized, this type of table may be needed more and more. Tables of this character have the decided advantage of providing to the forest owner specific information concerning the probable price value of his stumpage.

CHAPTER XIII

THE AGE OF FORESTS

159. The Importance of Age Determination. — The age of a forest is determined as the average age of the trees which compose it. Next to volume and area, age ranks as the most important factor concerning which it is necessary that the forester obtain definite information. The ultimate aim of all investigations into the age of timber is to determine when it is ripe or mature, and when the greatest amount of lumber per acre can be removed from the area. Volume determines the amount of the cut, area where it is cut, and the age when it is cut.

When the past history of a stand is completely known, age is a matter of record and no further investigation is necessary. In wild and unmanaged forests where no definite information regarding the origin and past development is available, the age of the stand can be determined only by the examination of individual trees of average characteristics accepted as representatives of the whole forest. The only accurate method of determining age under such circumstances is to fell the sample trees and make a count of their rings of annual growth. Growth rings are the result of the seasonal changes in the formation of the successive layers of woody tissue. The zone of light colored, large celled spring wood is usually sharply differentiated from the dark celled band of summer wood. A complete growth ring is composed of both spring wood and summer wood. In some trees, notably paper birch, aspen, hard maple, basswood, and others, there is so little difference between the spring and summer wood that a count can be accomplished only by the use of a hand lens or by applying coloring matter to bring out the contrast.

160. Methods of Determining Age in Trees. — The age of trees may be determined in four ways:

A. By an estimate or guess in the case of a particular tree taking into consideration:

1. The size and relative taper of the main stem. Young trees have a quickly tapering bole, and full grown trees tend to a cylindrical bole.

2. The form, size and shape of the crown. The ultimate crown development of that particular species under both open and forest conditions of growth must be known.

3. The color and condition of the bark of individual trees. The older the tree, the thicker the bark.

4. The effect of local conditions of the site itself on any one or all of the three preceding.

B. By a count of the whorls of branches or branch scars on the trunks of the trees from the top downward, adding to the count the number of years taken by the tree to reach the height of the last visible scar. This method is applicable only to certain coniferous trees and to those species whose lower limbs and branches are fairly persistent on the stem. Eastern white pine can be estimated in this manner up to an age of 10 to 20 years.

C. By the use of an increment borer. If care is used, and the boring is made through the growing center of the tree, the core, on removal, will show all the annual rings from the heart to the periphery, and a count will give the age of the portion of the tree *above the point of boring*. The true age can be determined by adding to the ring count of the core, the number of years that it took the tree to attain to the height of the point at which the boring was made.

D. The most accurate way to determine age is to fell a tree and count the annual rings* on the stump, each ring representing a year of growth. To the count on the stump, however, there must be added the number of years which it took the original seedling to reach stump height, for the count on the stump will give only the age of the portion of the tree *above that point*.

161. Even Aged Forests. — When trees of a group or stand are approximately of the same age and have been established on the site during the same period of reproduction, they are said to belong to one age class. The actual number of years included within the limits of an age class is variable. It is rarely that the age class interval is one year, and is more commonly 10, 20 or even 50 years. A stand or a forest which consists of but one single age class is said to be *Even Aged*. Stands are judged to be even aged if their crowns form practically a single canopy, or a

* Care must be taken to ascertain that these are true growth and not "false rings," produced by marked variation of the growing season.

one-storied forest. This fact will be found to be true if the reproduction period does not exceed one fifth of the maturing age of the species.

162. All Aged Forests. — When a stand or a forest is composed of several age classes it is said to be *All Aged* or *Uneven Aged.* The crown canopy of this type of forest consists of a number of layers or stories and this very fact is a demonstration of a grouping of several different age classes. All aged forests are sometimes termed "Selection Forests," that is, one in which all ages are scattered irregularly, and mature trees ready for cutting are "selected" on the basis of size and age. The usual example of this type of forest is one in which the trees of as many as several species, and of all sizes and ages are grouped together on the same site. There is a noted intermingling of crowns in the main canopy from stems of marked contrast in diameter dimensions.

163. The Age of Stands. — In the even aged type of forest all that is required is a careful selection of one or more sample trees whose size, appearance and form warrant them acceptable for the purpose.

TABLE XV

AGE CLASS VALUES ON WHICH TO BASE AGE

Age Class	Average Age of Test Trees in Years	Total Number of Trees in Age Group	Total Volume of Age Group	Average Yearly Increment for the Group in Board Feet
40- 60	53	60	1000	19
61- 80	63	50	1500	23
81-100	95	40	2000	21
101-120	117	30	2700	23

The task is much more difficult in the all aged forest. Here the age can be gaged only as the average or mean of the several age groups or age classes which compose the forest. Theoretically this forest is composed of all age classes, the main difficulty being to recognize them on the ground. Failing recognition on the identity of *age*, the next best thing seems to be to base recognition on the factor of size, that is, trees of the largest size must be of the greatest age, and those of young age have but small dimensions.

Four methods of determining the average age of this stand are available:

1. Granting that each of the age classes occupies equal* or approximately equal areas, the average may be determined by finding the total age of the test trees and determining the mean, thus:

$$(53 + 63 + 95 + 117) \div 4 = \frac{328}{4} = 82 \text{ years}$$

2. The main objection to the foregoing method of determining age is that it totally disregards the factor of numbers and of volume. It will be noted that the oldest age class which contains the fewest trees and contributes the largest volume to the total of the stand is given quite as much weight and consideration as the youngest age which has twice as many trees but contributes less than half as much volume. To meet this objection it might seem that the average should be based on a weighting at least by numbers.

When weighted by numbers,

$$\begin{array}{rcr} 53 \times 60 & = & 3,180 \\ 63 \times 50 & = & 3,150 \\ 95 \times 40 & = & 3,800 \\ 117 \times \underline{30} & = & \underline{3,510} \\ 180 & & 13,640 \end{array}$$

Then Mean Age = 13,640 ÷ 180 = 75.8,
= 76 years.

3. Objection is made to this computation on the basis that it stresses the mere presence of numbers and disregards the main effect of number within the age or size classes, namely, volume.

Weighting by volume,

$$\begin{array}{rcr} 53 \times 1,000 & = & 53,000 \\ 63 \times 1,500 & = & 94,500 \\ 95 \times 2,000 & = & 190,000 \\ 117 \times \underline{2,700} & = & \underline{315,900} \\ 7,200 & & 653,400 \end{array}$$

653,400 ÷ 7,200 = 90.7 = 91 years.

* If the areas are not approximately equal, in order to apply this method it will be necessary to determine the *area* of each class and *weight* average age by corresponding area to determine the mean.

4. Objection is taken to all the preceding computations in that they are based on purely arithmetical rather than geometric averages. That is, although age increases in arithmetical progression, volume — the immediate effect of the passing of time — follows a different trend and increases in geometric proportion.

Volume is very definitely correlated with age. It takes time to grow trees of desirable size, and conversely, trees of desirable dimensions must be of such age as is in proportion to their size. The outward and visible sign of the effect on a tree of the passing of a year, a decade or a century is an increase in dimensions or size, height and diameter, or by combining these dimensions of height and diameter, in volume. This increase in volume is known as its *Increment.* The grand total of all the annual increments is the *present volume* of the tree. The average yearly growth of the tree is found by dividing its present volume by its present age and is known as the *Mean Annual Increment.* If the present volume and mean annual increment of a tree are known, the age of the tree must be equal to the volume divided by the mean annual increment. The mean annual increment of the forest is equal to the sum of the several mean annual increments of the age classes. If the total volume of the forest is divided by the mean annual growth of the same forest, it is obvious that the resulting quotient must be the mean age of the forest.

$$\begin{array}{rcrcr} 1{,}000 & \div & 53 & = & 19 \\ 1{,}500 & \div & 63 & = & 23 \\ 2{,}000 & \div & 95 & = & 21 \\ \underline{2{,}700} & \div & 117 & = & \underline{20} \\ 7{,}200 & & & & 83 \end{array}$$

$$7{,}200 \div 83 = 86.7 = 87 \text{ years.}$$

5. A method of equal value and also on a geometric basis a computation of basal area values.

If trees have the same form height, that is, if the trees comprising the same age class diameter groups are approximately of similar height, the variations in their volumes are dependent on and are a factor of the variations of their diameters. These volumes, however, do not vary so much with the diameters as with the squares of these diameters. And the expression of the squares of these diameters is found in their respective basal areas.

Hence, if B is the total basal area of the plot or stand, and

b_1, b_2, b_3, and b_4 are the respective total basal areas of the four age classes, and a_1, a_2, a_3, and a_4 are the mean ages of these same age classes, then A, the mean age of the all aged forest, can be found from the formula:

$$A = \frac{(b_1 \times a_1) + (b_2 \times a_2) + (b_3 \times a_3) + (b_4 \times a_4)}{B}$$

164. Economic Age. — All the trees which start on a site during the same period of reproduction rarely grow up together. The tall, quick growing trees get the jump over the slower growers and spread their crowns in the main or upper canopy overtopping the laggards. Overtopping means that the laggards are deprived of a portion of the sunlight that they need for their growth. Their growth is then further retarded.

When intolerant* trees are overtopped and deprived of full light they die. When tolerant trees are overtopped they may persist for long periods of time in a perfectly healthy condition making little or no growth. This period is usually represented in the center of the tree by a narrow core or cylinder of hard, dense wood made up of many narrow growth rings. When the overwood is removed and the tree is given its full share of the light, the tree can and will recover its power for growth and mature in a perfectly normal way. This period of growth will not be marked by any abnormal rate of growth, nor is the length of the period of growth checked by any rapid maturing of the tree.

The question then arises as to the age of such a tree. It is perfectly obvious that the period of its growth since the recovery does not represent its age. Equally, it is obvious that the total age does not represent its possibilities for growth, since the years of suppression have contributed little or nothing to the volume value of the tree. If the tree had grown under normal conditions of shading and crowding, it would have reached its present size in a much shorter time. The economic age is the number of years required by a tree of the species to reach maturity size when it has the fullest uses of the resources of the site.

165. Age in Tropical Trees. — Tropical trees do not develop annual growth rings as do the trees of the temperate zones. The explanation of this is simply that where there is no contrast in the

* An intolerant tree may briefly be characterized as a "light demander," and a tolerant tree as a "shade endurer."

seasons, there will not be any contrast in the growth laid on at different periods of the year. The age of tropical trees can be determined only by special growth studies carried on over some length of time in which there is established the number of inches or tenths of inches that the tree increases in diameter over a period of five or ten years. The age of the tree is then accepted as the diameter multiplied by the number of years in the average one inch diameter increase growth period.

CHAPTER XIV

THE STUDY OF GROWTH

166. The Importance of Growth Studies. — Forest land, like agricultural land, is the potential producer of a crop grown from the soil. An essential difference between farm crops and forest crops is, that in the first case the crop is sowed, matured and harvested in a single year whereas the period necessary to produce a forest crop is many years. Just as the crop of farmer's wheat is made up from the individual heads growing on the thousands of stalks which are in the field, so the forest crop is made up from the individual trees which compose the forest. The effect of the passing of time on a tree is an increase in height and diameter, or, by a correlation between these two dimensions, an increase in volume. Among some of the writers in mensuration, the increase in diameter is known as *accretion* and the increase in volume as *increment.* For our purpose, however, we will use growth and increment interchangeably, prefixing the necessary descriptive term, as height growth, diameter growth, and volume growth, or increment, as the case may be.

Volume growth information is the ultimate aim of all growth studies. By making the study of the increment of individual trees or individual groups of trees, we can extend our results to whole stands or forests of which they are a part. There are two important elements in the making of growth studies.

(*a*) The present volume of the tree or stand.

(*b*) The time required to produce this volume.

When growth studies are applied to definite areas, the present volume of the stand is taken from the estimate. No growth study of this kind can be more accurate than the estimate is accurate. Correlative with the determination of volume is a determination of age. Age by itself means little, but coupling of age with growth, and growth with volume, and volume with area presupposes the whole body of facts upon which the practice of silviculture and the application of forest management depend. Studies of growth also answer important problems for the forester, such as the proper age at which timber should be cut in order to realize the greatest yield and the highest financial returns, the age to make thinnings, and the anticipated yields from plantings.

167. Purposes of Growth Studies. — The main purpose in the making of growth studies is to determine:

I. The increase in the volume of wood, such as:
  A. Total volume expressed generally in cubic feet.
  B. Merchantable volume expressed generally in board feet,
    (*a*) For a given period of time,
      1. For the total age of the tree.
      2. For any short period or current period, such as, a decade.
      3. For any one year.
    (*b*) On a given area.
      1. On a standard area such as the acre, when the growth in volume is expressed per acre.
      2. On the total area of the type, site, stand, or forest, which is the ultimate aim of all studies of growth.

II. A basis for predicting future forest growth, such as:
  A. The yields of wood possible on barren or unused land at present not forested.
  B. The yields of wood material on lands at present forested, but concerning the possibilities of which little is known.
    (*a*) With even aged stands thinned and unthinned.
    (*b*) With all aged stands thinned and unthinned.

168. Classification of Growth. — Actual growth, whether in diameter, height or volume may be classified as follows:

1. *Current Growth*, better known as *Current Annual Growth:* This is the actual growth for any one year. The summation of all the yearly individual items of growth is equal to the *Present Total Growth* or present volume.

2. *Periodic Growth:* — The total growth for any specified period of time, such as a decade, or a period of twenty years.

3. *Periodic Annual* Growth:* — The average annual growth for any one year during a specified period of time. It is obtained by dividing the *total* growth for the period by the number of years in the period.

* In ordinary growth studies involving a knowledge of annual increment, periodic annual is preferred to current annual growth due to the greater ease and greater certainty of its calculation.

4. *Mean Annual Growth:* — The *average* annual growth during the full period of the tree's existence. It is found by dividing present volume by present age.

169. Mean Annual vs. Current Annual Growth. — Mean annual growth, whether in a tree or in a stand, is an expression of

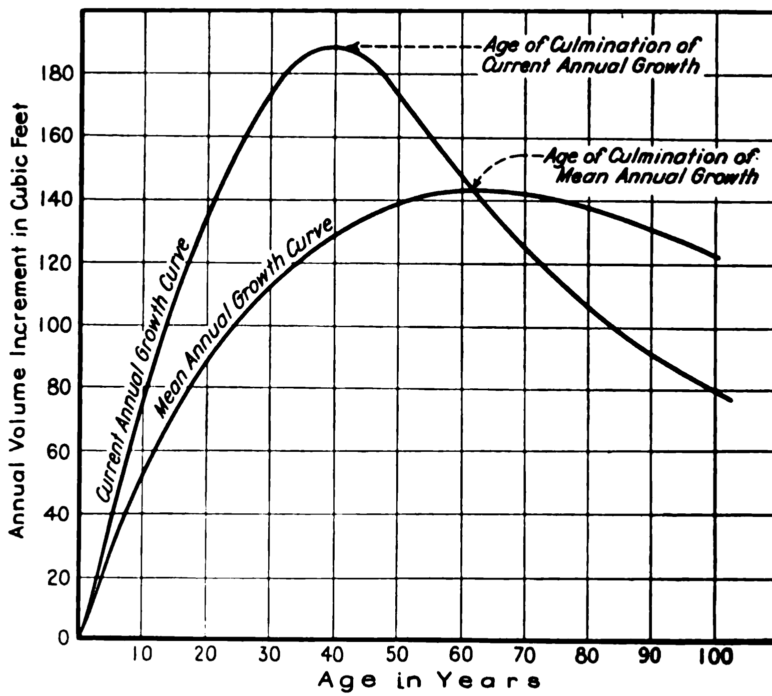


FIG. 74. — Current and Mean Annual Volume Growth of Eastern White Pine in Massachusetts. (Quality II Site.) Data Taken From U. S. Department of Agriculture Bulletin 13.

the average yearly response in the past to the growth factors of the site. Current annual growth is the actual response in any one year. There are two periods in the life of a tree or stand when these two coincide: one is at the end of the first year's growth and the other is at the year of culmination of the mean annual growth. This is true whether of growth in diameter, in basal area, in height, or in volume, and whether we study it for a single tree or for a group or stand of trees. The relation between total growth and mean annual growth is that of a sum to its average.

The relation of current annual growth and of mean annual growth is that of any component of the sum for any one year and that of the average up to and including that year. Reference is made to Fig. 74, which shows the curves of current annual growth and mean annual growth for white pine in Massachusetts.

Because the mean annual curve is a curve of averages its rise, rather than its projection as a straight line, is directly dependent upon a marked increase in the yearly accretions of growth during the early period of formation. Current annual growth constantly tends to increase up to a certain maximum, the effect of which is to raise, though at much slower rate, the curve representing the average growth for this period. It never can rise at a rate equal to current growth because in calculating mean annual growth the increase for any one year, great as it may be, must be prorated as an average back through all the years that the tree, or stand, has existed.

With the culmination of current annual growth, there is a marked slowing up and a rather abrupt retrogression in trend and in rate. When the current annual growth in amount drops to a value equal to that of the mean annual growth *for that one year*, the highest point in the development of the mean annual growth curve is definitely determined. This is the year of the culmination for mean annual growth. Subsequent to it, the annual growth will be less and less, with similar though more retarded effect on the mean annual growth curve.

In the growth of stands there are always present two powerful opposing groups of factors: those that build up, and those that tear down. The death of individual members of a stand is a constant thing from youth to old age. Nature's way of preparing for this is in the bountiful reproduction 2,000; 5,000; 10,000; 50,000; or even 100,000 seedlings to the acre, only a small fraction of which are expected to survive until the final harvest. The greater number of these disappear during the first few decades of the life of the stand. During the period of vigorous growth the loss in number is beneficial rather than harmful to the stand. Those which drop out are for the most part weaklings and their removal offers more space for the development of the survivors. This is the period when the forces building up the forest surpass in strength and effect the forces tearing down. But with the attainment of maturity, the strength of these two forces becomes more equal. The continuing loss of individual members not only removes units of accumulated volume but throws a heavier burden on the survivors. The forces tearing down are in the ascendant. The age of mean annual culmination marks their victory.

The importance of these facts, as far as predicting the growth of stands is concerned, lies in the accuracy of our knowledge of the future loss in numbers. We can study growth in a vigorous and healthy tree, and, on the basis of its past trend, predict with some accuracy the future growth of *that one tree* for the next few decades. But in extending that knowledge to the stand as a whole, unless we make definite allowances for the progressive loss in numbers over the period covered by the prediction, our statements regarding future growth and yield are of no great value.

170. Determination of Diameter Growth. — As a means of computing volume growth, it is necessary to measure and study diameter growth. Diameter growth figures should be in terms of D.B.H. since this, as a measurement of trees and forests, is practically standardized in all forestry work.

Studies of growth in diameter are made from actual examinations of cut cross-sections of felled sample trees, or from borings on standing trees. The former is the more accurate although it involves a wearisome and tedious process. Since trees are seldom cut at the D.B.H. point, which would involve not only impractical and needless refinement but also unnecessary waste, the usual custom is to study diameter growth on a section as near the D.B.H. point as is both practicable and convenient, and relate D.B.H. values to it. The process involves ring counts and age determinations, correlated with the diameter measurements. The steps in this process are three in number:

1. A diameter growth count measurement study made on the average radius of the stump.

2. A correction of this age so as to allow for the growth of the seedling to stump height.

3. A correlation of the corrected stump diameter age values to apply to D.B.H. measurements outside of the bark.

If the average stump ring count of a series of trees is corrected to true age and is plotted as in Fig. 75, by Diameter on Age, the desired correlation may be quickly and easily secured.

Two methods of counting and measuring the growth rings on the average radius are available:

1. Accept the pith as zero, count out from the center, marking off every ten years, or rings, as they occur. Measure in the same direction.

2. Begin at the cambium and count in toward the center, marking off every ten years, or rings, by a pencil dot on the radial line. Measure in the opposite direction.

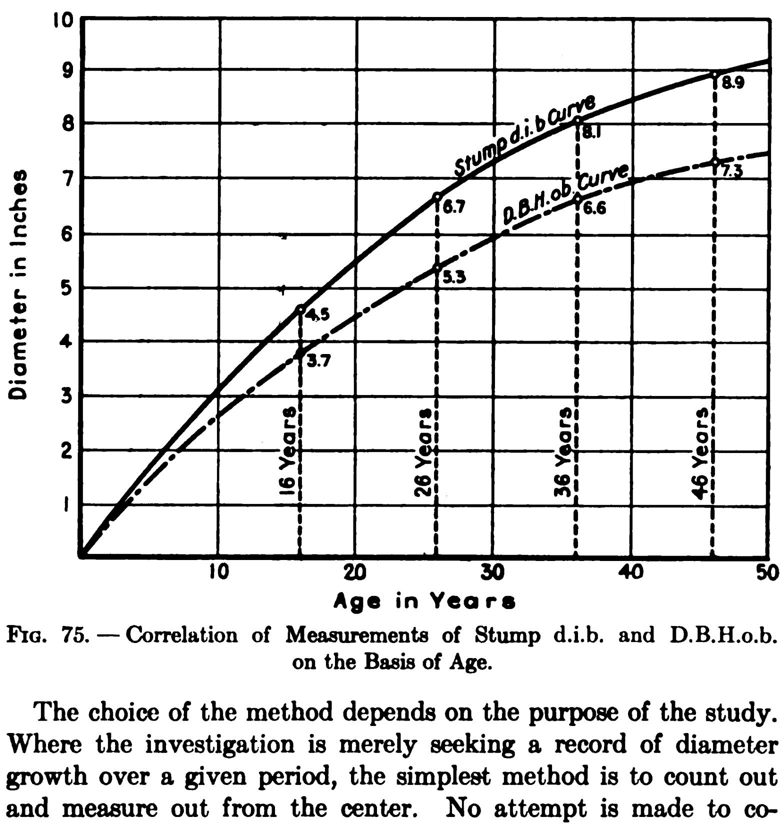
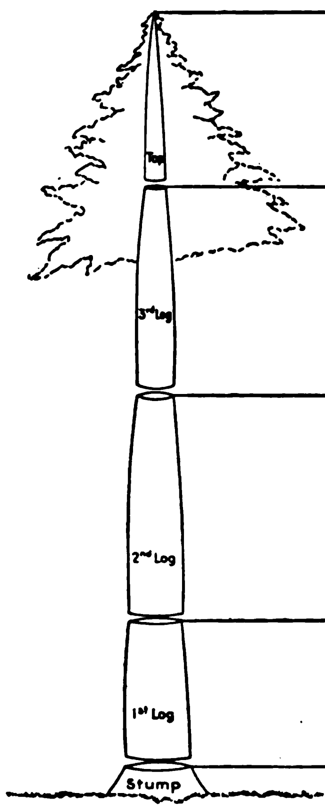


FIG. 75. — Correlation of Measurements of Stump d.i.b. and D.B.H.o.b. on the Basis of Age.

The choice of the method depends on the purpose of the study. Where the investigation is merely seeking a record of diameter growth over a given period, the simplest method is to count out and measure out from the center. No attempt is made to coördinate growth, volume and time.

171. Determination of Height Growth. — In some conifers, as long as their lower limbs persist, height growth can be measured directly from the distances between succeeding whorls of branches (Section 160). In older conifers and in all hardwood trees height growth can be measured only by making ring counts at successively higher points on the tree stem. Best results can be attained when the distances between sections are relatively short and when sections are spaced at more or less equidistant intervals. The crux of height growth determination lies within three points which can best be brought out by reference to Fig. 76:



1	2	3	4	5	6
Length of Section in Feet	Height above Ground in Feet	Ring Count on Top of Section	Years Required to Grow Length of Section	Years Required to attain Height of Section	Total Age (in Years) (A=3+5)
11.6	54.4	0	27	101	101
14.2	42.8	27	22	74	101
16.5	28.6	49	27	52	101
10.8	12.1	76	17	25	101
1.3	1.3	93	8	8	101

Fig. 76. — Diagram Showing the Relation of Height (length), Ring Count, and Age at Different Points on a Tree Stem.

1. No growth rings are produced at any point in a tree stem until the tree has lived long enough to attain, and has actually attained *that height above the ground.*

2. The number of years required for this period is equal to the

total age minus the number of years represented by the annual rings as counted on the section taken at the point of cutting.

3. The total age of a tree is equal to the ring count plus the number of years required to attain the height above ground at which the ring count was made.

The process of measuring height growth is a matter of determining the total age of the tree, the total ring counts at each point of section, and of coördinating the simple mathematical relations between them. Such work can be carried out only on felled trees.

172. Growth in Height and Growth in Diameter. — The growth relations of diameter to height in trees are not consistent. In intolerant species, the trees must maintain an average height growth, that is, must always be dominants, or die. Where an intolerant tree has a fairly assured place among the dominants, no great effort is required by the tree to equal or surpass competitors; and since, being a dominant, it has a relatively large digestive or assimilative mechanism of root and crown, a surplus of growing energy is secured. This is diverted toward developing an adequate support for the crown rather than in extending its height. Consequently, the diameter growth of these intolerant dominants may be *relatively* greater than the height growth. On the other hand, in a stand where there is constant danger of being overtopped, or where the species has lagged behind to the extent that the process of overtopping is already taking place, then the effort to retain or regain its position requires the entire energy of the tree, and the relative amount of height growth exceeds the relative growth in diameter. For this reason, a dominant tree in a crowded stand will probably be a stout, big butted tree with a slow taper and a high form quotient, whereas the overtopped tree will develop a slender stem of conical or of neiloid form (of low form quotient).

173. Reineke's Field Form for Growth Records. — The use of Reineke's field sheet, U. S. Forest Service Form* No. 558*a*, has received considerable advocacy in growth study work. The great advantage of the Reineke form is that it offers an easy and quick method for the computation of the volume, either by planimeter or transparencies, from taper diagrams representing approximations of the forms of actual trees. It also offers a ready method of

* Figure 71, page 220.

computing variations in volume to meet varying standards of utilization.

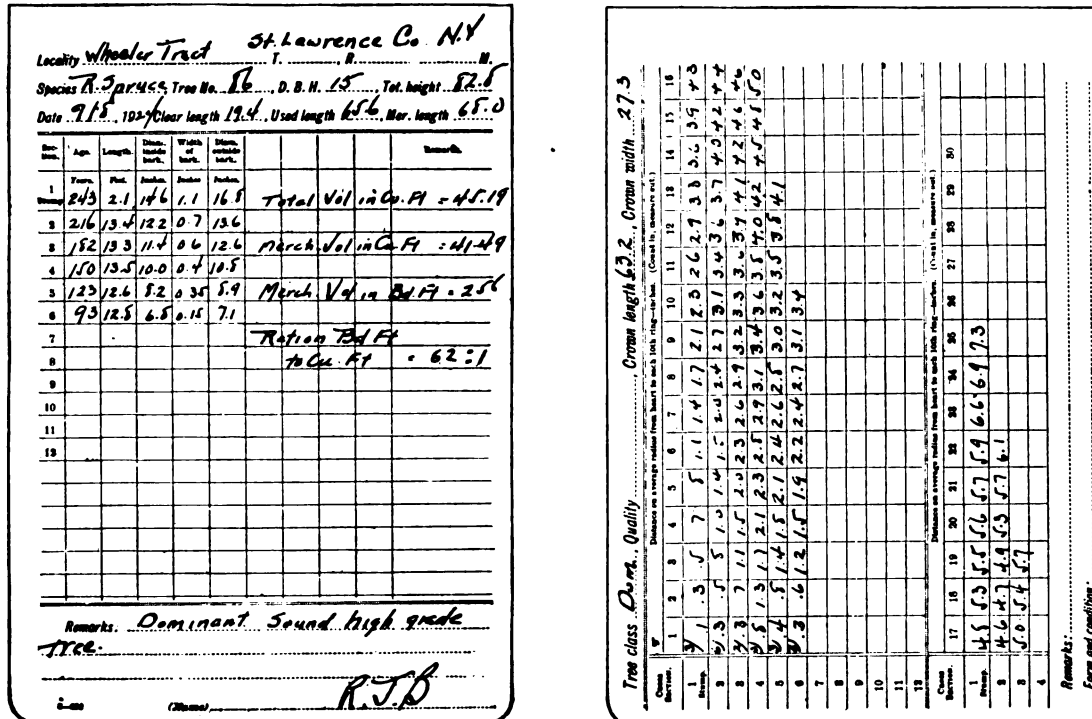
However, it is to be noted that the ratio of the ordinates to the abscissae is not of such value as to permit the construction of the best taper diagrams possible. The size of the sheet is too small* for an effective graph and the plotting of taper curves on age, though interesting as far as the individual tree is concerned, offers little information regarding the *average* growth of the average tree of that particular size or age class. The plotting in of individual curves is of assistance to the accuracy of the recorder and leads to the elimination of errors caused by any misunderstanding or oversight. But the subsequent reduction of these decade measurements into numerical values for purposes of averging becomes a tedious task, and adds to, rather than eliminates, the difficulties inherent in other methods of recording where the mathematical values are tallied directly.

When the mathematically averaged measurements for the tree which represent the group have been obtained, the Reineke form is of great value in the computation of volume by decades either by planimeter or by transparencies, depending upon the unit of measurement.

Any form of notes which adequately balances the problems of recording in the field with those of computation in the office will be found acceptable. In general, the U. S. Forest Service Form No. 334 with minor modification meets with general satisfaction. (See Fig. 77.)

174. Volume Growth of Stands Based on a Comparison of the Growth of Diameter Classes. — The response of the trees in a forest to their environment in terms of growth is considered to be a recurring series of phenomena, which are repeated by each size class in its normal progression from dimension to dimension. The past record of growth of a dimension class for a given period is to be accepted as an index of what is to be expected in the growth of the next lower dimension class through a similar period of development. If a tree now 19 inches in diameter has grown 1

* Mr. E. N. Munns suggests that some of the difficulties of plotting growth taper diagrams with the small scales might be obviated by using several sheets per tree with wider spacing, a larger scale and fewer charts per sheet. Since this adds to the number of sheets to take care of, it rather increases the troubles of plotting and computing.



Locality Wheeler Tract St. Lawrence Co. N.Y. T. ..... R. ..... M.
Species R. Spruce, Tree No. 16, D.B.H. 15, Tot. height 82.5
Date 9/15, 1924, Clear length 19.4, Used length 65.6, Mer. length 65.0

Sec-tion.	Age. Years.	Length. Feet.	Diam. inside bark. Inches.	Width of bark. Inches.	Diam. outside bark. Inches.	Remarks.
1 Stump	243	2.1	14.6	1.1	16.8	Total Vol in Cu. Ft = 45.19
2	216	13.4	12.2	0.7	13.6	
3	182	13.3	11.4	0.6	12.6	Merch. Vol in Cu. Ft : 41.49
4	150	13.5	10.0	0.4	10.8	
5	123	12.6	8.2	0.35	8.9	Merch. Vol in Bd. Ft = 256
6	93	12.5	6.5	0.15	7.1	
7						Ration Bd Ft
8						to Cu. Ft = 62:1
9						
10						
11						
12						

Remarks: Dominant Sound high grade tree.

(Name) R.J.B.

Tree class Dom., Quality ....., Crown length 63.2, Crown width 27.3

Distance on average radius from heart to each 10th ring—inches. (Count in, measure out.)

Cross Section.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1 Stump	.1	.3	.5	.7	.5	1.1	1.4	1.7	2.1	2.3	2.6	2.9	3.3	3.6	3.9	4.3
2	.3	.5	.5	1.0	1.4	1.5	2.0	2.4	2.7	3.1	3.4	3.6	3.7	4.0	4.2	4.4
3	.3	.7	1.1	1.5	2.0	2.3	2.6	2.9	3.2	3.3	3.6	3.9	4.1	4.2	4.6	4.6
4	.5	1.3	1.7	2.1	2.3	2.5	2.9	3.1	3.4	3.6	3.5	4.0	4.2	4.5	4.5	5.0
5	.4	.5	1.4	1.5	2.1	2.4	2.6	2.5	3.0	3.2	3.5	3.5	4.1			
6	.3	.6	1.2	1.5	1.9	2.2	2.4	2.7	3.1	3.4						
7																
8																
9																
10																
11																
12																

Distance on average radius from heart to each 10th ring—inches. (Count in, measure out.)

Cross Section.	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1 Stump	4.5	5.3	5.5	5.6	5.7	5.9	6.6	6.9	7.3					
2	4.6	4.7	4.9	5.3	5.7	6.1								
3	5.0	5.4	5.7											
4														

Remarks: .....
Form and condition: .....

Fig. 77. — U. S. Forest Service Form No. 334 for Recording Field Data in Growth Studies.

inch in the last 8 years, it is to be accepted that trees now 18 inches in diameter will require a similar period of years before they will have attained a diameter of 19 inches. This same idea extended to all the diameter classes within a stand provides a means of anticipating future growth on the basis of what has been demonstrated in the immediate past. It must not be assumed that every tree now 13 inches in diameter, for example, will grow to be 14 inch trees in a period equal to that shown by 14 inch trees in acquiring their last inch of diameter. Approximately as many 13 inch trees will do this as there are 14 inch trees standing at present. The 13 inch trees which do grow to 14 inch size will repeat the growth history of 14 inch trees. Similarly, in acquiring their next inch, they will repeat the growth history shown by trees now 15 inches in diameter, and so on.

Since this is the basis of the theory, it is necessary that the diameter growth of the several diameter classes be measured backward from the present diameter dimension for the period required to grow the last inch of diameter increase. A number of samples may be taken within each diameter class and averaged as in Table XVI.

The values as measured and averaged within the last half inch of radius (inch of diameter) will show considerable irregularity from diameter class to diameter class, which should be evened off by a single curve. The number of trees standing today is, of course, a matter of actual count from a stand table. The number of trees expected to be standing within each group is a matter of approximation and deduction. Inspection of the present stand may or may not record anticipated losses. But it is fairly safe to assume that there will be approximately as many 7 inch trees in the stand 20 years from date as there are 7 inch trees today, although *they will not be the same trees.* The recurring loss in numbers from diameter class to diameter class is quite as constant a thing as the recurring response to growth in the progressive increase from dimension to dimension over a similar period of time.

The application of the table in predicting growth is a relatively simple matter. Trees now 5.1 inches in diameter will grow to be 6.1 inches in 9.1 years. In another 8.4 years, or a total of 17.5 years, they will be 7.1 inches in size. In another 7.7 years they would increase to be 8.1 inches in size, and so on. But we are interested only in the growth for the next 2½ years and at the same

TABLE XVI

CURRENT DIAMETER GROWTH OF BALSAM FIR

Balsam Flat Type

St. Jacques, Madawaska County, N. B., August, 1916

Diameter Class in Inches	Number of Rings in Last Inch of Diameter		Diameter in Inches to Which Figures Apply	Number of Trees Per Acre at Present Time	Volume Per Acre in Cubic Feet, 1916	Diameter in Inches in 1936	Anticipated Number of Trees per Acre in 1936	Volume Per Acre in 1936, in Cubic Feet
	Measured	Curved						
6.0	9.2	9.1	5.1	16.0	11.2	7.4	10	50.0
7.0	8.8	8.4	6.0	10.0	31.0	8.5	8	57.6
8.0	7.1	7.7	6.9	6.0	30.0	9.6	5	62.0
9.0	7.3	7.3	8.1	4.3	31.6	10.9	3	46.8
10.0	6.8	7.0	9.2	3.1	29.6	12.1	2	39.0
11.0	6.9	6.8	9.9	3.0	37.2	12.9	2	48.0
12.0	7.3	6.6	11.1	2.0	31.6	14.9	1	28.7
Totals				44.4	192.0		31	332.1

rate, in which length of time they should have increased about $\frac{8}{10}$ of an inch, or have reached a size of 7.4 inches in 20 years. Similarly, trees 6.0 inches in size will in 8.4 years and in 7.7 more years, or a total of 16.1 years, be 8.0 inches, and at the end of 20 years will be 8.5 inches, and so on. It is then but a simple matter to interpolate from a volume table for the species, multiply by the anticipated number of trees, and compute the anticipated volume of the stand. The increase of 74 per cent by volume is accomplished in spite of a loss in numbers of some 13 of these trees, or 29½ per cent of the stems per acre. This is perhaps one of the easiest methods of growth prediction in all aged or uneven aged forests.

175. Volume Growth of Stands Based on a Graphic Projection of the Past Growth of Diameter Classes. — This method, which is adapted particularly to even aged stands of definite character and area, is based on the theory that the diameter growth of the im-

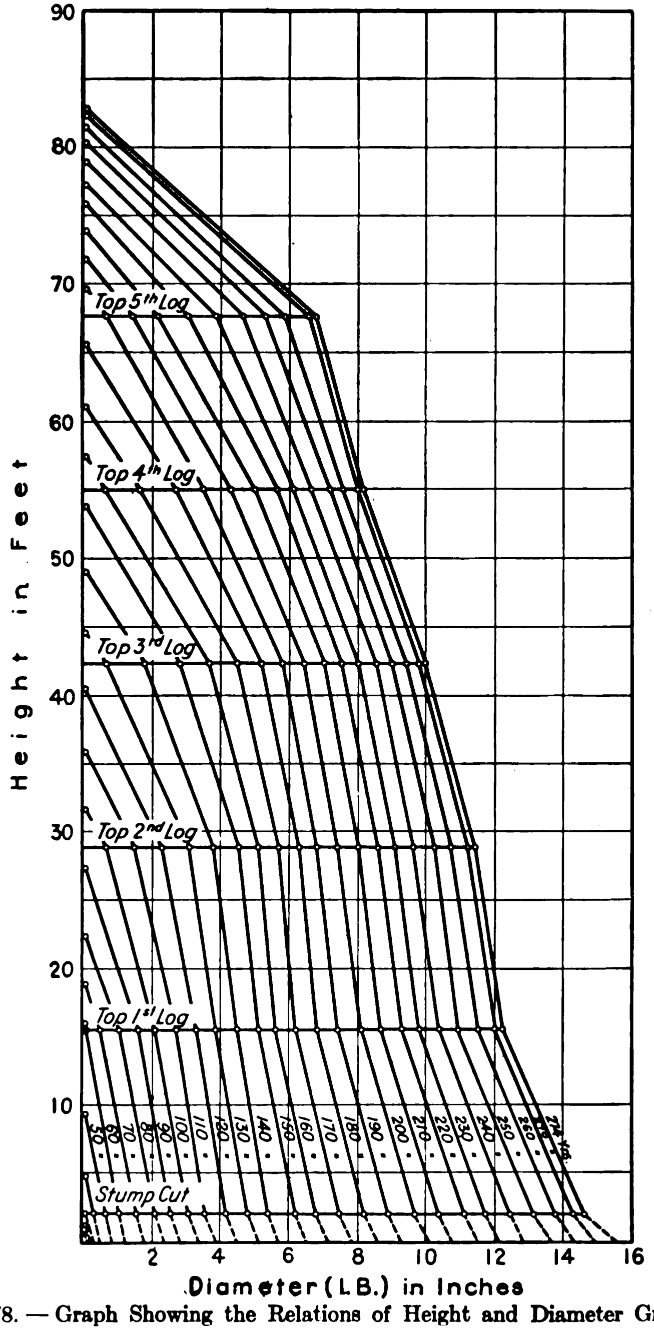


FIG. 78. — Graph Showing the Relations of Height and Diameter Growth by Decades. From this Diagram Volume Growth by Decades can be Easily Computed.

mediate past can be accepted as an index of the trend of the growth for the immediate future. Measure the past diameter growth of each diameter class for two, three, or four short periods. Graphically plot these values in a series of evened harmonized curves. The continuation of the curves on their trend for one, two or three periods of equal length in the future will offer means of determining growth data for such future periods.

Limitations of this method lie in the assumption of unchanging rate of diameter growth for definite periods in the future, and in the accuracy with which the anticipated numbers in the future stand are predicted. In projecting the curve in the future, on account of the uncertainty regarding the numbers of the future stand, it is not advisable to extend the projection over any greater length of time than it is based on in the past, and preferably restrict it to one period less.

TABLE XVII

CURRENT DIAMETER GROWTH OF BALSAM FIR

Upper Spruce Slope Type

Town of North Hudson, Essex County, New York

D.B.H. in Inches				Present D.B.H. Class, 1922	Diameters in Inches as Read from Curves		
1907	1912	1917	1922		1927	1932	1937
4.6	5.3	5.7	5.9	6	6.1	6.3	6.5
5.7	6.2	6.8	7.0	7	7.2	7.4	7.7
6.8	7.3	7.8	8.2	8	8.5	8.8	9.2
7.2	8.0	8.6	8.9	9	9.3	9.6	9.9
8.4	9.1	9.7	10.1	10	10.5	10.9	11.5
9.5	10.2	10.8	11.3	11	11.8	12.3	12.9
10.1	10.7	11.1	11.6	12	12.3	12.9	13.4

176. Volume Growth Based on the Analysis of Average Sample Trees. — It is sometimes the practice to make predictions of volume growth for areas of considerable extent from the analysis of sample trees of such character and form as are believed to represent the growth development of all of the trees on the area. These sample trees are felled and analyzed and the growth figures applied to the entire stand. The weakness of this method is that, although the trees picked may be average for the stand they represent, such condition is for *the present time only*. There is no guarantee nor knowledge that the trees picked will remain average for the stand for any period of time. This much is known, that they were not average trees for the entire past life of the stand, and no assumption can be made they will remain so in the future. The main effect of the progressive loss in numbers of the stand is such that there is a constant variation in the numerical values upon which the average is based. Consequently, any method of predicting the volume growth of a stand from data obtained from an average tree should be accepted only with considerable misgiving until more exact figures can be obtained.

177. Growth Studies and Forest Surveys. — Growth studies of themselves mean little or nothing. It is only when growth is combined with volume, and volume with the area which supports it, as well as with the time required to produce it, that the full significance of growth studies begin to appear. So close is this inter-relation, that any forest survey made on the basis of and for purposes of future forest management should concurrently undertake a complementary set of growth studies. The intensity of this study may vary all the way from the detailing of a separate party to make intensive analysis down to the more or less casual boring of sample trees at indeterminate intervals and with corresponding dependability.

The use of the increment borer as an aid for acquiring growth data over extended forest areas has not received that full attention it deserves. When employed with care and consistency it gives sufficiently good results* within certain well recognized limitations. The significant data are those outlined in following paragraphs.

Volume. — The total present volume per acre of the several species is needed as the basis for study and analysis. For purpose of comparison and prediction the desired goal is a set of volume growth tables constructed on the basis of diameters alone for the area under examination. These should be checked by hypsometer measurements over D.B.H. on standard volume tables for the several species.

* Volume Increment on Cut Over Pulp Woodlands, by E. F. McCarthy and W. M. Robertson, Jour. of For., Vol. XIX, No. 6, Oct. 1921.

Cruise and Yield Study for Management, by R. R. Fenska and D. E. Lauderburn, Jour. of For., Vol. XXII, No. 1, Jan. 1924, pp. 75–80.

*Stand Tables.** — What is required are records of maximum dependability that will adequately express the distribution of the stand through the various size classes for each type, site and age class division recognized. If such stand tables are based on sufficient acreage actually tallied, there should be little or no need to resort to curves for the purpose of evening off irregularities. The upper limits of such stand tables will be fixed by the maximum size of the trees, and the lower limit should be extended downward one or two inches below the minimum merchantability dimensions.

Mortality Record. — Some of the trees within a given dimension class must normally be expected to die in the growth progression to the next larger diameter group. Such natural mortality of greater or less amount is known to be a common factor to all dimension groups. It thus becomes of paramount importance in studying the growth on large, as well as small, areas, that definite data shall be secured and allowance shall be made for anticipated mortality. Some allowance may be made for this in the tree tallies, on which the stand tables are based, by separately classifying those trees which in the best judgment of the observer are not expected to be alive or standing at the end of the next decade, or at the most, the next two decades. These figures should be modified by the probable resistance or liability of the species to windthrow, insect attack or fungus decay as influenced by size, age, and likelihood of occurrence. The normal loss of natural forests will be greatly affected by logging operations. Although annual studies of mortality are greatly to be desired, their cost is prohibitive and a quite satisfactory substitute will be found in the determination of loss in successive decades.

Diameter Growth. — Increment borings by diameter classes are made in conjunction with the tally record. Of necessity these borings are restricted to healthy, well formed, well founded trees which give every promise of standing until the end of the next decade. Borings† are made at D.B.H. using the same distribution

* A stand table is a tabulated statement showing for the average acre within the type the number of trees of each species in each D.B.H. class. This form of stand table varies greatly in form and derivation from the one described in Section 187.

† Major W. G. Wright reports that in studies of this character, he has found it more satisfactory to cut small chips from the tree of sufficient size to contain the significant data. The chips can be preserved for later study with more success than can increment cores.

of type, age, site and tree class as was observed in the timber tally. The borings should be well distributed throughout the diameter range in direct proportion,* at least, to the numerical values in the stand table.

Measurements are made of the number of rings in the last inch or half inch of radius. Another way is to measure the radial growth for a given period such as 10 years (10 growth rings). What is desired is an expression of diameter increment per D.B.H.

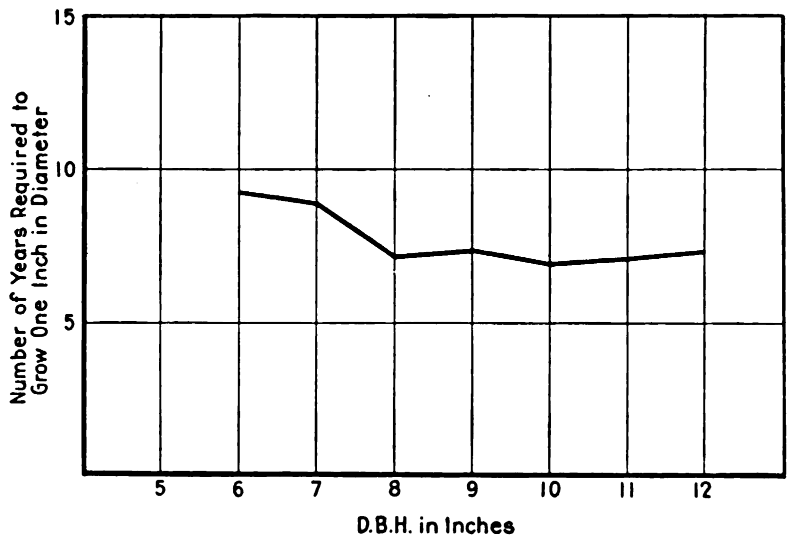


FIG. 79. — Diameter Growth of Balsam Fir, Balsam Flat Type, St. Jacques, N. B., August 1916.

class as a basis of determining volume increment. The relation of diameter to height in bored trees may be checked by hypsometer measurement.

Application of Growth Data. — Measurements showing the number of rings in the last half inch of radius are mathematically averaged by diameter classes. These values are then plotted on the Number of Years required to grow 1 inch in Diameter on D.B.H., as in Fig. 79. This will bring out irregularities due to error, or insufficient data, as well as serve as a basis of comparison of growth by diameter classes. These values are then taken and plotted in a " cumulative curve " of D.B.H. on the years required

* McCarthy and Robertson, advise 100 borings per inch diameter class.

for growth from class to class, as in Fig. 80. The purpose of this curve is to visualize the continuous growth process of each diameter class, and the plotted points represent for each dimension the sums of all the years passed through by it to attain its present size. This curve does not represent the diameter growth of a single tree, nor is it the average for the stand through any such period of

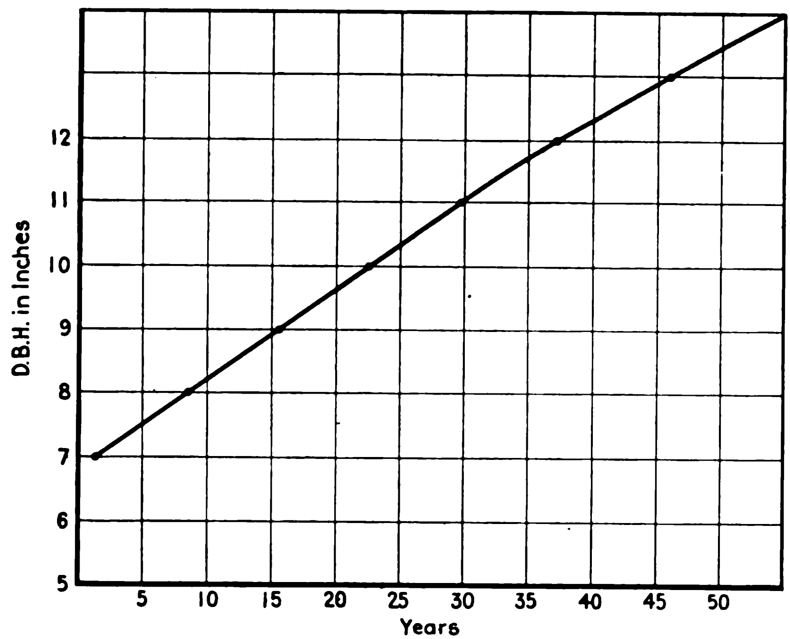


FIG. 80. — Diameter Growth of Balsam Fir (Fig. 79) Extended as a Continuous Growth Curve.

years. It merely shows the contemporary growth for a short period up to the time of measurement.

Application to the stand and volume tables will determine the volume present in any given inch class at the time of measurement. Future diameter values are then translated into terms of volume, and thence, with a decision as to the probable number of stems per acre as can be anticipated from the mortality record, into terms of volume per acre. The total volume for all of the diameter classes is the completed record.

178. Growth Per Cent. — An easy and convenient way to express volume growth or increment is as a percentage. The relation between growth and volume is thus to be considered in exactly the same way as the relation between the interest and the principal on which it is earned. The volume present for any specific year will be the principal, and the growth or increase in volume (or diameter) is the interest. The ratio between them is the percentage. Due to the fact that the actual growth for any one year becomes a part of the capital on which the next year's growth, and percentage, is computed, the value of the growth percentage figures are never constant but vary from year to year. The problem is further complicated by the variation in the *rate* of growth in trees from youth to old age.

In spite of their limitations, growth percentage calculations have the advantage of commending themselves to business men whose common practice it is to gage the acceptability of all enterprises by the relation between the income or revenue and the capital or principal on which it is earned.

Growth percentage may be expressed on the basis of mean annual growth, current annual growth or periodic annual growth. The last of these is very commonly used. Growth percentage is to be accepted only as an index of the present status or capacity of the stand.

Growth percentage may be calculated either on the basis of differences in volumes or in diameters over a definite period, by application of one of the following formulas:

A. Formulas based on volume determination:

1. *Compound interest formula,*

$$p = \left(\sqrt[n]{\frac{V}{v}} - 1\right) 100$$

2. *Pressler's (volume) formula,*

$$p = \left(\frac{V - v}{V + v}\right) \frac{200}{n}$$

3. *Kunze's formula,*

$$p = \frac{(V - v)\, 200}{V(n - 1) + v\,(n + 1)}$$

where V = the present volume of the tree or stand.
v = the volume n years ago.
n = the number of years covered by the study.

The first of these formulas is seldom used due to the necessity of using either logarithms or compound interest tables in the calculation. The second represents an endeavor to overcome the difficulty of deciding whether to use V or v as the base or principal. By assuming the mean volume for the period names as $\frac{V+v}{2}$ and stating the proportion as $x : 100 :: \frac{V-v}{n} : \frac{V+v}{2}$, the difficulty of decision was eliminated. But in use the formula consistently underestimates, and in an endeavor to overcome this discrepancy, Kunze elaborated his formula as a direct derivative from Pressler.

B. Formulas based on diameter determination:

4. *Pressler's (diameter) formula,*

$$p = \left(\frac{d_1^3 - d_2^3}{d_1^3 + d_2^3}\right)\frac{200}{n}$$

where d_1 = present diameter (D.B.H.).
d_2 = diameter n years ago.
n = number of years in period.

5. *Gevorkiantz formula,*

$$p = \frac{228}{n}\left(\frac{D-d}{d}\right)$$

where D = present D.B.H.
d = D.B.H. n years ago.
n = number of years covered by the study.

6. *Schneider's formula,*

$$p = \frac{400}{nd}$$

where d = present D.B.H.o.b. in inches.
n = number of rings in the last inch of radius.

The particular advantage of this group of formulas is that they can be applied directly to the standing tree with no further data needed than that which can be supplied with a common increment borer. Pressler's diameter formula is an endeavor to utilize the coördination of the ring count and diameter dimensions. The use of the cube, rather than the square of the diameter as an expression of the relative variation of trees in volume, is a resultant of the recognition of height as a variable of volume as well as of diameter.

The Gevorkiantz* formula is one in which an endeavor is made to correlate form with diameter and height as a variation of volume. Its weakness lies in the assumption that form relations are constant in all trees. Its advantage lies in its simplicity, only the changes in diameter over the period being required. Results calculated by it fall very close to those of admittedly more accurate methods. It is as yet an untried quantity, but its relative simplicity invites its wider adoption.

Schneider's formula, which tends to underestimate, though based on a rather ingenious statement of the mathematical relationships of height, diameter, and form in trees, ignores the first as a variable, and hence is perhaps adaptable only with mature trees where height growth has practically ceased. Its simplicity, which requires only a measurement of D.B.H. outside the bark, and a boring or cut at the D.B.H. point which will penetrate the tree far enough to enable rings to be counted on the last (outside) inch, commends its use during the initial stages of forest management when the approximations of easily and quickly secured growth figures will suffice until more accurate data can be secured.

* E. J. Hanzlak suggests that a variable factor *F* be substituted for the 228 in the Gevorkiantz formula. The value of *F* will vary according to the species and the unit of volume used. Jour. of For., Vol. XXV, No. 4, April, 1927, p. 443.

CHAPTER XV

THE YIELDS OF FORESTS

179. The Problems of Stand Increment. — The study of volume growth or increment in individual trees is only a means to an end, namely that of determining volume growth in forests, or rather, of determining the amount of wood material per acre which can be expected to grow in forests during definite periods of time. The application of growth figures to forested areas is of importance, since knowledge regarding volume yields per acre on the basis of the time required to produce them forms a means for determining their value. In addition, fundamental problems of management are settled, such as the rotation, that is, the age, at which the timber should be cut in order to realize on its greatest volume production per acre; the time for, the intensity of, and the effect on growth, of silvicultural thinnings; the choice of methods of silvicultural treatment, and the degree of benefit to be derived therefrom. In short, the application of growth figures to forested areas is an endeavor to remove all of the operations of forest ownership and forest management from the field of speculation, and place it on a foundation of good, sound business.

In the application of growth figures to forested areas, a number of baffling and complex factors are met. Any one of these, though producing due effect on the growth of individual trees, exercises a much greater, more far-reaching influence upon the composite growth of stands. Among the most important that may be mentioned are:

(*a*) The differentiation in the *character* of the stand, represented by its constant variation in proportion of species and diminution in numbers, the ever increasing size of the trees that are left, and the consequent change in the individual influence of the species that remain.

(*b*) The influence of variations in *composition*. Stands are rarely pure, in that they consist in their entirety of but a single species. More tolerant associates creep in, and, in time, assume a position of some importance in the stand. This tendency is so common that stands 80 per cent pure or better are commonly accepted as pure stands.

(*c*) The intrusion of two, or of several, age classes presents a separate set of problems of great complexity. This is known as variation in *form*. The form of a stand is either even aged or all aged.

(*d*) The *density* of the stocking produces a profound influence on the growth of individual members of the stand. This is another way of determining the degree of efficiency with which the individual trees are utilizing the resources of the site. Trees react on each other's growth in that they may be so crowded, due to competition of both root and crown, that individuals are getting neither as much food nor as much room as they require. At the other extreme, open or grass areas occur in the forest. The desirable mean is when the site is just supporting that number of stems which will most fully utilize its resources without undue deprivation of food or of room to individual members. Such a stand may be known as a *fully stocked stand.*

(*e*) Finally, there is the variation in the *capacity* of the site itself to produce tree growth. The soils in which forest trees are found are not uniform. There are good soils and poor soils, deep soils and shallow soils, dry soils and wet soils, sweet soils and sour soils, loamy soils and rocky soils. Variations of atmospheric warmth, precipitation and length of growing season cause profound differences in different regions. Needless to say, under a given set of climatic conditions, deep, well drained, superior soils that will produce, for a given species, larger trees within a given time than can be produced on a dry, shallow, rocky soil, will be judged to be of superior quality. The forester's endeavor to classify soils or sites on the basis of their desirability for tree production is termed **site quality** classification, or, more briefly, **site class.** Site quality, then, is the expression of the reaction of the trees and stands themselves to their environment, and is of utmost importance in studying the growth of forests.

From the foregoing, it is seen that the difficulties of studying growth and yield in stands are many and complex. The solution can best be accomplished by considering their application in four broad and distinct aspects:

1. Yields in pure even aged forests.
2. Yields in mixed even aged forests.
3. Yields in pure many aged forests.
4. Yields in mixed many aged forests.

180. Yields in Pure Even Aged Forests. — The distinguishing characteristics of this type of forest are: It is comprised of units consisting mainly of but a single species, and all the trees within a specified area are of a single age group or class. Thus, on the basis of age, the individual trees of each unit show a distinct gradation in development and size from the trees of other units. If there is shown such gradation in size and development, it follows that there will be shown gradation in volume. Thus, if stands of uniform density are arranged in order of increasing age, they should show increasing gradation in their volumes per acre, and the record could be used as a criterion of the development of all other stands of similar species and character.

When such a record is arranged in tabulated form, preferably by even decades, it is known as a **yield table.** Growth in even aged stands can best be studied and gaged by comparing the volumes per acre taken from arbitrarily selected and established sample plots within each age group unit or stand, with the values of volume production or yield at similar ages as taken from a standard yield table.

There are a number of points which should be emphasized in regard to yield tables:

1. A separate yield table must be constructed for every separate species. There is no such thing as a "universal" yield table applicable to all species and to all conditions.

2. A yield table is usually constructed in terms of a given unit of measurement and to meet a given standard of utilization (stump height, used length, merchantable top d.i.b., etc.). With the adoption of a new unit of measurement, cubic feet instead of board feet, and the initiation of closer standards of utilization, it is probable that an entirely new yield table would have to be constructed.

3. Yield tables are local in character and application, so that different tables may have to be constructed for different sections of a tree's natural range.

4. Yield tables are always based on three essential factors: *volume, area, and age.* Area is standardized on the basis of the unit acre, hence all volume values have to be reduced to their terms of volume per acre and their age values based on terms of average age.

5. Yield tables endeavor to show ideal forest growth rather than actual growth.

6. Yield tables, so constructed, become established standards. They are accepted as standards because they endeavor, as far as possible, to eliminate all of the variables which so affect yields, as follows:

(*a*) *Character*, by requiring separate yield tables for separate species.

(*b*) *Form*, by requiring separate yield studies for even aged stands and for all aged stands.

(*c*) *Composition*, by requiring construction of separate yield tables by distinctly different methods for pure stands and for mixed stands.

(*d*) *Density*, by requiring that all stands upon which construction and compilation of the yield table are based shall be of " normal " stocking, that is, fully stocked stands of such condition at their age that they are most efficiently utilizing all of the resources of the site.

(*e*) *Site Quality Variation*, by requiring that for each decade separate yields per acre must be computed for each site quality.

Such a table as this is known as a **Normal Yield Table.** The chief uses in which a normal yield table can be employed are:

1. To determine the approximate final yield per acre and for the stand as a whole at any age when definite information regarding site quality distribution is known.

2. As a basis for determining the site quality value of any stand when the average age of the stand, the volume per acre and its standard of normality, that is, its density, as gaged by its relative total basal area per acre, are known.

3. When the average age of the stand is known, its relative density (basal area per acre) stated, and its site index determined, the yield table may be used directly as a means of measuring the volume of the stand.

4. Site quality values may also be approximated by using the average total height of dominant trees as tabulated in the table when the age and density of the stand are known.

5. To standardize actual yields on specific areas by comparing them with the ideal yields of the yield table.

6. As a basis of comparing the mean annual and the periodic annual increments of the actual growth of a stand with the normal increments of the yield table.

7. For prophesying anticipated returns on bare forest lands,

with a definite species and for different rotations. These values can be only approximated and should be based on figures of average site quality. It is assumed that a desirable intensity of silvicultural treatment and forest management will be maintained.

181. Yield Table* Construction — Field Work. — From 100 to 300 pure even aged stands of the desired species (80 per cent pure or better) are selected for measurement. These stands should meet such conditions of being as fully stocked as will satisfy some liberality in a strict definition† of "normality." Both understocked and overstocked stands are to be avoided. The former may be recognized by such factors as holes in the crown cover, by a noticeably low number of trees per acre, by abnormally high diameters (D.B.H.) for their age, and by an excessive amount of ground cover, underbrush or reproduction on the forest floor. When the species is excessively intolerant, holes in the crown and a large amount of reproduction become of minor importance. Overstocking is harder to identify. It is characterized by an excessively high percentage of suppressed trees, and by a state of stagnation as evidenced by very slow diameter and height growth, even among the dominants. As a rule, overstocked stands are rareties, and unless there is absolutely no uncertainty as to such identification, any stand approaching overstocking should be accepted and be carefully measured.

* In the preparation of the material incorporated into this chapter, the following authors have been consulted to whom due acknowledgment is hereby made:

1. Report of the Committee on the Standardization of Methods of Preparing Volume and Yield Tables. Jour. of For., Vol. XXIV, No. 6, Oct. 1926, p. 653.
2. A Method of Preparing Timber Yield Tables, by Donald Bruce, Jour. of Agric. Research, Vol. XXXII, No. 6, March 15, 1926, p. 543.
3. A Modification of Bruce's Method of Preparing Timber Yield Tables, by L. H. Reineke, Jour. of Agric. Research, Vol. XXXV, No. 9, Nov. 1, 1927, p. 843.
4. Preliminary Normal Yield Tables for Second Growth Western Yellow Pine in Northern Idaho and Adjacent Areas, by C. E. Behre, Jour. of Agric. Research, Vol. XXXVII, No. 7, Oct. 1, 1928.
5. Volume, Yield and Stand Tables for Second Growth Southern Pines U. S. Dept Agri., Misc. Pub. 50, Washington, D. C., Sept., 1929.

† A "normal stand" may be defined as one fully stocked and in proper growing condition, and producing the maximum volume (in cubic feet) possible for a given age and site quality.

Stands which show pronounced variation of density within their limits should always be excluded from consideration.

The stands accepted for study and measurement should be well scattered over the range of age and site qualities involved, and should be of such size as will permit the establishment of a sample plot containing from 100 to 300 trees. In addition, they should allow the maintenance of an entirely surrounding isolation strip of timber similar in character, form, age, density, and site quality to that on the plot. The width or depth of this isolation strip should be sufficient to protect and adequately define the plot area as being representative of a given set of conditions. It will vary from a few feet to a hundred feet or more depending upon the species, the site, slope, exposure, uniformity and extent of the stand. Occasionally it may be necessary to have a plot with perhaps one side exposed. In such cases extreme care is to be taken in laying out this boundary to include the entire area occupied by border trees.

Sample plots are laid out with staff compass and chain tape, with all slope distances reduced to their true horizontal value for computation of area. Plots should be four-sided but not necessarily rectangular. All angles, however, should be greater than 60 degrees. At least one corner stake should be permanently located and the plot record should be accompanied by a sketch map showing its location, form, and size.

Within each plot, diameter, height, age, site quality, and volume are measured as follows:

Diameter. — The diameter breast high of every tree 0.5 inches D.B.H. and over on the plot is measured with calipers (or diameter* tape) to the nearest $\frac{1}{10}$ inch. Trees are tallied by standard inch classes (from 0.6 inches below to 0.5 inches above), by species and by crown classes. Usually three crown classes will be found sufficient. All species whether of subordinate importance or not should be recorded. The custom of tallying minor species as belonging to a " Miscellaneous " class should not be followed unless the tally identifies the species of each tree for which such record is made.

* Care should be taken to use the same diameter measuring instrument as was used in the construction of the corresponding volume table.

TABLE XVIII*

AVERAGE D.B.H. AND AVERAGE TOTAL HEIGHT OF DOMINANT TREES, SLASH PINE

Age, Years	Dimensions of Average Trees — Site Index in Feet									
	60		70		80		90		100	
	D.B.H., Inches	Total Height, Feet	D.B.H., Inches	Total Height, Feet	D.B.H., Inches	Total Height, Feet	D.B.H., Inches	Total Height, Feet	D.B.H., Inches	Total Height, Feet
15	3.9	29	4.6	34	5.2	39	5.9	43	6.6	48
20	4.4	36	5.2	42	6.0	48	6.8	54	7.7	61
25	5.3	42	6.3	49	7.2	56	8.1	63	9.2	71
30	6.1	48	7.3	56	8.3	63	9.4	71	10.5	79
35	6.9	52	8.1	61	9.2	69	10.5	77	11.7	86
40	7.6	55	8.9	64	10.1	73	11.4	83	12.8	92
45	8.1	68	9.5	67	10.8	77	12.2	87	13.7	96
50	8.6	60	10.0	70	11.4	80	12.9	90	14.5	100
55	9.0	62	10.4	72	11.8	83	13.4	93	15.0	103
60	9.3	64	10.8	74	12.2	85	13.9	95	15.5	106

* From Miscellaneous Publication 50, Volume Yield and Stand Tables for Second Growth Southern Pines, U. S. Dept. Agr., Washington, D. C., Sept. 1929.

Height. — Heights are measured with U. S. Forest Service hysometer, Faustman hypsometer, or Improved Abney level, all distances being carefully taped. On each plot a sufficient number of total heights of sample trees are measured as will enable the drawing of a height curve over diameter, which will include the complete range of diameters found on that plot. The number of trees measured is not fixed, nor is it a constant ratio of the total number of trees calipered. Enough measurements to enable a satisfactory curve for the plot will suffice.

Separate height on D.B.H. curves are needed for all subordinate species which may be present, unless they are so unimportant or proportionally of such equal form height as to make the application of the major species height curve not only practicable but desirable.

The total height of from 5 to 10 dominants per plot, either on or immediately adjacent thereto, should be measured for the purpose of site class identification. (Section 183.) These heights should be read to the nearest foot. Care is to be taken to see that such trees are truly dominants.* In case of doubt, it is better to disregard them and seek further.

The total height from 10 to 15 dominant trees of the species growing in obviously open or over-dense stands on areas adjacent to the plot and on similar sites hould also be measured for the purpose of observing the effect of stand density on height growth. It may not be possible nor desirable to do this with every plot, but a sufficient number of such measurements should be taken to enable the determination of this important check.

All trees measured for height should be of normal form and condition. Forked trees and trees with broken tops are to be avoided. The degree of their acceptability for volume table work is a good criterion.

Age. — Age usually will be determined from increment borings from dominant trees on each plot or from trees approximately dominant. These borings will be taken at breast height and are to be accepted only when they penetrate the growing center of the tree. The ring count as read from the increment core must be coördinated with, and increased by, the number of years required by a seedling of the species to grow to D.B.H. height, in order to get total age. A sufficient number of borings should be made to obtain an average which is to be accepted as the age of the plot. Exceptionally large trees, when present, should be tested to see if they are older than the main stand, and occasional small trees should be bored to determine whether the stand is essentially even aged.

In some cases, it may be desirable to determine age from ring counts on freshly cut stumps on areas immediately adjacent to the plot, granted that similar conditions of character, form and site quality are being maintained. The stump ring count must be increased by the number of years required for a seedling to grow to stump height, in order to get total age.

When seedling height growth is studied for the purpose of determining the age height interval to add to such ring counts, whether taken on the stump or at D.B.H., measurements should

* The committee of the Society of American Foresters defines dominants as follows: "Trees with crowns extending above the general level of the forest canopy and receiving full light above and partly from the side; larger than the average trees in the stand and with crowns well developed, but possibly somewhat crowded on the sides." Jour. of For. Vol. XV, No. 1, Jan. 1917, p. 74.

be taken on seedlings growing on similar sites. On such similar sites, selection should be made of open growing healthy seedlings which show every promise of ultimately developing into dominants, or at least into trees of average dominant character.

All increment cores made for the purpose of determining age should be numbered in sequence in order to permit ready identification when preserved for later study and analysis.

Site Quality. — Direct site quality classification of each plot by inspection in the field on any absolute basis is undersirable on account of its relativity and inaccuracy. Such classification should be left for office determination. Nevertheless, there is definite need of determining whether the plots as taken are satisfactorily distributed through the full range of available site classes. This is accomplished by drawing a tentative series of height age curves (site index curves — see Section 183) on the basis of the first 15 to 20 sample plots. These are used to designate a temporary site quality classification for each plot pending subsequent confirmation or re-classification.

Volume. — The volume of each plot is computed directly from volume tables in application to the tree tally. If adequate volume tables of the species are not readily available, it will be necessary to compile them, following the method suggested in Sections 152 and 153. The method of determining volume by felling one or two dominants per plot is both undesirable and inaccurate, since the range of diameter values is not fully coördinated nor covered.

The usual field party for collecting yield table data consists of one or more two-man crews working in conjunction. When more than one such party is engaged in this work, it is of utmost importance to coördinate their methods particularly in regard to a common conception of "normality." The party leader should be responsible for and have direct charge of both field and office work. It would seem best that the leader should start with a single two-man party, the members of which should be carefully trained in all methods of field technique, until a desirable standardization of thought and action is attained. This party can be split and resplit as newer members become proficient. In a party of any size, the leader should spend a considerable amount of time with each group in order to see that the uniformities are being maintained.

A complete description should be made for each plot covering the following points:

(*a*) Plot number.

(*b*) Area in acres or decimals of an acre.

(*c*) Timber, type, occurrence, and distribution.

(*d*) Species.

(*e*) Site Class (entered tentatively in field, permanently in office).

(*f*) Age — average age of stand, and ages of sample trees.

(*g*) Timber tally — diameters tallied by species and crown classes.

(*h*) Height measurements — as taken on sample trees.

(*i*) Normality.

(*j*) Per cent of boundary isolated.

(*k*) Slope and aspect.

(*l*) Altitude, relative and absolute.

(*m*) Soil and rock.

(*n*) Soil cover, underbrush, herbaceous material, litter and reproduction.

(*o*) History with dates of known occurrences (fires, cuttings, etc).

(*p*) Date and names of party.

(*q*) Survey notes and sketch map.

Any form of notes and tally sheet that can meet the requirements as previously laid down will suffice. Minor variations would be necessary for different species and different regions. The standard form of the U. S. Forest Service Form No. 547 as illustrated in Fig. 81 has served with considerable success and wide application. Computations of basal area, volume, etc. should be filed with the original notes for each plot.

182. Yield Table Construction — Office Work. — The first step is to scrutinize all field sheets carefully for omissions and possible error. Within each plot the field measurements and data are summarized as follows:

(*a*) The number of trees tallied is totaled by species and crown classes.

(*b*) The total basal area is computed to three significant figures and is totaled by species and crown classes.

(*c*) The average D.B.H. for each species is obtained by dividing its total basal area by its total number of trees and determining the corresponding diameter.

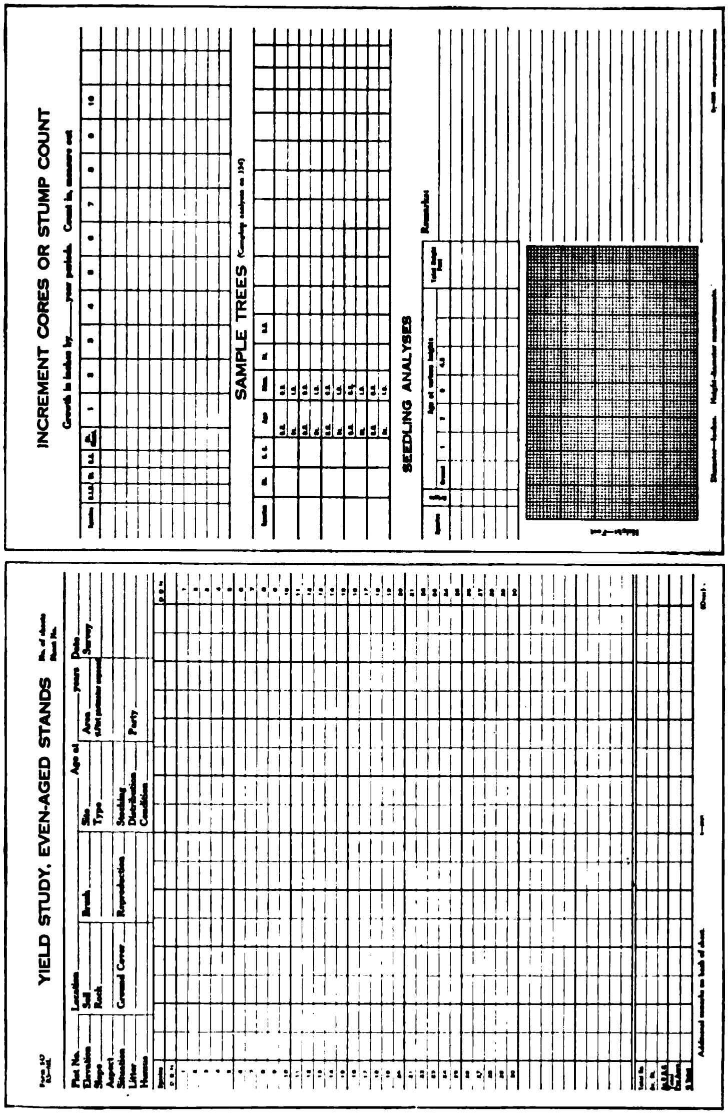


FIG. 81. — U. S. Forest Service Form No. 547. This may be Used in Recording Yield Study Data in Even-aged Stands.

(*d*) A curve of total height over D.B.H. is drawn for each plot.

(*e*) The average height is read for the plot from this curve as the height value corresponding to the average D.B.H.

(*f*) The true average total age is determined by adding seedling height age to the ring count of the boring or stump.

(*g*) Volume is computed in the cubic unit as based on the total contents of the trees and is totaled for the plot. Board foot volume varies with the log rule used and the standards of utilization as expressed in the volume tables employed.

TABLE XIX*

NUMBER OF TREES PER ACRE IN DOMINANT STAND, SLASH PINE

Age, Years	Number of Trees Per Acre				
	Site Index in Feet				
	60	70	80	90	100
15	1,550	780	605	475	385
20	865	640	500	450	320
25	685	510	400	320	255
30	530	400	310	250	205
35	420	315	250	205	165
40	350	265	210	175	140
45	315	235	190	155	125
50	285	220	175	140	115
55	265	205	160	130	110
60	255	195	155	125	105

* From Miscellaneous Publication 50, Volume Yield and Stand Tables for Second Growth Southern Pines, U. S. Dept. Agr., Washington, D. C., 1929.

The area of each plot is obtained and the basal area figures are computed and summarized in terms per unit acre.

The next step is the rejection or retention of doubtful plots. All plots taken are submitted to this test, which is based on the deviation of plot basal area per acre figures for each age and site class from corresponding values as read from a series of tentative curves of " normal " acre basal area over age and site constructed according to the method described in Section 184, interpolating to the nearest year of age and the nearest foot of site class. Ordinarily a deviation exceeding twice the standard error (Section 114) should be the standard of exclusion, but for more precise work this standard may be based on three times the standard deviation (Section 111).

The remaining plots are then permanently assigned a final site class index classification according to the method described in

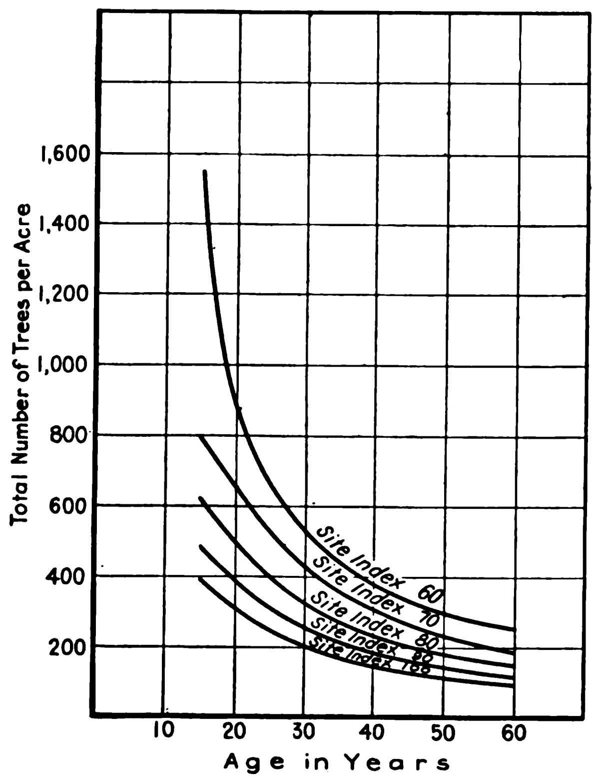


FIG. 82. — Total Number of Trees Per Acre on Age by Site Index Classes; Slash Pine. Data Taken from Miscellaneous Publication No. 50, U. S. Department of Agriculture, 1929.

Section 183. For these site index curves use average height of dominant trees and read the plot values to the nearest foot at the site classification age. In some cases, it may be desirable to base this classification on ultimate (final rotation age) heights, in which case careful coördination should be made with height, and the usual classification age and site values should be assigned on *the latter basis.*

Plots are now sorted by age and site index class. The age class interval should be 5, 10, or 20 years depending upon the extreme range of ages, and the site index interval may be 5, 10, 15, 20 feet depending upon the range of heights at the classification age. As a rule, 10 year age classes and 10 foot site index classes will be found most satisfactory.

With the data now available, the following information should be secured and summarized from a series of harmonized curves constructed originally on an anamorphic basis, but which may subsequently be presented in conventional form. This data should be tabulated by age and site classes:

(*a*) Total number of trees per acre, Fig. 82, Table XIX.

(*b*) Total basal area per acre, Fig. 84, Table XX.

(*c*) Average basal area per acre.

(*d*) Average D.B.H. per acre accepted as the diameter corresponding to average basal area, Table XVIII.

(*e*) Average height of dominant trees, Fig. 83.

(*f*) Volumes per acre in cubic feet, board feet, or any other unit desired. Fig. 86, Table XXI.

Two self-contained cross checks should be applied to test the accuracy of this work:

(*a*) The total number of trees per acre multiplied by the average basal area for a given age and site class should be equal to the total basal area per acre.

(*b*) The total number of trees per acre for a given age and site class multiplied by the volume taken from a volume table for dimensions corresponding to the average D.B.H. and average total height should be equal to the total volume per acre.

All of these figures should be obtained and checked both for the entire stand and for the dominant stand, and for that portion of the stand above the diameter limit fixed by the board foot volume table employed.

Average deviation of the individual plots from the curves should be obtained for the following:

(*a*) Basal area per acre.

(*b*) Number of trees per acre.

(*c*) Volume in board feet per acre.

(*d*) Volume in cubic feet per acre.

The frequency distribution of the diameter classes should be analyzed and if found " normal," that is, showing conformity to the general trend of a normal frequency curve, the standard deviation should be obtained. (See Section 111.) This standard deviation should then be curved over D.B.H. as a step in the preparation of stand graphs for any age and site as described in Section 187.

The following information should be immediately available in any set of standard yield tables:

(*a*) Species, common and scientific name.

(*b*) Forest type and region.

(*c*) Author and date; and names of collectors of field data.

(*d*) Number of plots used.

(*e*) Average deviation of plot data from yield table figures in basal area, number of trees and volume on a per acre basis.

(*f*) Average total height of dominants.

(*g*) Average D.B.H., and average total height of entire stand.

(*h*) Total number of trees per acre.

(*i*) Total basal area per acre.

(*j*) Yield per acre for the entire stand by age and site class in terms of any desired unit of volume.

183. The Determination of Site Quality. — The rate of growth and the yield of a species per acre at any given age varies with the productivity of the soil, or, as it is more commonly known, with its *site quality.* There are a number of different ways by which site quality may be indicated and classified. The method that seems most satisfactory is an arbitrary system of numerical values known as the site indexes. These numerical values are the average total heights of the dominant trees in a stand at 50 years (or 100 years), the index age being arbitrarily chosen in accordance with the length of the rotation.

This site index system is to be accepted as akin to the farmer's method of classifying agricultural land on the basis of its being 27 bushel potato land, or 35 bushel wheat land, as the case may be, the reproductive period of one crop year and the average acre being implied. But whereas farm land is classed by actual yield, forest land, because of the effect of density of stocking on yields per acre, must be rated by some other factor in which the effect of density or stand crowding is least shown. Such is found in the acceptance of the average height of the dominants. Height, also, is effected least by the character of the stand and the basing of the classification on dominants only, eliminates the effects of stand form and relative position in the crown canopy. Thus 68 site index white pine land means simply that the productive powers of the site and soil are such that at 50 years the dominants will have an average total height of 68 feet. Better sites will show greater heights and will be so rated numerically, and poorer sites will have smaller values.

Site index values can best be constructed by graphic methods. What is desired is not a set of height growth on age figures for individual trees, but rather a set of figures, or curves, which will show the normal development in height growth of the average dominants of the stand. The average height of dominant trees and the total age of each stand as taken from the field record of sample plots in a yield study, form the basis of construction. These, if thrown into a graph of total height on age, will develop a comet-shaped band with ill-defined margins, which is incapable of accurate zonation into site class divisions.

Solution of this vexing problem can be found in a system of anamorphic* curves (See Chapter X). The average† total height and the average age of each age class are mathematically computed. These are thrown into a curve of total height on age. This curve is then used as the graduating curve for anamorphosis, similar to the method described in Section 137. Accepting 50 years as the index age for classification, the ordinate for 50 years is then followed horizontally to the graduating curve, and thence vertically as the abscissa of the anamorphosed graph as was illustrated by the heavy dotted lines in Fig. 60. Through points where this 50 year abscissa intersects ordinates at 60, 70, 80, 90 and 100‡ feet above ground, straight lines radiating from the origin are drawn. These are the site index curves in anamorphosed form. For convenience in handling, these straight line graphs may now be

* "A Method of Preparing Timber Yield Tables" by Donald Bruce, Jour. of Agr. Research, Vol. XXXII, No. 6, March 15, 1926, p. 543.

† The problem can also be solved by application of alinement charts. See "A Modification of Bruce's Method of Preparing Yield Tables," by L. H. Reineke, Jour. of Agr. Research, Vol. XXXV, No. 9, Nov. 1, 1927, p. 843.

‡ Usually site index values are constructed on 20 or on 10 foot intervals. This avoids confusion in construction and bulkiness in handling. Values could be determined, curved and tabulated for every foot of site index, or even for smaller values if desired. By actual experience, it has been found that intervals less than 1 foot are not practical.

reconstructed on a system of regular horizontal graduations as in Fig. 83, thus giving a set of curves in conventional form.

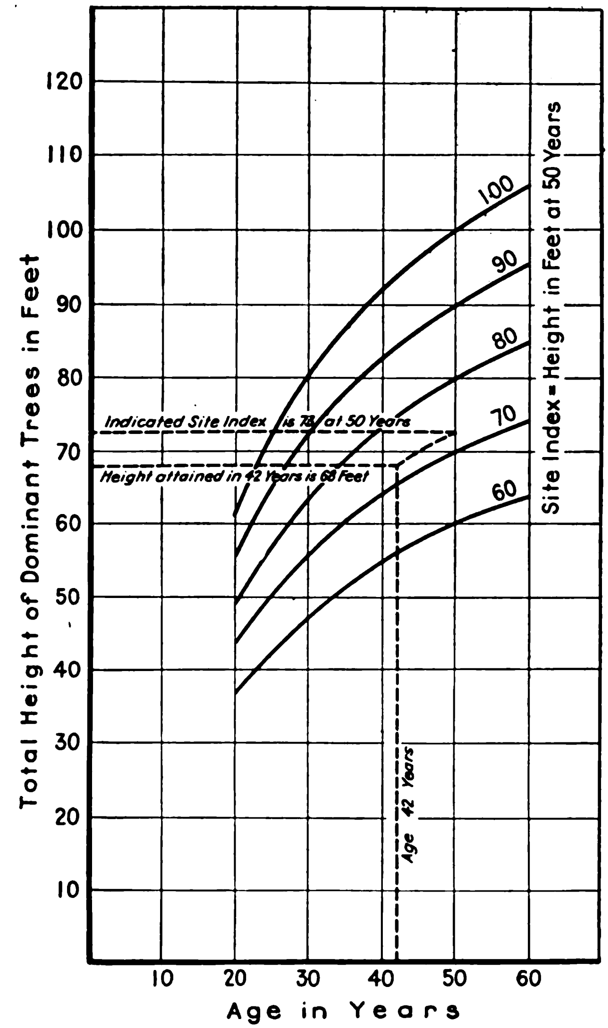


FIG. 83. — Site Index Curves for Slash Pine. Data Taken from Miscellaneous Publication No. 50, U. S. Department of Agriculture, 1929.

In order to apply the site index values so determined to any given stand or plot, it is necessary to average mathematically the total heights of from 5 to 10 dominants on such area. Suppose in a plot with an assigned total age of 42 years, the total heights of 5 dominant trees are found to be 78, 63, 61, 75, and 65 feet respectively. The average is 68.4, evened off to 68 feet. Follow vertically to the conventional curve (see the dotted lines in Fig. 83) the ordinate representing 42 years until it intersects a secondary abscissa, or horizontal distance projected at a height of 68 feet. This point will be $\frac{2}{8}$ of the distance above the 70 foot curve where it crosses the 42 year ordinate. However, our classification is based not on height at 42 years, but on height development normally attained by dominants at 50 *years*. Two eighths of the distance above the 70 foot curve at 50 years gives us a height of 73 feet, or a 73 foot site index.

184. Rejection of Abnormal Plots. — Plots are rejected in yield table construction if they do not conform to standards of density accepted as " normal." The best available basis for testing the density of a stand is its total basal area per acre. To this end, preliminary curves showing the basal area per acre for each site index class and for each age are first prepared.

TABLE XX*

BASAL AREA AT BREAST HEIGHT OF DOMINANT STAND, SLASH PINE

Age, in Years	Basal Area in Square Feet Per Acre				
	Site Index in Feet				
	60	70	80	90	100
15	82	85	87	89	90
20	96	99	102	105	108
25	101	106	109	112	115
30	105	111	114	118	122
35	108	114	118	123	127
40	111	117	122	127	131
45	113	119	124	130	134
50	115	121	126	132	137
55	116	123	128	134	139
60	117	124	130	135	141

* From Miscellaneous Publication 50, Volume Yield and Stand Tables for Second Growth Southern Pines, U. S. Dept. Agr., Washington, D. C., Sept. 1929.

These curves may be constructed on the anamorphic principle in which the mathematically averaged basal areas per acre for all site classes on age may be used as the graduating curve. Averages are then obtained by 10 year age classes for all plots, the site indexes of which fall within the arbitrarily determined limits of 40 to 49,

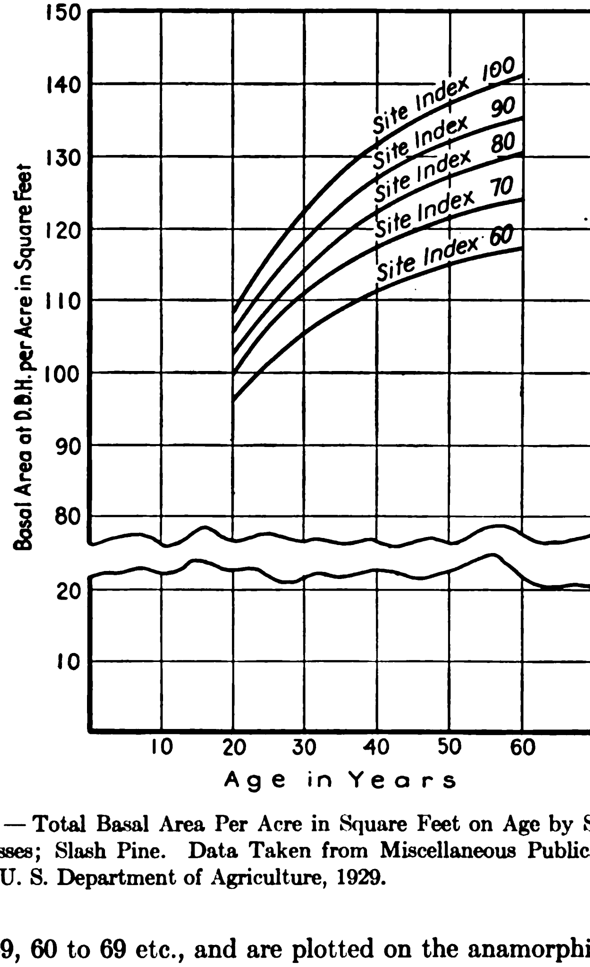


FIG. 84. — Total Basal Area Per Acre in Square Feet on Age by Site Index Classes; Slash Pine. Data Taken from Miscellaneous Publication No. 50, U. S. Department of Agriculture, 1929.

50 to 59, 60 to 69 etc., and are plotted on the anamorphic graph. Through each set of points, straight lines radiating from the origin are drawn, care being taken to balance plus and minus deviation of the points from their corresponding lines. There will be as many of these lines as there are site index values.

The proper spacing of these curves* on the graph is accomplished by means of " intercept " curves.

The entire plot data is grouped in 10 foot site index classes. For each plot the total basal area as read from the graduating curve at the age corresponding to that of the plot is compared with the actual total basal area of the plot, and a percentage ratio of curved values into actual values is obtained. These percentage ratios are then plotted over site index value and a smooth even curve obtained. By applying the percentages from the intercept curves to the group a final series in proper spacing is drawn showing total basal area per acre on age in conventional form (Fig. 84).

The total basal area per acre for each plot, each age and site class is then compared to the total basal area per acre, as read from the curves for that age and site quality. All plots which exceed twice the standard error should be rejected. However where standards of normality have not been uniform or consistently maintained, this may permit too liberal a degree of retention and it may be advisable to fall back on Bruce's† more precise standard of rejection based on the average deviation.

185. Alinement Chart Yield Tables. — Yield tables can be compiled from alinement charts constructed on principles discussed in Chapter X. If the data measured for the construction of normal yield tables area ssembled on the basis of the fully stocked acre and scaled as the graduations on a series of parallel axes as in Fig. 85, there will be offered ready and easy means for the computation of yield. The availability of such a chart depends upon the assumption that all values so graphed represent for the various sites constant percentages of average conditions for all ages.

On the left of the chart a series of six equispaced scales for age represent per acre values for volume (in cubic feet), basal area, total number of trees, average D.B.H., average height for the entire stand, and average height of the dominant trees. Similar values are scaled for site index on a set of six scales at the right. A central axis midway between the two groups carries average

* Though drawn as straight lines they are really representatives of curves, that is, are amorphosed curves.

† Bruce, though advising that all plots which exceed twice the standard deviation (S.D. = 1.25 A.D.) should be rejected, also suggests that all plots falling close to the line should be further scrutinized before final rejection.

D.B.H. in inches on one side and volume (cubic feet) on the other.

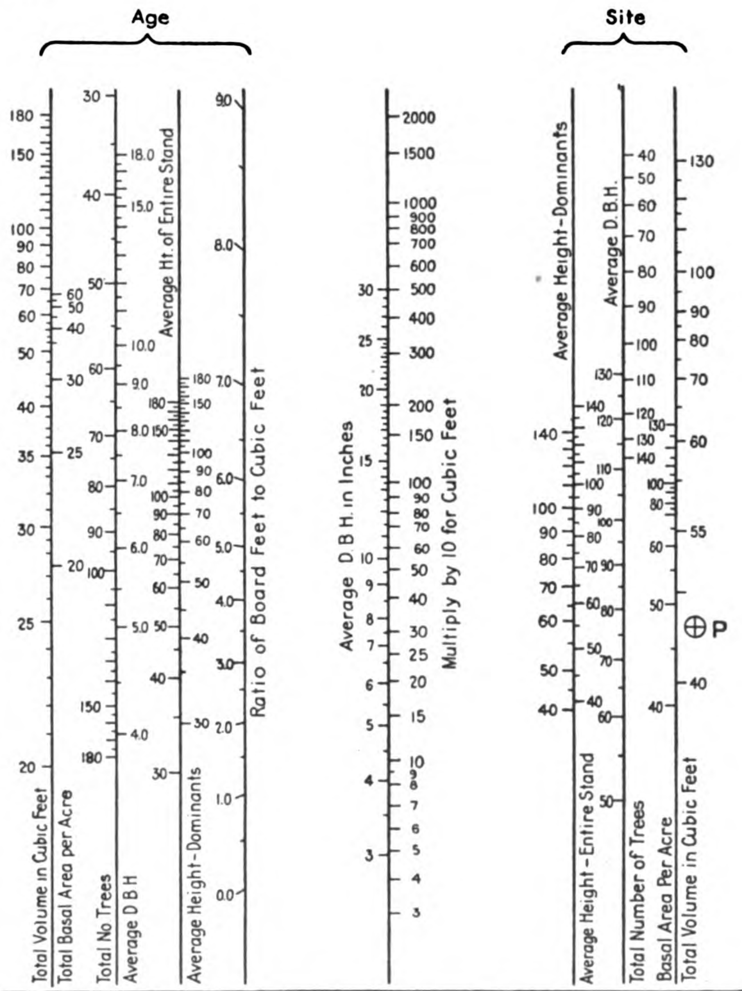


FIG. 85. — Alinement Chart Yield Table for Western Yellow Pine (After Behre).

By joining or alining the proper age point on a given scale at the left with the proper site index point on the corresponding scale at the right, values may read on the central axis. Readings on the central scale must be multiplied by 10, for cubic foot volumes.

Board foot values are obtained by pivoting on the point marked at the lower right, *P*, so as to aline the average D.B.H. as scaled on the central axis to obtain a proper cubic foot-board foot conversion factor as scaled on a supplemental central axis. This conversion factor multiplied by the cubic foot volumes will give board foot volumes.

186. Adaptation of Yield Tables to Local Use. — Even when site quality is accurately identified, it is advisable to check yield tables for local use by the actual measurement of some 20 to 25 plots. A more serious discrepancy will be found when the local standards of utilization do not conform to those under which the table was originally constructed. To compensate for such difference proceed as follows:

Examine the stand table both on an absolute and percentage basis for the entire stand. Determine the number of trees whose present size class shows that they will be affected by the proposed change. Deduct (or add) their volume from the volume yield for that age and site per acre. Determine by measurement and comparison the reduced average difference in yield per tree under the changed standard of utilization on the trees that are left available for cutting. Multiply this by the number of trees left and deduct from the tabular yield.

Suppose, for example, the tabular yield for a 55 year stand on an 80 foot site of second growth slash pine is 26,000 board feet. Yield table construction is based on utilization to a 5 inch top which presupposes the idea that any trees capable of yielding one 16 foot log with a 5 inch top were included in the yield computations. When a 7 inch tree is considered the smallest capable of so doing, it means that the computations were based on all trees in the stand that have 6.6 inches D.B.H. and over. The total number of trees per acre is 258 and the average diameter is 11.4 inches. Suppose a standard of utilization demanding an 8 inch top d.i.b. Obviously the smallest tree capable of carrying one 16 foot log with an 8 inch top will probably have 10 inches D.B.H. This means a lessened yield from a stand of this age, the exact amount of which is determined by the solution of two factors:

(*a*) The reduction in yield occasioned by the exclusion of volume now found on trees 7, 8, and 9 inches breast high.

(*b*) The reduction in yield per tree occasioned by cutting to an 8 inch rather than a 5 inch top.

TABLE XXI*

YIELD TABLE IN BOARD FEET PER ACRE, SLASH PINE

Dominant Stand

International (¼ inch Kerf) Rule

Age, Years	Yields Per Acre, Board Feet				
	Site Index in Feet				
	60	70	80	90	100
15		700	1,500	3,100	5,400
20	500	2,000	4,000	6,500	9,000
25	2,000	4,500	7,500	12,000	15,000
30	4,000	7,500	12,000	16,500	20,500
35	6,000	11,000	16,000	20,500	25,500
40	8,500	14,000	19,000	24,000	30,000
45	10,500	16,000	22,000	27,500	33,500
50	12,000	18,000	24,000	30,500	36,000
55	13,500	19,500	26,000	32,500	38,500
60	14,000	21,000	27,500	34,500	40,500

* From Miscellaneous Publication 50, Volume, Yield, and Stand Tables for Second Growth Southern Pines, U. S. Dept. Agr., Washington, D. C., Sept. 1929.

Examination of the stand table shows that 70 per cent of the stand or 180 trees are of 10 inches D.B.H. and over, and that 15 per cent or 39 trees are to be found in the 7, 8, and 9 inch classes. These 39 trees by reference to the volume table carry a volume for the age and site of 1,760 board feet.

The lessened volume on the 180 trees which are 10 inches and over, due to the cutting of an 8 inch rather than a 5 inch top, by actual inspection and checking on trees of average diameter among them, averages 10 board feet per tree. This when multiplied by the 180 trees gives a total of 1,800 board feet. The total of 1,760 + 1,800 or 3,560 board feet is then deducted from the tabular yield of 26,100 to give 22,540 board feet as the yield for an 80 foot 60 year old stand under the changed standard of utilization.

187. Yield Table Stand Graphs and Stand Tables. — The application of yield tables to specific stands is made easier when

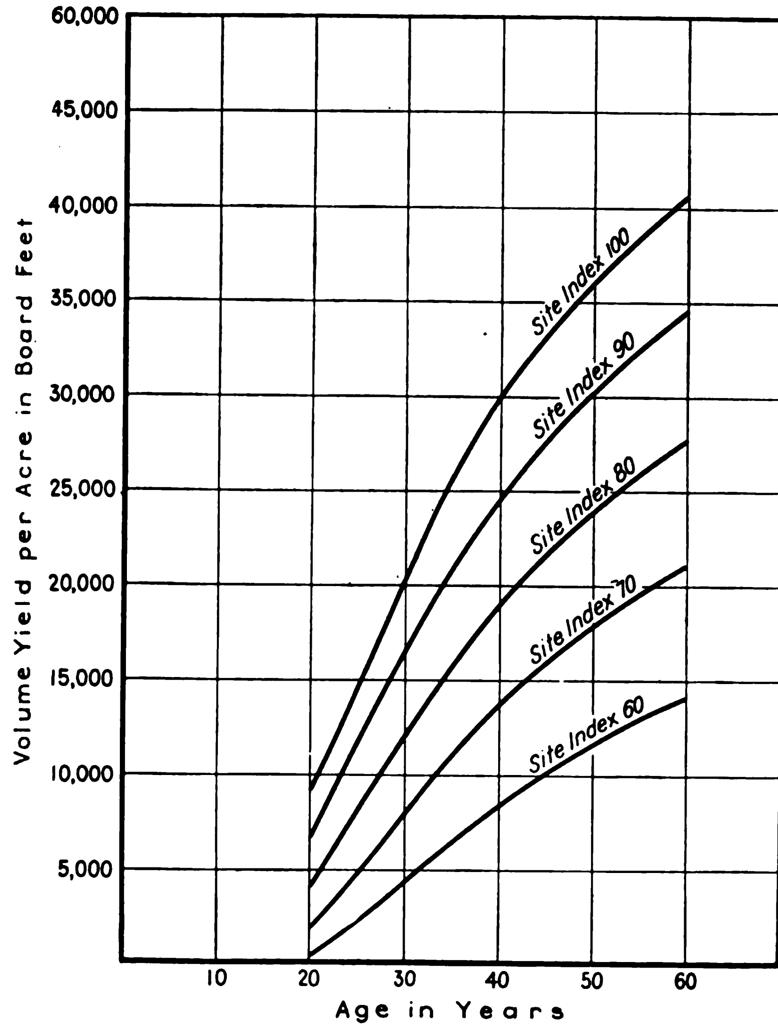


FIG. 86. — Normal Yield Table in Board Feet for Slash Pine. Data Taken from Miscellaneous Publication No. 50, U. S. Department of Agriculture, 1919.

there is available some definite information, not only in regard to the range of diameters and sizes, but also to the number of trees in each size class. This information is best represented in a stand table, which, however, differs considerably in form and application from that previously described in Section 177.

Investigation* of the distribution of size classes (diameters) in even aged stands has established the fact that in their distribution they approximate closely the bell-shaped form exemplified in a " normal " frequency curve, (Fig. 36) and that with the determination of a " coefficient of variability " for the species, the frequency, that is, the number of items, at any point in the curve may be determined by tables of mathematical† computation. If frequency coördinate paper is taken, which is so graduated that the frequencies are cumulatively expressed, the frequency distribution will be shown as a straight line, and from it may be read the *relative* distribution above and below any given point.

Following the methods suggested by Bruce‡ and Reineke§ further developed by Behre‖ and Meyer,¶ these may be developed directly from the field data. Consideration is restricted to those plots which show easily identified normality of purity and density. That is, only plots are considered which are practically 100 per cent pure and which show all the uniformities of a fully stocked stand (no large holes in the canopy, a minimum of undersized or suppressed trees and no exceptionally large diametered trees.

These plots are collected by size classes based on the D.B.H. of the average trees. This establishes a series of groups each containing a range of diameter classes. Mathematical averaging establishes in each group the number of trees within each of these diameter classes. Cumulative sums are then built up for the full range of diameters present, and are reduced to corresponding percentage values based on the group total. These percentage

* Notes on the Composition of Even-Aged Stands by F. S. Baker, Jour. of For., Vol. XXI, No. 7, Nov. 1923, p. 712.

† $y = \frac{n}{\alpha\sqrt{2\pi}} \cdot \frac{1}{e^{x^2}/2\alpha^2}$

‡ Donald Bruce, *op. cit.*, No. 21.

§ L. H. Reineke, *op. cit.*, No. 58.

‖ C. E. Behre, *op. cit.*, No. 14.

¶ Walter H. Meyer, Yields of Second Growth Spruce and Fir in the Northeast. Tech. Bull. 142, U. S. Dept. Agr., Washington, D. C., November, 1929.

TABLE XXII

NUMBER OF TREES AS DISTRIBUTED THROUGH THE DIAMETER CLASSES IN A 70 YEAR 70 FOOT SITE INDEX STAND OF WESTERN YELLOW PINE

Containing 362 Stems Per Acre when the Diameter of the Average Tree is 9 Inches

D.B.H. Class in Inches	Percentage of Trees Included within D.B.H. and all Smaller Diameter Classes	Percentage of Trees within Each D.B.H. Class	Number of Trees within Each D.B.H. Class
16	100.00	0.03	0.11
15	99.97	0.07	0.25
14	99.90	0.27	0.98
13	99.70	0.83	3.00
12	98.87	2.57	9.31
11	96.30	5.80	21.00
10	90.50	10.50	39.71
9	80.00	17.50	63.11
8	62.50	20.00	72.00
7	42.50	20.75	74.34
6	21.75	12.75	45.51
5	9.00	6.00	21.81
4	3.00	2.35	8.51
3	0.65	0.53	1.93
2	0.12	0.12	0.43
1	0.00		

values are plotted on arithmetic probability paper, the points being connected by straight lines. Readings of diameter values at arbitrarily selected percentage points such as 99.9, 99, 95, 90, 80, 70, 60, 50, 10 and 2 derive a set of values of average diameter of the stand on diameter class. These derived percentage values are then plotted on rectangular coördinate paper, average diameter of the stand being accepted as the independent variable and scaled on the abscissa axis. Straight lines are drawn. One through all of the 99.9 per cent points, one each through the 99 per cent points, the 95 per cent points, and so on. From these set of curves, values are read which are now plotted on logarithmic probability paper as in Fig. 87. It will be found advisable to expand the diameter class scale in this last chart to include a larger range of diameter class values.

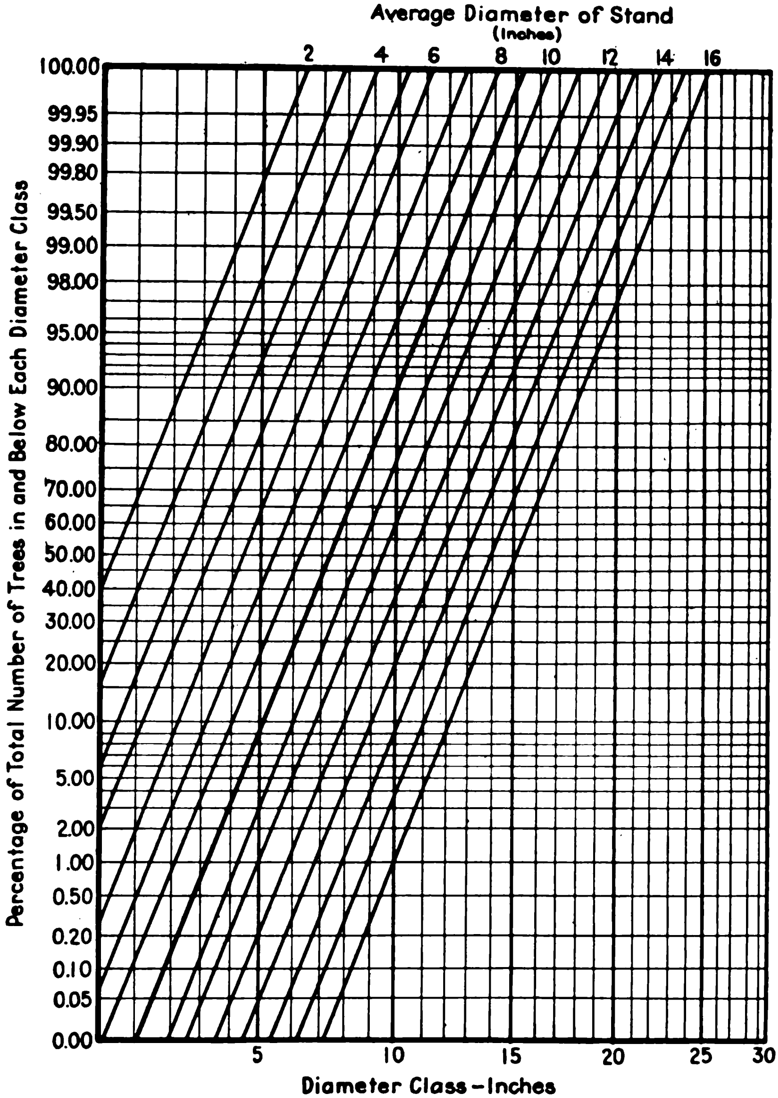


FIG. 87. — Stand Graph for 70 Year 70 Foot Site Index Stand of Western Yellow Pine with an Average Diameter of 9 Inches.

The bottom horizontal scale of the chart represents Diameter Class in Inches. The top horizontal scale shows the average diameter of the stand. The vertical scale indicates percentages of the total number of trees. The diagonals represent the distribution of the average D.B.H. of the trees in dimension classes ranging from 2 to 16 inches. The point at which a diagonal intersects a diameter class ordinate shows on the percentage scale the percentage of the total number of trees included in that and all smaller diameter classes. Thus a stand of given site and age class shows an average D.B.H. of 9 inches. There are 362 trees per acre for this age and site. By following the diagonal downward, are derived the data of Table XXII. A complete stand table for slash pine constructed on a similar percentage basis is presented as Table XXIII.

TABLE XXIII*

STAND TABLE ON A PERCENTAGE BASIS, SLASH PINE

D.B.H. (inches) of the Average Tree	Percentages of all Trees in Given D.B.H. Class								
	2 Inches	4 Inches	6 Inches	8 Inches	10 Inches	12 Inches	14 Inches	16 Inches	18 Inches
3	100	28	2						
4	100	56	13	1					
5	100	74	32	6					
6	100	86	53	19	3				
7	100	92	69	35	10	1			
8	100	96	81	52	20	5	1		
9	100	98	89	66	35	12	2		
10	100	99	95	80	51	22	6	1	
11		100	98	89	66	36	13	3	
12		100	99	94	79	52	22	6	1
13			100	98	89	68	36	13	3

* From Miscellaneous Publication 50, Volume, Yield and Stand Tables for Second Growth Southern Pines, U. S. Dept. Agr., Washington, D. C., Sept. 1929.

188. Normal and Empirical Yield. — The construction of yield tables, with their insistence on uniformity, particularly of density and composition, represents a set of normal or ideal conditions rarely found in nature. The tendency of natural forests is away from normality. Consequently, the values in a normal yield table should never be applied to a stand until its nearness to, or departure from, normality has been reliably ascertained, and the values of the normal table have been discounted or reduced by a proper factor to the value which is in direct proportion to the relative normality of the actual stand.

The value of the reducing factor or index of normality is the ratio between the volume of the actual stand and the volume of the normal stand in terms of the yield at the same age. Volume per acre is a relatively difficult thing to determine and will not be preferred if a simpler substitute is available. The best is the expression of the square of the diameter as found in basal area. The ratio existing between the total basal area per acre of the actual stand and the tabulated per acre basal area for the same age and site quality, as taken from a normal yield table, determines the value of a discount factor or a reducing factor which, applied to the per acre yields of the normal table, reduces them in amount proportional to the conditions of the actual forest.

An actual stand of 40 year loblolly pine in North Carolina shows from a tally of all the trees a total basal area of 206 square feet. A normal yield table for the same species and site, and at the same age of 40 years, tabulates 244 square feet. The value of the reducing factor as an index of normality is the ratio of the actual yield to the normal yield for that particular age site, or 206 divided by 244, or 0.84. The normal yield should then be discounted to 84 per cent of its value, and this figure can be used for predicting yields over rather limited periods of time not to exceed 10, or at the most 20 years; for the ages then attained. Such yields are known as *empirical* rather than normal yields.

189. Yields in Mixed Even Aged Forests. — Natural forests are rarely pure but consist of two or more species in varying proportions. A stand whose composition shows 80 per cent of one species is generally regarded as a pure stand. When even aged stands consist of two or more species in proportions of sufficient significance to deserve the term " mixed " there are two distinct methods of handling the situation.

Yield tables are prepared for pure, fully stocked stands of each species. Discount these tables to correspond with the percentage value of each species in the mixture. The sum of the discounted yields per acre for the year specified gives the yield desired. The limitation of this method lies in the fact that composition varies with age; and in using such a yield table to predict yields, some knowledge must be in hand as to the degree and effect of future variations in composition.

Another method disregards the variation in composition as expressed by the mixture percentages, and meets the problem by a descriptive item such as "Mixed Hardwoods." Yield tables are then prepared for the mixed stands on the same basis as if they were pure.*

190. Yields in Pure All Aged Forests. — When the forest consists of several age classes, the problem of determining yields is not so easy to solve. Each age class exerts an effect upon the total volume per acre in proportion to its age and development. The difficulty lies in the proper identification of the age class as a group within the stand on one hand, and the correct appraisal of its influence on the stand as a whole on the other. The desired goal is yields per acre at different ages, tabulated in regular form. The steps to accomplish this are brief:

Sample plots (sample areas) of convenient size are established in pure all aged stands. Particular care is to be taken to maintain uniform composition and site within the plot and to note that the stands are as fully stocked as possible. On each sample area obtain a full tally of all the trees by D.B.H. classes. The tally should include all trees down to 2 inch D.B.H.o.b. Seedlings and saplings are to be studied on smaller sample plots within the area. All values are subsequently reduced to terms of the unit acre. The purpose of this step is to obtain data which will show for the area the average number of trees per acre within each size class.

The average area or growing space occupied by a tree of each size class is assumed to be in direct proportion to its crown spread. The usual procedure is to accept the horizontal projection of tree crowns to be roughly circular in shape, and to measure *one diameter only*. This diameter value is squared† and the result is accepted as the area of the crown spread. From 10 to 20 trees within each D.B.H. class are measured and the average obtained for the class.

The average values are then plotted on cross-section paper.

* See "Growth Study and Normal Yield Tables for Second Growth Hardwoods," by J. Nelson Spaeth, Harvard Forest Bulletin 1, Cambridge, Mass., 1921.

† If tree crowns were perfect circles and were continually touching one another, undoubtedly preference would be given to $A = \frac{\pi D^2}{4}$ as the expression of area. Crowns are never circular and never wholly contiguous hence $A = D^2$ is preferred.

"Crown Spread Area in Square Feet" is the ordinate and "D.B.H.o.b. in inches" is the abscissa. A smooth, even curve is drawn to eliminate irregularities. The results tabulated by D.B.H. classes are accepted as indexes of the *relative growing space* required by each D.B.H. class.

The average age of each D.B.H. class is obtained. This may be done by making a study of diameter growth on from 50 to 100 trees of the species and by casting the results into a curve of diameter growth on age. Stump d.i.b. values must be correlated to D.B.H.o.b. values.

Each D.B.H. class is assumed to be an age class. Within each such age class the following are determined:

I. *Volume.* — The volume within the age class is computed from standard volume tables for the species.

II. *Age.* — The average for each diameter class equals the age as taken from the diameter growth age curve.

III. *Area.* — The area occupied by each age class on the average acre is computed by multiplying the average curved crown spread area for the diameter class by the corresponding number of trees from the stand table. The area should be stated in acres or decimals of an acre.

The volume of each age class (D.B.H. class) is reduced to terms of the unit acre. With "Volume per Acre" as the ordinate, and "Age in Years" the abscissa, a curve is plotted. The smooth curve eliminates irregularity through the plotted points and is read at decade intervals. These decade interval values are tabulated. A separate, complete calculation is made for each site quality.

The method is not without its limitations when applied to predicting yields in actual stands. As tabulated, the table shows conditions in fully stocked stands of even age. In application, it is used with stands which are not fully stocked and not even aged. Dependence on crown spread as an index of area is not safe since the present area which is being occupied by the age class is relative only, and does not indicate how much more area has been occupied in its past development. At best, the method seems to establish an index of relativity rather than a standard of actuality. Values in the table must be discounted by a percentage proportional to the deviation of the actual forest from the standards established by the table. Even then, it is somewhat speculative as compared with the reliability of measurements based on pure even aged sample plots.

191. Yields in Mixed All Aged Forests. — This is the type of forest which consists of several more or less tolerant species growing together on the same site, with all ages and all sizes, and varying degrees of composition represented in the mixture. As stated previously, it sometimes is spoken of as a "selection forest," although, strictly speaking, it does not meet all the requirements of the definition.

The construction of a yield table for an all aged selection forest is an impossibility, and the growth or yields cannot be predicted by any known yield tables. The varying character, behavior, and reaction of several associated species to the factors of the site, the effect of individual increase in size and age, is so complicated as to render impossible a proper weighting of their relative importance and an accurate method of their elimination.

There is only one way to study growth and yield in all aged mixed stands; that is by the establishment of permanent sample plots under conditions where all items of past history are ascertained and where all details of future development can be carefully observed and accurately measured. These plots may be of any size from 1 to 40 acres, or less, or greater, according to the uniformity of the stand, and the time and methods available for their establishment and measurement. They may be irregular or rectangular in shape. Whatever their size or shape, all measurements will eventually be reduced to terms of the unit acre.

The most convenient size and shape is from $\frac{1}{10}$ to 1.0 acres square or rectangular, granted that the conditions of uniformity in form, density, composition, quality, and character are maintained within its boundaries. A noted variation in density, composition, character, or form is reason enough for a smaller plot, or for establishing a new plot. The plot must be carefully posted with permanent corner stakes, and must be capable of being easily located again after an interval of 5 or even 10 years.

On the sample plot every living tree within the calipers, that is, every tree which has a height of more than 4.5 feet and a diameter greater than 0.5 inch, is accurately located on a map of the plot, given a serial number and tallied. This serial number may be stamped on an aluminum tag which is attached to the tree, preferably at or near the D.B.H. point. Another method is to paint the number on the tree above a painted + exactly at the D.B.H. point. This works all right with large trees, but is scarcely applicable in the smaller sizes. The purpose of so carefully noting the D.B.H. point on each tree is to establish a standard point, not only for this first measurement but for all subsequent measurements.

For trees below the calipers, that is, smaller than 4.5 feet high, reproduction counts are made on smaller sample plots, such as a square rod. From 8 to 16 of these reproduction plots per acre are taken, their average determined and reduced to terms of the unit acre.

In addition, height studies correlated with diameter are made, the light intensity is measured, and accurate data are collected regarding all other vegetation, woody and herbaceous, which may be on the ground. Photographs are taken, carefully oriented from predetermined points accurately located by stakes, so that in later measurements photographs with exactly the same background will show the relative developments.

The usual period between two measurements on an established permanent sample plot is 5 years, but it may be as long as 10 years. In consequence, a long period of years must elapse before any significant conclusions can be drawn from the several records of measurements. Yet it is the only reliable method of studying growth and determining yields in stands of this character.

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* Indicates those which have been freely consulted.

APPENDIX

TABLES USEFUL IN FOREST MEASUREMENT

TABLE XXIV

AREA OF CIRCLES IN SQUARE FEET

Diameter	Tenths of Inches									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	AREA — SQUARE FEET									
Inches										
1	0.006	0.007	0.008	0.009	0.011	0.012	0.014	0.016	0.018	0.020
2	.022	.024	.026	.029	.031	.034	.037	.040	.043	.046
3	.049	.052	.056	.059	.063	.067	.071	.075	.079	.083
4	.087	.092	.096	.101	.106	.111	.115	.121	.126	.131
5	.136	.142	.147	.153	.159	.165	.171	.177	.184	.190
6	.196	.203	.210	.216	.223	.230	.238	.245	.252	.260
7	.267	.275	.283	.291	.299	.307	.315	.323	.332	.340
8	.349	.358	.367	.376	.385	.394	.403	.413	.422	.432
9	.442	.452	.462	.472	.482	.492	.503	.513	.524	.535
10	.545	.556	.568	.579	.590	.601	.613	.625	.636	.648
11	.660	.672	.684	.697	.709	.721	.734	.747	.760	.772
12	.785	.799	.812	.825	.839	.852	.866	.880	.894	.908
13	.922	.936	.950	.965	.979	.994	1.009	1.024	1.039	1.054
14	1.069	1.084	1.100	1.115	1.131	1.147	1.163	1.179	1.195	1.211
15	1.227	1.244	1.260	1.277	1.294	1.310	1.327	1.344	1.362	1.379
16	1.396	1.414	1.431	1.449	1.467	1.485	1.503	1.521	1.539	1.558
17	1.576	1.595	1.614	1.632	1.651	1.670	1.689	1.709	1.728	1.748
18	1.767	1.787	1.807	1.827	1.847	1.867	1.887	1.907	1.928	1.948
19	1.969	1.990	2.011	2.032	2.053	2.074	2.095	2.117	2.138	2.160
20	2.181	2.204	2.226	2.248	2.270	2.292	2.315	2.337	2.360	2.383
21	2.405	2.428	2.451	2.475	2.498	2.521	2.545	2.568	2.592	2.616
22	2.640	2.664	2.688	2.712	2.737	2.761	2.786	2.810	2.835	2.860
23	2.885	2.910	2.936	2.961	2.986	3.012	3.038	3.064	3.089	3.115
24	3.142	3.168	3.194	3.221	3.247	3.275	3.301	3.328	3.355	3.382

Diameter	Area	Diameter	Area	Diameter	Area	Diameter	Area	Diameter	Area
Inches	*Sq. ft.*	*Inches*	*Sq. ft.*	*Inches*	*Sq. ft.*	*Inches*	*Sq. ft.*	*Inches*	*Sq. ft.*
25	3.41	32	5.59	39	8.30	46	11.54	53	15.32
26	3.69	33	5.94	40	8.73	47	12.05	54	15.90
27	3.98	34	6.30	41	9.17	48	12.57	55	16.50
28	4.28	35	6.68	42	9.62	49	13.10	56	17.10
29	4.59	36	7.07	43	10.08	50	13.64	57	17.72
30	4.91	37	7.47	44	10.56	51	14.19	58	18.35
31	5.24	38	7.88	45	11.04	52	14.75	59	18.99

TABLE XXV

VALUES FOR SCHIFFEL'S FORMULA FOR CUBIC VOLUMES OF ENTIRE STEMS

Diameter, Inches	0.16 of the Area of a Circle at Breast Height (0.16B)									
	0.0 Sq. ft.	0.1 Sq. ft.	0.2 Sq. ft.	0.3 Sq. ft.	0.4 Sq. ft.	0.5 Sq. ft.	0.6 Sq. ft.	0.7 Sq. ft.	0.8 Sq. ft.	0.9 Sq. ft.
1	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003
2	.003	.004	.004	.005	.005	.005	.006	.006	.007	.007
3	.008	.008	.009	.010	.010	.011	.011	.012	.013	.013
4	.014	.015	.015	.016	.017	.018	.018	.019	.020	.021
5	.022	.023	.024	.025	.025	.026	.027	.028	.029	.030
6	.031	.032	.034	.035	.036	.037	.038	.039	.040	.042
7	.043	.044	.045	.047	.048	.049	.050	.052	.053	.054
8	.056	.057	.059	.060	.062	.063	.065	.066	.068	.069
9	.071	.072	.074	.075	.077	.079	.080	.082	.084	.086
10	.087	.089	.091	.093	.094	.096	.098	.100	.102	.104
11	.106	.108	.109	.111	.113	.115	.117	.119	.122	.124
12	.126	.128	.130	.132	.134	.136	.139	.141	.143	.145
13	.147	.150	.152	.154	.157	.159	.161	.164	.166	.169
14	.171	.173	.176	.178	.181	.183	.186	.189	.191	.194
15	.196	.199	.202	.204	.207	.210	.212	.215	.218	.221
16	.223	.226	.229	.232	.235	.238	.240	.243	.246	.249
17	.252	.255	.258	.261	.264	.267	.270	.273	.276	.280
18	.283	.286	.289	.292	.295	.299	.302	.305	.308	.312
19	.351	.318	.322	.325	.328	.332	.335	.339	.342	.346
20	.349	.353	.356	.360	.363	.367	.370	.374	.378	.381
21	.385	.389	.392	.396	.400	.403	.407	.411	.415	.419
22	.422	.426	.430	.434	.438	.442	.446	.450	.454	.458
23	.462	.466	.470	.474	.478	.482	.486	.490	.494	.498
24	.503	.507	.511	.515	.520	.524	.528	.532	.537	.541
25	.545	.550	.554	.559	.563	.567	.572	.576	.581	.585
26	.590	.594	.599	.604	.608	.613	.617	.622	.627	.631
27	.636	.641	.646	.650	.655	.660	.665	.670	674	.679
28	.684	.689	.694	.699	.704	.709	.714	.719	.724	.729
29	.734	.739	.744	.749	.754	.759	.765	.770	.775	.780
30	.785	.791	.796	.801	.806	.812	.817	.822	.828	.833
31	.839	.844	.849	.855	.860	.866	.871	.877	.882	.888
32	.894	.899	.905	.910	.916	.922	.927	.933	.939	.945
33	.950	.956	.962	.968	.974	.979	.985	.991	.997	1.003
34	1.009	1.015	1.021	1.027	1.033	1.039	1.045	1.051	1.057	1.063
35	1.069	1.075	1.081	1.087	1.094	1.100	1.106	1.112	1.118	1.125

TABLE XXV — *Continued*

VALUES FOR SCHIFFEL'S FORMULA FOR CUBIC VOLUMES OF ENTIRE STEMS

Diameter, Inches	0.16 of the Area of a Circle at Breast Height (0.16B)									
	0.0 Sq. ft.	0.1 Sq. ft.	0.2 Sq. ft.	0.3 Sq. ft.	0.4 Sq. ft.	0.5 Sq. ft.	0.6 Sq. ft.	0.7 Sq. ft.	0.8 Sq. ft.	0.9 Sq. ft.
36	1.131	1.137	1.114	1.105	1.156	1.163	1.169	1.175	1.182	1.188
37	1.195	1.201	1.208	1.214	1.221	1.227	1.234	1.240	1.247	1.254
38	1.260	1.267	1.273	1.280	1.287	1.294	1.300	1.307	1.314	1.321
39	1.327	1.334	1.341	1.348	1.355	1.362	1.368	1.375	1.382	1.386
40	1.396	1.403	1.410	1.417	1.424	1.431	1.438	1.446	1.453	1.460
41	1.467	1.474	1.481	1.488	1.496	1.503	1.510	1.517	1.525	1.532
42	1.539	1.547	1.554	1.561	1.569	1.576	1.584	1.591	1.599	1.606
43	1.614	1.621	1.629	1.636	1.644	1.651	1.659	1.667	1.674	1.682
44	1.689	1.697	1.705	1.713	1.720	1.728	1.736	1.744	1.751	1.759
45	1.767	1.775	1.783	1.791	1.799	1.807	1.815	1.823	1.831	1.839
46	1.847	1.855	1.863	1.871	1.879	1.887	1.895	1.903	1.911	1.920
47	1.928	1.936	1.944	1.952	1.961	1.969	1.977	1.986	1.994	2.002
48	2.011	2.019	2.027	2.037	2.044	2.053	2.061	2.070	2.078	2.087
49	2.095	2.104	2.112	2.121	2.130	2.138	2.147	2.156	2.164	2.173
50	2.182	2.190	2.199	2.208	2.217	2.226	2.234	2.243	2.252	2.261
51	2.270	2.279	2.288	2.297	2.306	2.315	2.324	2.333	2.342	2.351
52	2.360	2.369	2.378	2.387	2.396	2.405	2.414	2.424	2.433	2.442
53	2.451	2.461	2.470	2.479	2.488	2.498	2.507	2.516	2.526	2.535
54	2.545	2.554	2.564	2.573	2.583	2.592	2.602	2.611	2.621	2.630
55	2.640	2.649	2.659	2.669	2.678	2.688	2.698	2.707	2.717	2.727
56	2.737	2.746	2.756	2.766	2.776	2.786	2.796	2.806	2.815	2.825
57	2.835	2.845	2.855	2.865	2.875	2.885	2.898	2.905	2.915	2.926
58	2.936	2.946	2.956	2.966	2.976	2.986	2.997	3.007	3.017	3.027
59	3.038	3.048	3.058	3.069	3.079	3.089	3.100	3.110	3.121	3.131
60	3.142	3.125	3.163	3.173	3.184	3.194	3.205	3.215	3.226	3.237
61	3.247	3.258	3.269	3.279	3.290	3.301	3.311	3.322	3.333	3.344
62	3.355	3.365	3.376	3.387	3.398	3.409	3.420	3.431	3.442	3.453
63	3.464	3.475	3.486	3.497	3.508	3.519	3.530	3.541	3.552	3.563
64	3.574	3.586	3.597	3.608	3.619	3.630	3.642	3.653	3.664	3.676
65	3.687	3.698	3.710	3.721	3.733	3.744	3.755	3.767	3.778	3.790
66	3.801	3.813	3.824	3.836	3.848	3.859	3.871	3.882	3.894	3.906
67	3.917	3.929	3.941	3.953	3.964	3.976	3.988	4.000	4.012	4.023
68	4.035	4.047	4.059	4.071	4.083	4.095	4.107	4.119	4.131	4.143
69	4.155	4.167	4.179	4.191	4.203	4.215	4.227	4.239	4.252	4.264
70	4.276	4.288	4.301	4.313	4.325	4.337	4.350	4.362	4.374	4.387

TABLE XXV — *Continued*

VALUES FOR SCHIFFEL'S FORMULA FOR CUBIC VOLUMES OF ENTIRE STEMS

Diameter, Inches	0.16 of the Area of a Circle at Breast Height (0.16B)									
	0.0 Sq. ft.	0.1 Sq. ft.	0.2 Sq. ft.	0.3 Sq. ft.	0.4 Sq. ft.	0.5 Sq. ft.	0.6 Sq. ft.	0.7 Sq. ft.	0.8 Sq. ft.	0.9 Sq. ft.
71	4.399	4.412	4.424	4.436	4.449	4.461	4.474	4.486	4.499	4.511
72	4.524	4.536	4.549	4.562	4.574	4.587	4.600	4.612	4.625	4.638
73	4.650	4.663	4.676	4.689	4.702	4.714	4.727	4.740	4.753	4.766
74	4.779	4.792	4.805	4.818	4.831	4.844	4.857	4.870	4.883	4.896
75	4.909	4.922	4.935	4.948	4.961	4.975	4.988	5.001	5.014	5.027
76	5.041	5.054	5.067	5.080	5.094	5.107	5.120	5.134	5.147	5.161
77	5.174	5.187	5.201	5.214	5.228	5.241	5.255	5.269	5.2 2	5.296
78	5.309	5.323	5.337	5.350	5.364	5.378	5.391	5.405	5.419	5.433
79	5.446	5.460	5.474	5.488	5.502	5.515	5.529	5.543	5.557	5.571
80	5.585	5.599	5.613	5.627	5.641	5.655	5.669	5.683	5.697	5.711

TABLE XXV — *Continued*

VALUES FOR SCHIFFEL'S FORMULA FOR CUBIC VOLUMES OF ENTIRE STEMS

Diameter, Inches	0.66 of the Area of a Circle at the Middle Height of the Tree (0.66b)									
	0.0 Sq. ft.	0.1 Sq. ft.	0.2 Sq. ft.	0.3 Sq. ft.	0.4 Sq. ft.	0.5 Sq. ft.	0.6 Sq. ft.	0.7 Sq. ft.	0.8 Sq. ft.	0.9 Sq. ft.
1	0.004	0.004	0.005	0.006	0.007	0.008	0.009	0.010	0.012	0.013
2	.014	.016	.017	.019	.021	.023	.024	.026	.028	.030
3	.032	.035	.037	.039	.042	.044	.047	.049	.052	.055
4	.058	.061	.064	.067	.070	.073	.076	.080	.083	.086
5	.090	.094	.097	.101	.105	.109	.113	.117	.121	.125
6	.130	.134	.138	.143	.147	.152	.157	.162	.166	.171
7	.176	.182	.187	.192	.197	.202	.208	.213	.219	.225
8	.230	.236	.242	.248	.254	.260	.266	.273	.279	.285
9	.292	.298	.305	.311	.318	.325	.332	.339	.346	.353
10	.360	.367	.375	.382	.389	.397	.405	.412	.420	.428
11	.436	.444	.452	.460	.468	.476	.484	.493	.501	.510
12	.518	.527	.536	.545	.554	.563	.572	.581	.590	.599
13	.608	.618	.627	.637	.646	.656	.666	.676	.686	.696
14	.706	.716	.726	.736	.746	.757	.767	.778	.788	.799
15	.810	.821	.832	.843	.854	.865	.876	.887	.899	.910
16	.922	.933	.945	.956	.968	.980	.992	1.004	1.016	1.028
17	1.040	1.053	1.065	1.077	1.090	1.102	1.115	1.128	1.140	1.153
18	1.166	1.179	1.192	1.205	1.219	1.232	1.245	1.259	1.272	1.286
19	1.299	1.313	1.327	1.341	1.355	1.369	1.383	1.397	1.441	1.426
20	1.440	1.454	1.469	1.483	1.498	1.513	1.528	1.542	1.557	1.572
21	1.587	1.603	1.618	1.633	1.649	1.664	1.680	1.695	1.711	1.726
22	1.742	1.758	1.774	1.790	1.806	1.822	1.839	1.855	1.871	1.888
23	1.904	1.921	1.937	1.954	1.971	1.988	2.005	2.022	2.039	2.056
24	2.073	2.091	2.108	2.126	2.143	2.161	2.178	2.196	2.214	2.232
25	2.250	2.268	2.286	2.304	2.322	2.341	2.359	2.378	2.396	2.415

TABLE XXV — *Continued*

VALUES FOR SCHIFFEL'S FORMULA FOR CUBIC VOLUMES OF ENTIRE STEMS

Diameter, Inches	0.66 of the Area of a Circle at the Middle Height of the Tree (0.66b)									
	0.0 Sq. ft.	0.1 Sq. ft.	0.2 Sq. ft.	0.3 Sq. ft.	0.4 Sq. ft.	0.5 Sq. ft.	0.6 Sq. ft.	0.7 Sq. ft.	0.8 Sq. ft.	0.9 Sq. ft.
26	2.433	2.452	2.471	2.490	2.509	2.528	2.547	2.566	2.585	2.605
27	2.624	2.644	2.663	2.683	2.703	2.722	2.742	2.762	2.782	2.802
28	2.822	2.842	2.863	2.883	2.903	2.924	2.944	2.965	2.986	3.006
29	3.027	3.048	3.069	3.090	3.111	3.133	3.154	3.175	3.197	3.218
30	3.240	3.261	3.283	3.305	3.327	3.349	3.371	3.393	3.415	3.437
31	3.459	3.482	3.504	3.527	3.549	3.572	3.595	3.617	3.640	3.663
32	3.686	3.709	3.732	3.756	3.779	3.802	3.826	3.849	3.873	3.896
33	3.920	3.944	3.968	3.992	4.016	4.040	4.064	4.088	4.112	4.137
34	4.161	4.186	4.210	4.235	4.260	4.285	4.309	4.334	4.359	4.385
35	4.410	4.435	4.460	4.486	4.511	4.537	4.562	4.588	4.614	4.639
36	4.665	4.691	4.717	4.743	4.769	4.796	4.822	4.848	4.875	4.901
37	4.928	4.955	4.981	5.008	5.035	5.062	5.089	5.116	5.143	5.171
38	5.198	5.225	5.253	5.280	5.308	5.336	5.363	5.391	5.419	5.447
39	5.475	5.503	5.532	5.560	5.588	5.616	5.645	5.673	5.702	5.731
40	5.760	5.788	5.817	5.846	5.875	5.904	5.934	5.963	5.992	6.022
41	6.051	6.081	6.110	6.140	6.170	6.200	6.230	6.260	6.290	6.320
42	6.350	6.380	6.411	6.441	6.471	6.502	6.533	6.563	6.594	6.625
43	6.656	6.687	6.718	6.749	6.780	6.812	6.843	6.874	6.906	6.937
44	6.969	7.001	7.033	7.064	7.096	7.128	7.160	7.193	7.225	7.257
45	7.290	7.322	7.354	7.387	7.420	7.452	7.485	7.518	7.551	7.584
46	7.617	7.650	7.683	7.717	7.750	7.784	7.817	7.851	7.884	7.918
47	7.952	7.986	8.020	8.054	8.088	8.122	8.156	8.190	8.225	8.259
48	8.294	8.328	8.363	8.404	8.433	8.467	8.205	8.537	8.573	8.608
49	8.643	8.678	8.714	8.749	8.785	8.820	8.856	8.892	8.927	8.963
50	8.999	9.035	9.072	9.108	9.144	9.180	9.217	9.253	9.290	9.326

TABLE XXVI

DOYLE LOG RULE

Top diameter — Inches	Contents in Board Feet — Length of log in feet: 6	8	10	12	14	16
6	1.5	2.0	2.5	3.0	3.5	4
7	3.4	4.5	5.6	6.8	7.9	9
8	6	8	10	12	14	16
9	9	12	16	19	22	25
10	13	18	22	27	31	36
11	18	24	31	37	43	49
12	24	32	40	48	56	64
13	30	40	51	61	71	81
14	37	50	62	75	87	100
15	45	60	76	91	106	121
16	54	72	90	108	126	144
17	63	84	106	127	148	169
18	73	98	122	147	171	196
19	84	112	141	169	197	225
20	96	128	160	192	224	256
21	108	144	181	217	253	289
22	121	162	202	243	283	324
23	135	180	226	271	316	361
24	150	200	250	300	350	400
25	165	220	276	331	386	441
26	181	242	302	363	423	484
27	198	264	331	397	463	529
28	216	288	360	432	504	576
29	234	312	391	469	547	625
30	253	338	422	507	591	676
31	273	364	456	547	638	729
32	294	392	490	588	686	784
33	315	420	526	631	736	841
34	337	450	562	675	787	900
35	359	480	601	721	841	961
36	383	512	640	768	896	1024
37	408	544	681	817	953	1089
38	433	578	722	867	1011	1156
39	459	612	766	919	1072	1225
40	486	648	810	972	1134	1296

TABLE XXVII
Scribner Log Rule*

Top Diameter, Inches	Volume in Board Feet					
	12 feet long	14 feet long	16 feet long	18 feet long	20 feet long	22 feet long
6	12	14	18	22	24	28
7	18	24	28	32	34	38
8	24	28	32	40	44	48
9	30	35	40	45	50	55
10	40	45	50	55	65	70
11	50	55	65	70	80	90
12	59	69	79	88	98	108
13	73	85	97	109	122	134
14	86	100	114	129	143	157
15	107	125	142	160	178	196
16	119	139	159	178	198	218
17	139	162	185	208	232	255
18	160	187	213	240	267	293
19	180	210	240	270	300	330
20	210	245	280	315	350	385
21	228	266	304	342	380	418
22	251	292	334	376	418	460
23	283	330	377	424	470	518
24	303	353	404	454	505	555
25	344	401	459	516	573	631
26	375	439	500	562	625	688
27	411	479	548	616	684	753
28	436	509	582	654	728	800
29	457	539	609	685	761	838
30	493	575	657	739	821	904
31	532	622	710	799	888	976
32	552	644	736	828	920	1012
33	588	686	784	882	980	1078
34	600	700	800	900	1000	1100
35	657	766	876	985	1095	1204
36	692	807	923	1038	1152	1268
37	772	901	1029	1158	1287	
38	801	934	1068	1201	1335	
39	840	980	1120	1260	1400	
40	903	1053	1204	1354	1505	
41	954	1113	1272	1431	1590	
42	1007	1175	1343	1511	1679	
43	1046	1222	1396	1571	1745	
44	1110	1295	1480	1665	1850	
45	1139	1329	1518	1707	1898	
46	1190	1388	1587	1785	1983	
47	1242	1449	1656	1862	2070	
48	1296	1512	1728	1944	2160	

* Courtesy of Office of Minnesota Department of Conservation.

TABLE XXVIII

The International Log Rule
(For Saws Cutting ¼ inch Kerf)

Top diameter — Inches	Contents in Board Feet						
	Length of log in feet						
	8	10	12	14	16	18	20
6	10	10	15	15	20	25	25
7	10	15	20	25	30	35	40
8	15	20	25	35	40	45	50
9	20	30	35	45	50	60	70
10	30	35	45	55	65	75	85
11	35	45	55	70	80	95	105
12	45	55	70	85	95	110	125
13	55	70	85	100	115	135	150
14	65	80	100	115	135	155	170
15	75	95	115	135	160	180	205
16	85	110	130	155	180	205	235
17	95	125	150	180	205	235	265
18	110	140	170	200	230	265	300
19	125	155	190	225	260	300	335
20	135	175	210	250	290	330	370
21	155	195	235	280	320	365	410
22	170	215	260	305	355	405	455
23	185	235	285	335	390	445	495
24	205	255	310	370	425	485	545
25	220	280	340	400	460	525	590
26	240	305	370	435	500	570	640
27	260	330	400	470	540	615	690
28	280	355	430	510	585	665	745
29	305	385	465	545	630	715	800
30	325	410	495	585	675	765	860
31	350	440	530	625	720	820	915
32	375	470	570	670	770	875	980
33	400	500	605	715	820	930	1045
34	425	535	645	760	875	990	1110
35	450	565	685	805	925	1050	1175
36	475	600	725	855	980	1115	1245

TABLE XXIX

SCRIBNER DECIMAL C LOG RULE

Top diameter — Inches	Contents in Board Feet						
	Length of log in feet						
	6	8	10	12	14	16	18
6	5	5	10	10	10	20	20
7	5	10	10	20	20	30	30
8	10	10	20	20	20	30	30
9	10	20	30	30	30	40	40
10	20	30	30	30	40	60	60
11	20	30	40	40	50	70	80
12	30	40	50	60	70	80	90
13	40	50	60	70	80	100	110
14	40	60	70	90	100	110	130
15	50	70	90	110	120	140	160
16	60	80	100	120	140	160	180
17	70	90	120	140	160	180	210
18	80	110	130	160	190	210	240
19	90	120	150	180	210	240	270
20	110	140	170	210	240	280	310
21	120	150	190	230	270	300	340
22	130	170	210	250	290	330	380
23	140	190	230	280	330	380	420
24	150	210	250	300	350	400	450
25	170	230	290	340	400	460	520
26	190	250	310	370	440	500	560
27	210	270	340	400	480	550	620
28	220	290	360	440	510	580	650
29	230	310	380	460	530	610	680
30	250	330	410	490	570	660	740
31	270	360	440	530	620	710	800
32	280	370	460	550	640	740	830
33	290	390	490	590	690	780	880
34	300	400	500	600	700	800	900
35	330	440	550	660	770	880	980
36	350	460	580	690	810	920	1040
37	390	510	640	770	900	1030	1160
38	400	540	670	800	930	1070	1200
39	420	560	700	840	980	1120	1260
40	450	600	750	900	1050	1200	1350

TABLE XXX

Scaled Values in Inches for Graduating the Biltmore Stick

$$*S = \frac{d(a - t)}{\sqrt{a(a + d)}}$$

	Values for S when $t = \frac{1}{4}$ inch				
	$a = 23''$	$a = 24''$	$a = 25''$	$a = 26''$	$a = 27''$
6	5.29	5.31	5.34	5.36	5.38
7	6.06	6.09	6.13	6.15	6.17
8	6.82	6.85	6.90	6.93	6.96
9	7.54	7.59	7.64	7.71	7.78
10	8.26	8.31	8.36	8.41	8.46
11	8.94	9.01	9.07	9.12	9.17
12	9.62	9.69	9.76	9.83	9.89
13	10.28	10.35	10.42	10.51	10.59
14	10.92	11.01	11.09	11.17	11.25
15	11.54	11.63	11.72	11.82	11.91
16	12.15	12.26	12.36	12.46	12.56
17	12.75	12.86	12.98	13.08	13.19
18	13.34	13.47	13.59	13.70	13.81
19	13.90	14.04	14.18	14.31	14.44
20	14.46	14.61	14.75	14.89	15.02
22	15.55	15.72	15.89	16.05	16.19
24	16.60	16.79	16.95	17.11	17.30
26	17.62	17.82	17.99	18.20	18.38
28	18.59	18.82	19.04	19.43	19.44
30	19.55	19.79	20.02	20.24	20.46
32	20.47	20.72	20.97	21.21	21.45
34	21.36	21.64	21.91	22.16	22.42
36	22.23	22.52	22.81	23.08	23.35
38	23.07	23.38	23.69	23.99	24.27
40	23.91	24.24	24.56	24.86	25.16
42	24.71	25.05	25.38	25.71	26.03
44	25.50	25.87	26.23	26.55	26.89
46	26.26	26.65	27.01	27.36	27.72
48	27.01	27.41	27.80	28.17	28.54
50	27.76	28.18	28.57	28.96	29.34
52	28.48	28.91	29.32	29.72	30.11
54	29.18	29.63	30.06	30.48	30.89
56	29.88	30.35	30.79	31.22	31.64
58	30.56	31.04	31.49	31.94	32.38
60	31.23	31.73	32.20	32.67	33.12

* S = Scale value of graduations on stick in inches. d = diameter dimension of the tree.
t = thickness of stick in inches. a = arm's length, or reach, measured in inches.

TABLE XXXI

Scale Value for Chrysten Hypsometer

(Based on use of a 10′ Pole)

Height of the tree in feet	Distance in inches for corresponding graduations on the face of the hypsometer as scaled from the edge of the lower lip.	
	When the distance between the two protruding lips of the hypsometer is 15 inches	When the distance between the two protruding lips of the hypsometer is 30 inches
10	15.00	30.00
12	12.50	25.00
14	10.71	21.40
16	9.38	18.76
18	8.33	16.66
20	7.50	15.00
25	6.00	12.00
30	5.00	10.00
40	3.75	7.50
50	3.00	6.00
60	2.50	5.00
70	2.15	4.29
80	1.88	3.75
100	1.50	3.00
110	1.36	2.73
120	1.25	2.50
150	1.00	2.00

INDEX

C

D

E

M

T